1. Consider a discrete-time system whose input $x[n]$ and output $y[n]$ are related by

$$y[n] = \sum_{k=-\infty}^{n} 2^{k-n} x[k + 1].$$

Is the system: (a) linear? (b) time invariant? (c) causal? (d) stable? (25 points for each)

(a) Yes - it's a sum over n.

(b) $y[n] = \sum_{k=n}^{\infty} 2^{k-n} x[k+1]$. 

(c) Impulse response: $h[n] = \sum_{k=-\infty}^{n} 2^{k-n} \delta[k+1] = \sum_{k=-\infty}^{n} 2^{k-n-n-1} = \sum_{k=-\infty}^{n} 2^{k-n-1}$. Not causal since $h[-1] = 2^{-1} u[-1] = \frac{1}{2} \neq 0$.

(d) Stable? $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} 2^{n-n} u[n+1] = \sum_{n=-\infty}^{\infty} 2^{n} = \sum_{n=0}^{\infty} 2^{n} = 2 \sum_{n=0}^{\infty} 2^{n} = \frac{2}{1-2} = \infty$.

Not stable.

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ECE 313 Discussion #2 Quiz #6 - 11/02/2012

1. Consider a discrete-time system whose input $x[n]$ and output $y[n]$ are related by

$$y[n] = \sum_{k=-\infty}^{n} 2^{n-k} x[k + 1].$$

Is the system: (a) linear? (b) time invariant? (c) causal? (d) stable? (25 points for each)

(a) Yes - see above.

(b) Yes - see above.

(c) Impulse response: $h[n] = \sum_{k=-\infty}^{n} 2^{n-k} \delta[k+1] = \sum_{k=-\infty}^{n} 2^{n-k-k} \delta[k+1] = 2^{n} u[n+1]$. Not causal since $h[-1] = 2^{-1} u[-1] = \frac{1}{2} \neq 0$.

(d) Stable? $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} 2^{n-n} u[n+1] = \sum_{n=-\infty}^{\infty} 2^{n} = \sum_{n=0}^{\infty} 2^{n} = 2 \sum_{n=0}^{\infty} 2^{n} = \frac{2}{1-2} = \infty$.

Not stable.
1. Consider a discrete-time system whose input $x[n]$ and output $y[n]$ are related by

$$y[n] = \sum_{k=-\infty}^{n} 1^{k-n} x[k+1].$$

Is the system: (a) linear? (b) time invariant? (c) causal? (d) stable? (25 points for each)

(a) Linear (see above)

(b) Time Invariant (see above)

(c) Causal? No since $h[-1] = u(-1) = 1 \neq 0.$

(d) Stable? $\sum_{n=0}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 1^{n+1} = \sum_{n=1}^{\infty} 1 = \infty \text{ so not stable.}$