Complexity-Adaptive Universal Signal Estimation for Compressed Sensing

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Motivation

Noisy compressed sensing (CS) [1]

- Universal CS algorithms
- Conventional CS algorithms
- Noisy compressed sensing (CS)

Universal MAP estimation

- Goal: reconstruct x given y and Φ

Conventional CS algorithms

- Assume sparsity or compressibility
- Need prior knowledge about signal structure

What if prior knowledge not available?

Universal CS algorithms

- Focus on recovery of stationary ergodic non-i.i.d. signals with unknown statistics

Background

Universal MAP estimation [2]

- \[ x_{\text{MAP}} = \arg \max_v f_x(v)f_y|_x(y|v) = \arg \min_v \Psi^X(v) \]
- \[ \Psi^X(v) = -\ln(f_x(v)) + \frac{|y-\Phi v|^2}{2\sigma^2} \text{; optimal risk: } \Psi^X(x_{\text{MAP}}) \]
- Work on discretized space to reduce complexity
  1. Map indices \( j \in \{1, \ldots, Z\} \) to \( \mathbb{R} \) via discretizer \( Q(j) \)
  2. Estimate \( v = Q(w) \), \( w = [w_1, \ldots, w_N] \in \{1, \ldots, Z\}^N \)
- Universal prior [3] \( p_0(w) = 2^{-H_q(w)} \)
- \( q \)-depth conditional empirical entropy \( H_q(w) [4] \)

MCMC and enhancements

Markov chain Monte Carlo (MCMC) [2,4]

- Boltzmann PMF \( p_0(w) = \frac{1}{Z} \exp(-s\Psi^H_q(w)) \)
- Generate random samples with Gibbs sampler
- Iteration: Process one entry of \( w \) with Gibbs sampler
- Super-iteration: Process all entries of \( w \)

Does it reconstruct signals well?

- No, fixed quantizer slows down convergence

Level-adaptive (LA-MCMC) [2]

- \( Q_{\text{opt}} = \arg \min_q |y - \Phi Q(w)|_2^2 \)

How to minimize \( l_2 \) error with fixed number of levels?

How many levels to use?

What if prior knowledge not available?

Experimental settings:

- Compare SLA-MCMC with LA-MCMC, CoSaMP, and turboGAMP
- Signal length \( N=10000, M=2000-7000 \), AWGN
- SNR=5 and 10 dB
- Error metric: Mean signal-to-distortion ratio

\[ \text{MSDR} = 10 \log_{10} \frac{\text{Var}(x)}{\text{MSE}} \]

SLA-MCMC estimation results for a four-state Markov switching source (generates pattern \( +1,+1,-1,+1,-1, \) with 3% glitches)

Numerical results

SLA-MCMC, LA-MCMC, turboGAMP, and CoSaMP estimation results for a two-state Markov source with non-zero entries drawn from a uniform distribution \( U[0,1] \)

References and acknowledgements

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