Conduction Convection Radiation processes of a solar collector using FEA

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ABSTRACT
Conduction, Convection and Radiation processes are considered for a flat plate solar collector using finite element analysis. Radiation dominates the other two processes. It being non-linear phenomena requires an iterative procedure to solve problems analytically, which is quite difficult. So, I tried to find the temperature distribution of the solar collector using FEA. Radiation in ANSYS requires the use of superelements.

INTRODUCTION
A typical flat plate solar collector is a metal box with a glass or plastic cover on top and a dark colored absorber plate on the bottom. The sides and the bottom of the collector are usually insulated to minimize heat loss. Sunlight passes through the glass and strikes the absorber plate, which heats up and thereby changing solar energy into heat energy (though some part of the energy is lost by radiation and convection). The heat is transferred to liquid passing through pipes attached to the absorber plate.

In locations with average available solar energy, flat plate collectors are sized approximately one-half to one-square foot per gallon of one days hot water use.

APPLICATIONS:
The main use of this technology is in residential buildings where the demand for hot water has a large impact on energy bills. Commercial applications include Laundromats, car washes, military laundry facilities and eating establishments. The technology can also be used for space heating if the building is located off-grid or if utility power is subject to frequent outages. Solar collectors are cost-effective in sunny temperate areas.

NOMENCLATURE
1-Absorber, Aluminum surface
2-Glass
3-Ambient Air
J-Radiosity
\( E_{b_2} \)-energy radiated per unit time per unit area by the radiating surface 2
q-heat flux
k-thermal conductivity
T-Temperature
A-cross sectional area
F-viewfactor
h-convective heat transfer coefficient
\( \varepsilon \)-emissivity
\( \sigma \)-stefen Boltzmann constant

Figure 1: simplified 2 dimensional model of a flat plate solar collector

The figure 1 shows the cross-section of a flat plate solar collector. The air space in between the glass surface and the absorber (Aluminum in this case) is assumed not to flow for the model but instead is treated with an effective thermal conductivity, ke. I considered two analyses.

a) All the outside surfaces of the insulation are adiabatic
b) This is the more realistic case. All the outside surfaces of insulation may convect and radiate to the surroundings.

Objective:
Such analysis is performed to determine material temperatures. The temperature distribution may also be needed in order to perform an analysis for thermally induced stresses.

ANALYSIS:
Heat loss due to Conduction:
Conduction is the transfer of heat through a material. For a flat plate collector, conduction transfer is found at a) the absorber plate b) through the air gap c) heat losses through the back of the collector.

Heat loss by Convection:
Heat transfer by convection is due to the free convection to the ambient air.
\[ Q = hA\Delta T \] (1)

Heat loss by Radiation:
Radiation is the energy transferred by electromagnetic waves that originate from a system because of the temperature
of the system. The total radiation from a perfect black body is proportional to the fourth power of the absolute temperature of the body.

\[ Q_{12} = A F_{12} \sigma T^4 \]  

(2)

Rate of useful energy delivered can be estimated by knowing the solar radiation input to absorber and the various losses that occur.

<table>
<thead>
<tr>
<th>Physical Properties of materials:</th>
<th>Thermal conductivity (W/m/C)</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>204</td>
<td>1.0</td>
</tr>
<tr>
<td>Glass</td>
<td>0.78</td>
<td>0.9</td>
</tr>
<tr>
<td>Insulation</td>
<td>0.048</td>
<td>0.95</td>
</tr>
<tr>
<td>Air</td>
<td>0.042</td>
<td></td>
</tr>
</tbody>
</table>

Elements chosen for the problem:

a) Link32:
   It is a uniaxial element with the ability to conduct heat between its nodes. The element has a single degree of freedom, temperature at each node point. The conducting bar is applicable to a two-dimensional steady state or transient thermal analysis.

b) Plane55:
   This element can be used as a plane element or as an axisymmetric ring element with a two-dimensional thermal conduction capability. The element has four nodes with a single degree of freedom, temperature, at each node. The element is applicable to a two-dimensional, steady state or transient thermal analysis.

c) SURF151:
   This element may be used for various load and surface effect applications. It may be overlaid onto a face of any 2-D thermal solid element. The element is applicable to two-dimensional thermal analyses.

d) Matrix 50:
   This superelement is a group of previously assembled elements that is treated as a single element. The superelement, once generated, may be included in any ANSYS model and used in any analysis type for which it is applicable. The superelement can greatly decrease the cost of many analyses. Once the superelement matrices have been formed, they are stored in a file and can be used in other analyses the same way any other ANSYS elements are used.

By looking at the geometry, we might think of considering an axisymmetric model. But Radiation Matrix Utility Method (/AUX12) doesn’t support axisymmetric models.

Radiation is a non-linear phenomena. It involves the variation of fourth power of absolute temperature. When modeling a flat plate solar collector, the first thought of including Radiation would be applying a boundary condition, like what we do for convection. But this is not the case. Radiation in ANSYS can be done by the two methods.

a) Radiation Matrix Utility Method.

b) Radiosity Solver Method.

For my case, I chose the Radiation Matrix utility Method. This method (/AUX12) consists of three steps.

1) Define the radiating surfaces

2) Generate the radiation matrix.

   This generates the viewfactors of all the radiating surfaces. Radiation view factors are defined as \( F_{12} \) is the fraction of energy leaving the surface 1 which reaches surface 2. So, the radiant energy leaving a surface 1 and arriving at surface 2 is \( E_{21} A_1 F_{12} \) and the energy leaving surface 2 and arriving at surface 1 is \( E_{12} A_2 F_{21} \).

   We have to decide if we need a SPACE NODE. In my case, for the radiation on the inner surfaces of the model I don’t require a space node. I chose a non-hidden method of calculating viewfactors as all the surfaces can see each other.

3) Use the radiation matrix in the thermal analysis.

   Care must be taken that the skin elements be removed and the key options of the superelement be changed to radiation options.

   For a radiation dominant problem between surfaces, the radiation superelement (MATRIX50) produced by AUX12 is still the most direct and accurate approach. The superelement represents the heat transfer between each element face and all the other element faces included in the radiation superelement. The orientation of the faces, their area, and their emissivity are all taken into account. This method can also account for whether surfaces are hidden from each other. Other features of the AUX12 method are an exact calculation of form factors for non-hidden applications. The main drawback of the AUX12 method is that it is computationally intensive and may take several hours and a large amount of memory for large models.

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Figure2: The meshed model

LOADS:

1) I applied a constant heat flux of 750W/sq meters on the absorber. This physically represents the amount incident sun’s rays onto the absorber.

2) A convective boundary condition on the glass plate with a heat transfer coefficient of 20W/sq m°C and an ambient temperature of 303K.
3) Radiation from the outside surface of glass to the space node (the temperature of the space node being 303K)

4) Adiabatic wall condition on the insulation.

**Analytical Solution:**

For the absorber (Aluminum):

\[ J_1 = E_{b1} \]  

(3)

The convection and solar energy delivered to the absorber (surface 1):

\[ \frac{q}{A_1} = \frac{k_c}{\Delta x} (T_1 - T_2) + \frac{q}{A_s} \]  

(4)

So, we can write,

\[ E_{b1} - F_{12} J_{2i} = \frac{k_c}{\Delta x} (T_1 - T_2) + \frac{q}{A_s} \]  

(5)

The overall energy balance for surface 2 is:

\[ \frac{\varepsilon_2}{1 - \varepsilon_2} (2E_{b2} - J_{2i} - J_{2o}) = \frac{k_c}{\Delta x} (T_1 - T_2) + h(T_3 - T_2) \]  

(6)

For the overall system, the solar energy absorbed must eventually be lost by convection and radiation from the outside surface of the glass. Thus,

\[ \frac{q}{A_s} = h(T_2 - T_3) + \varepsilon_2 (E_{b2} - E_{b3}) \]  

(7)

Finally, the radiation lost from the outside of the glass can be written as:

\[ \frac{q}{A_{rad}} = \frac{\varepsilon_2}{1 - \varepsilon_2} (E_{b2} - E_{b3}) = (E_{b2} - J_{2o}) \frac{\varepsilon_2}{1 - \varepsilon_2} \]  

(8)

We solve equation (5) for \( E_{b2} \) and \( T_2 \) by iteration.

Then, we solve equation (8) for \( J_{2o} \)

Solving equation (5) for \( J_{2i} \) and substituting in equation (6) gives:

\[ \frac{\varepsilon_2}{1 - \varepsilon_1} \left[ 2E_{b2} - J_{2i} - \frac{k_c}{\Delta x} (T_1 - T_2) + \frac{q}{A_s} - J_{2o} \right] = \frac{k_c}{\Delta x} (T_1 - T_2) + h(T_3 - T_2) \]  

(9)

Then we can solve equation (9) iteratively, to get the temperature of the absorber. Solving this problem analytically is difficult. Hence the need for a finite element model.

**RESULTS AND DISCUSSIONS:**

The maximum temperature of the model is about 612K from the finite element model. The absorber plate has the maximum temperature. This is obvious from the fact that the thermal conductivity of the absorber (aluminum surface) plate is very large compared to the other materials in the model. One might doubt that the temperature gradients seen in the figure (4) might be due to conduction rather than that of radiation. So, I tried checking with the thermal conductivity of the air (which in the physical sense, implies replacing air with some other material that has a very low thermal conductivity) to be 1e-20. Even then, there is a temperature gradient. This proves that in a solar collector, radiation process dominates conduction. It can be seen that the the maximum temperature is more when the thermal conductivity of air is reduced (figure(4)). This might due to the fact that heat is not lost by conduction through the air gap. So, the absorber plate has a higher temperature (in the case where thermal conductivity of air is reduced).

I also tried reducing the emissivities of the radiating surfaces but then I ended up getting error messages (I mean the
singularity case). If the emissivities of the radiating surfaces are too low then we can neglect radiation, which means we just have conduction and convection. So, in that case the problem won’t be this complicated, as there is no need of using a super element. We can incorporate convection by applying as a load, which I guess is too simple.

While doing Radiation problems, we have to keep in mind that axisymmetric models don’t work with the Radiation Matrix Utility (/AUX12) method. This /AUX 12 has two ways of calculating the view factors. a) Hidden type b) Non hidden type. The non-hidden method calculates the form factors from every element to every other element regardless of any blocking elements. The hidden method (default) first uses a hidden line algorithm to determine which elements are “visible” to every other element.

**ACKNOWLEDGMENTS**

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