Problem 1: Calculate the axial strain energy in the column of the structure shown below. Use the equivalence of work and energy to calculate the amount that the column shortens. Hint: There is no need to calculate the reactions.
Problem 2: For each of the five beams/frames shown below, decide whether the bending moment diagram shown is correct in shape. If not, state briefly why not. Write your answers next to each moment diagram.
Problem 3: Calculate the bending strain energy in segment 2 of the beam shown below.

\[ M(x_2) = \omega_0 \frac{x_2^4}{4} \]
Problem 4: For the beam shown below:
(a) Express the distributed load, which has a maximum value of \( w_0 \), as a function of \( x_1 \).
(b) Set up the integral you would use in calculating the work done by the distributed load in moving through the given displacement. Do not evaluate the integral.

\[
S(x) = \frac{w_0}{120EI} \left( x_1^5 - 5x_1^4 + 4I_0^5 \right)
\]
Problem 5: Use Castigliano’s theorem to calculate the rotation at point A.
Problem 6: For the indeterminate structure shown below, use the vertical reaction at point C as your redundant force and do the following:
(a) Sketch the primary structure labeling relevant displacements and showing the deflected shape.
(b) Sketch the redundant structure labeling relevant displacements and showing the deflected shape.
(c) Write the compatibility equation resulting from these structures.

Do not calculate the reaction $C_y$ and do not calculate the values of any of the relevant displacements.
Problem 7: Calculate $B_y$ by considering it a redundant force in this problem. Do all calculations in kips and feet.

Problem Structure

Useful Solutions

\[
\delta(x) = -\frac{\omega_0}{24EI} \left( x^4 - 4x^3 + 3x^2 \right)
\]

\[
\delta(x) = \frac{Pb}{6EI} \left( 3x^3 - 3x^2 - b \right) \quad x \leq a
\]

\[
l = a + b
\]
Workspace
Problem 8: Shown below is a beam for which the goal is to calculate the rotation at point A using Castigliano’s theorem. The solution given is incorrect. Find and circle the error and describe concisely what mistake has been made. You do not need to present the correct solution.

1. Reactions

\[ \begin{align*}
\sum Y &= 0 \quad A_y = \frac{w_0 l}{2} \\
\sum M &= 0 \quad B_y = \frac{w_0 l^2}{2}
\end{align*} \]

2. Moment functions

\[ M'(x) = M(x) \]

\[ \text{fictitious moment} \]

\[ \begin{align*}
\sum M &= 0 \\
M(x) + w_0 x_1^2 + M' - \frac{w_0 x_1^3}{2} &= 0
\end{align*} \]

\[ M(x_1) = \frac{w_0 l x_1}{2} - M' - \frac{w_0 x_1^3}{2} \]

\[ \text{take } \frac{\partial M}{\partial M'} = -1 \]

\[ \text{set } M' = 0 \]

\[ M(x) = \frac{w_0 l x_1}{2} - \frac{w_0 x_1^3}{2} \]

3. Apply Castigliano

\[ \Theta_A = \int_0^l M(x) \frac{\partial M}{\partial M'} \, dx_1 \]

\[ = \int_0^l \left( \frac{w_0 l x_1}{2} - \frac{w_0 x_1^3}{2} \right)(-1) \, dx_1 \]

\[ = -\frac{w_0 l^3}{12 EI} \]
Reference equations

Equilibrium equations

\[ \sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0 \]

Strain Energy

\[ U_{i,\text{bending}} = \int \frac{M(x)^2}{2EI} \, dx \]

\[ U_{i,\text{axial}} = \int \frac{N(x)^2}{2EA} \, dx = \frac{N^2 L}{2EA} \quad \text{if } N(x) \text{ is constant} \]

Work done by loading

\[ U_c = \frac{1}{2} P \Delta \text{ for a point load} \]

\[ U_c = \int \frac{1}{2} w(x) \delta(x) \, dx \text{ for a distributed load} \]

Castiglano’s theorem

\[ \Delta_i = \frac{\partial U_i}{\partial P_i} \]

\[ \Theta_i = \frac{\partial U_i}{\partial M'_i} \]

Useful extra formulas for using Castiglano

\[ \frac{\partial}{\partial P_i} \int \frac{M(x)^2}{2EI} \, dx = \int \frac{M(x) \partial M(x)}{EI} \, dx \]

\[ \frac{\partial}{\partial P_i} \int \frac{N(x)^2}{2EA} \, dx = \int \frac{N(x) \partial N(x)}{EA} \, dx \]

\[ \frac{\partial}{\partial M'_i} \int \frac{M(x)^2}{2EI} \, dx = \int \frac{M(x) \partial M(x)}{EI} \, dx \]

\[ \frac{\partial}{\partial M'_i} \int \frac{N(x)^2}{2EA} \, dx = \int \frac{N(x) \partial N(x)}{EA} \, dx \]