A method to determine the $R$-curve of flexible materials using a monotonic loading of a single trouser tear specimen

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Abstract. It is shown that the $R$-curve of flexible materials can be determined using a monotonic loading of a single trouser tear specimen. The load-displacement records of trouser tear specimens are analyzed based on the locus method which determines the critical $J$-integral value ($J_c$) using the locus line of crack initiation points on load-displacement curves of specimens which differ only in initial crack length. Based on this analysis, it is shown that the crack resistance ($R$) during crack growth including crack initiation and steady state crack propagation can be expressed in terms of quantities that are directly related to the load-displacement plane, and that $R$ can be simplified to a linear function of load. It is also shown that the crack growth ($C$) can be expressed as a linear function of load-point displacement. The load-displacement record of trouser tearing is then converted to an $R$-curve simply by changing the scale on each axis. The effectiveness of this method is illustrated experimentally by constructing the $R$-curve directly from the load-displacement record of a trouser tear specimen made from a thermoplastic rubber sheet.

1. Introduction

The $R$-curve [1–3] which describes crack resistance as a function of crack growth has been used to explain fracture instability and to characterize crack growth behavior beyond crack initiation. The experimental determination of the $R$-curve in terms of energetic parameters such as the $J$-integral [4] or the energy release rate has been discussed by several investigators [5–8]. The use of the $J$-integral $R$-curve for ductile materials has become common in spite of the limited applicability [6] of the $J$-integral for extensive crack growth. This common usage may be attributed to the relatively simple experimental procedure of determining the $J$-integral value from a single specimen which has a predictable ratio $\Phi$ [9–12] of the $J$-integral to the total work done per unit uncracked area. Recently, some investigators [7, 8] have tried to determine the $R$-curve in terms of the energy release rate using repeated loading and unloading of a single specimen.

In this study, it will be shown that in the case of flexible materials the $R$-curve can be determined directly from a monotonic loading of a single trouser tear specimen [13, 14]. The load-displacement records of trouser tear specimens which vary only in initial crack length will be analyzed utilizing the locus method [15–17] which partitions the fracture energy along the locus line of crack initiation points. Based on this analysis, it will be shown that the load-displacement record can be directly converted to the $R$-curve simply by changing the scale on the load and displacement axes.
2. Analysis of the load-displacement record of trouser tearing based on the locus method

Shown in Fig. 1 is a trouser tear specimen where the hatched portion is for gripping and the initial crack length \( a \) is defined as the length between the hatched portion and the crack tip. The length of each leg of this specimen is the initial crack length. The fracture mode in this trouser tear specimen is a combination of mode I and III [18], and the major governing mode of the two is dependent upon the material. In flexible materials, the fracture mode would be predominantly in mode I as can be seen from the experimental results of rubber tearing shown by Rivlin and Thomas [13]. Even if the fracture mode is not clearly distinguishable, the crack resistance determined from the trouser tearing bears notice since similar modes of tearing frequently occur in practical application of flexible materials.

The energy required to propagate a crack in a trouser tear specimen may include the energy for the localized bending in the legs along the folding line which occurs across the crack tip perpendicular to the crack propagation direction during tearing. Hence, the resistance value determined using the trouser tear loading configuration may be dependent on the width of the legs of the specimen if the localized bending in the leg occurs in a plastic fashion. However, in this paper we confine our discussion to materials in which bending of the legs can be considered as an elastic deformation. In the case of flexible materials where the plastic energy consumption due to the bending of the legs is negligible, our discussion is applicable within this confinement.

Previously we have reported [15-17] that the critical \( J \)-integral value \( (J_c) \) at crack initiation can be determined from

\[
J_c = -\frac{1}{B} \frac{\Delta U_c}{\Delta a},
\]

where \( B \) is the thickness of the specimen, \( U_c \) is the area enclosed by a loading curve, the displacement axis, and the locus line of the crack initiation points on the load-displacement records of identical specimens with different initial crack lengths as shown in Fig. 2

![Fig. 1. A trouser tear specimen in undeformed and deformed state. The hatched portion is for gripping.](image-url)
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Fig. 2. A schematic load-displacement record showing $U_i$ as a function of initial crack length $a$.

Fig. 3. Schematic load-displacement curves of trouser tear specimens which differ only in initial crack length. The solid circle on each loading curve denotes the crack initiation point.

schematically, and $a$ is the initial crack length. This locus method is applicable to virtually any loading configurations which may yield different types of crack initiation locus lines.

Schematic load-displacement curves of trouser tear specimens which vary only in initial crack length are shown in Fig. 3. Since the variation in initial crack size causes a difference in leg length only, the crack initiation load and the steady state crack propagation load are not dependent on the initial crack length unless the initial crack length becomes extremely short. The solid circle on each loading curve denotes the crack initiation point, and the dotted horizontal line is the locus line of the crack initiation points. The hatched area ODEF is $-\Delta U_i$ for the two loading curves of initial cracks $a_1$ and $a_4$. Hence, (1) can be rewritten in this case as

$$J_C = \frac{1}{B} \frac{\text{AREA(ODEF)}}{(a_4 - a_1)}. \quad (2)$$
Based on (2), \( J_c \) values can be directly determined using two trouser tear specimens which differ only in initial crack length.

In Fig. 4, the loading line O-D-L is transferred horizontally to yield the broken line F-H-I. Since the base length O-F is twice the difference in initial crack length, the \( \text{AREA(ODHF)} \) can be written as \( \text{AREA(ODHF)} = 2P_c(a_4 - a_1) \) where \( P_c \) is the load at the crack initiation point. The difference in slope between the two loading lines O-D and F-E is due to the difference in the initial length of the legs. The specimen of longer initial crack length shows larger displacement at an equal load due to its longer legs. Since the difference in total leg length in this case is \( 2(a_4 - a_1) \), it can be shown that \( \text{AREA(FHE)} = 2(a_4 - a_1) \times U_0 \) where \( U_0 \) is the hatched area shown in Fig. 5. The line O-M in Fig. 5 is the loading line of a tensile specimen of unit length whose width is \( W/2 \). Since \( \text{AREA(ODEF)} \) in Fig. 4 is the sum of \( \text{AREA(ODHF)} \) and \( \text{AREA(FHE)} \), (2) can be rewritten as

\[
J_c = \frac{2P_c}{B} + \frac{2U_0}{B}.
\]
Based on (3), \( J_c \) can be determined using one trouser tear specimen and a regular tensile specimen without a precrack. An expression developed by the tearing energy approach [13, 14] for trouser tearing is

\[
T = \frac{2P_c \lambda_c}{B} - W_c W,
\]  

where \( T \) is the tearing energy, \( \lambda_c \) is the extension ratio of the leg at the critical load \( P_c \), and \( W_c \) is the strain energy density in the leg at the same load. Equation (3) which has been derived based on the locus method can be compared with (4) as follows. The area \( U_0 \) in Fig. 5 can be expressed as

\[
U_0 = P_c X_c - \int_0^{X_c} P(X) \, dX = P_c X_c - W_c W_B/2.
\]

Substitution of (5) to (3) yields \( J_c = 2P_c (1 + X_c)/B - W_c/W \). But it can be shown that \( 1 + X_c = (1 + X_c)/1 = \lambda_c \). Hence, (3) is equivalent to (4). The significance of (3), however, is that each term can be directly related to the load-displacement record. The first term of the right side of (3) came from the area ODHF shown in Fig. 4, and the second term came from the area FHE. When the area FHE is small compared to the area ODHF, which is the case for most materials, the second term of the right side of (3) can be neglected, and \( J_c \) can be determined using one trouser tear specimen.

3. Determination of the R-curve using a trouser tear test

Recently we have reported [19] that the resistance (\( R \)) to crack growth during extensive crack growth can be determined from

\[
R = \frac{\tilde{G}}{B} = -\frac{1}{B} \frac{dU_L}{da},
\]

where \( \tilde{G} \) is the total energy release rate [20], and \( U_L \) is the area enclosed by a loading curve and the locus line of a set of characteristic points on the load-displacement initial crack length. The characteristic points can be the maximum load points or the points at which the same amount of crack growth occurs, or any other set of points of interest on the loading curves. Since crack initiation points can also be considered as characteristic points, (6) can be used to determine the crack resistance at crack initiation points by taking \( U_L \) along the locus of crack initiation points. If it can be surmised that the crack resistance is constant along a locus line, then the derivative \( dU_L/da \) in (6) can be replaced by \( \Delta U_L/\Delta a \), which gives the same form as (1). When it is uncertain whether the resistance along a locus line is constant, (6) can be used to investigate the variance of the crack resistance by plotting \( U_L \) with respect to the initial crack length \( a \). This generalized locus method is also applicable to virtually any loading configurations.

The crack resistance \( R \) during crack growth may depend on the speed of the crack propagation. The crack propagation speed \( u \) is generally a function of \( C, a \) and \( \nu \), where \( C \)
is the amount of the crack propagated after crack initiation, \(a\) is the initial crack length, and \(v\) is the loading rate. The crack propagation speed in trouser tearing, however, can be regarded as being independent of the initial crack length. A difference in initial crack length causes only a difference in initial leg length and the difference in the speed of crack propagation due to different initial leg lengths is usually negligible. Hence, the crack resistance during crack growth in trouser tearing is not a function of the initial crack length even if the material's crack resistance is sensitive to the crack propagation speed.

Since the crack resistance during crack growth in trouser tearing is not a function of the initial crack length, it can be surmised that the load at which the same amount of crack growth occurs is the same even for specimens with different initial crack lengths. Hence, the locus line of the points at which the same amount of crack growth occurs in trouser tearing is a horizontal line, and the crack resistance along the locus line is a constant. Therefore, the resistance during crack growth can be determined based on (6) utilizing the horizontal locus line and the constant crack resistance along the locus line. Consequently, it can be shown that the resistance to crack propagation during crack growth in trouser tearing is

\[
R = \frac{2P}{B} + \frac{2U_p}{B},
\]

(7)

where \(P\) is the load, and \(U_p\) is the strain energy stored up to the load \(P\) in a specimen of unit length with the same thickness and width as the leg of the trouser tear specimen. When \(U_p\) is negligible, the crack resistance \(R\) during crack growth including crack initiation and steady state crack propagation can be determined from

\[
R = \frac{2P}{B}.
\]

(8)

Equation (8) describes the crack resistance \(R\) as a function of load \(P\). Since a load-displacement record is the plot of load \(P\) as a function of displacement \(X\), the crack resistance \(R\) in (8) can be described as a function of \(X\) utilizing load-displacement records. Shown in Fig. 6 is a schematic load-displacement record of a trouser tear specimen, where the point \(N\) denotes the crack initiation. The solid vertical line is the load axis, and the broken vertical line is the resistance axis. Since the crack resistance \(R\) in (8) is a linear function of \(P\), the resistance axis can be constructed directly on the load-displacement record by adjusting the scale linearly. Hence, it is seen that the load-displacement curve in trouser tearing describes the crack resistance \(R\) as a function of \(X\). If we can relate the displacement \(X\) to the crack growth \(C\), the load-displacement curve can be converted into an \(R\)-curve which describes \(R\) as a function of \(C\).

The load-point displacement in trouser tearing includes the displacements due to the longitudinal extension of the legs, the compliance of the bent legs, the stretching of the material in the vicinity of the crack tip, and the displacement due to crack growth. From geometric considerations, it can be shown that the displacement due to the crack growth is twice the amount of the crack propagation. After reaching steady state crack propagation, the displacement due to factors other than crack growth can be considered as being constant.
during further crack growth when $U_p$ is negligible. Hence, the total increment in load-point displacement during steady state crack propagation is equal to twice the increment in crack growth. Based on this, the amount of crack growth up to the point where steady state crack growth begins can be determined experimentally. For instance, if the crack growth up to point $S$ in Fig. 6 is known as $C_s$, then the crack growth up to $Q$ where the steady state crack growth begins can be written as

$$C_Q = C_s - \frac{(X_S - X_Q)}{2}. \quad (9)$$

The total crack growth $C_S$ from crack initiation to a point $S$ can easily be determined experimentally by marking the point $S$ on the loading curve during the testing when the actual crack growth in the specimen reaches a certain value. Hence, the crack growth $C_Q$ up to the point where the steady state begins is determinable, and the crack growth in the range between $N$ and $Q$ may be linearly approximated as

$$C = \frac{C_Q}{X_Q} X', \quad (10)$$

where $X'$ is defined to be $X' = X - X_a$ for mathematical simplicity. The scale of the $C$ axis after point $Q$ in Fig. 6 is not significant because the crack resistance after point $Q$ is constant. Hence, (10) can be applied without limiting the range. Therefore, the crack growth axis can also be constructed directly on the load-displacement record by adjusting the scale linearly by a factor of $C_Q / X_Q'$ for the entire range. Consequently, the load-displacement record of a trouser tear specimen can be regarded as an $R$-curve which describes the crack resistance as a function of crack growth.
4. Experimental

The material chosen for the experimental study was Santoprene® 201-73, a thermoplastic rubber manufactured by the Monsanto Chemical Company. Trouser tear specimens (Fig. 1) were made from extruded sheets. The width \((W)\), length \((L)\), and thickness \((B)\) of the specimens were 40, 80, and 1.6 mm respectively. The initial cracks were created using a sharp razor blade, and the crack lengths \((a)\) were 20, 30, 40, and 50 mm. From a preliminary test, it was found that the extruded sheet has a tendency to tear along the extrusion direction. Hence, the specimens were made along the extruded direction so that the crack propagation would occur following the initial direction of the crack.

Tests were performed at a cross head speed of 50 mm min\(^{-1}\). The ambient temperature was 24°C, and the relative humidity was 63 percent during the test. Load vs load-point displacement graphs were recorded and the crack initiation points were marked on each load-displacement curve during the test. The areas under the loading curves were calculated numerically from the data points taken from the load displacement records.

5. Result and discussion

Shown in Fig. 7 are the load vs load-point displacement records of trouser tear specimens which differ only in initial crack length, and a tensile specimen of 60\((L)\) \(\times\) 20\((W)\) \(\times\) 1.6\((B)\) mm without crack. The loading line of the tensile specimen shows small displacement compared to other loading lines of trouser tear specimens. Although the cracks in trouser tear specimens were intended to grow straightforward following the initial direction, the actual crack growth deviated from the intended straight line. Due to this deviation, the load during extensive crack growth fluctuated. Solid circles on the loading lines of the trouser tear specimens denote the observed crack initiation points.

![Graph showing load vs load-point displacement](image)

*Fig. 7.* Load vs load-point displacement records of trouser tear specimens, and a tensile specimen of 60\((L)\) \(\times\) 20\((W)\) \(\times\) 1.6\((B)\) mm without crack. (Santoprene® 201-73, a thermoplastic rubber).
Based on the load-displacement records shown in Fig. 7, $U_c$ which is the area enclosed by a loading curve, the displacement axis, and the locus line of the crack initiation points was calculated for the trouser tear specimens. For the locus line after the crack initiation point of the largest crack size, a vertical line was used for convenience since the absolute value of $U_c$ does not affect the slope of $U_c$ with respect to $a$ [16]. Shown in Fig. 8 is the plot of $U_c/B$ as a function of initial crack length $a$. The upper and lower bars denote maximum and minimum values for each initial crack length, and the empty circle denotes the mean value. The slope of the least squares linear fitted line was taken as $J_c$ based on (1), and the resulting value was 4.0 kJ m$^{-2}$.

It is seen in Fig. 7 that the amount of load-point displacement which occurred in the tensile specimen is not significant although the length of the tensile specimen is 60 times unit length. Hence, $U_0$ in the second term of the right side of (3) is negligible in this material. Therefore, the crack initiation resistance $J_c$ was calculated based on (3) ignoring the second term, and the plot of $J_c$ vs $a$ is shown in Fig. 9. The average $J_c$ value is 4.4 kJ m$^{-2}$ which is slightly higher, although one positive term in (3) has been neglected, than the $J_c$ value from the locus method. From this, it is seen that the effect of ignoring the second term is not significant compared to possible experimental errors.

Shown in Fig. 10 is one of the load-displacement records of trouser tear specimens in Fig. 7. This load-displacement curve was chosen for constructing the $R$-curve because
Fig. 9. $J_t$ vs initial crack length $a$. The horizontal dotted line shows the average $J_t$ value of 4.4 kJ m$^{-2}$.

Fig. 10. Load vs load-point displacement record of a trouser tear specimen. (A thermoplastic rubber.)
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![R-curve graph](image)

Fig. 11. An R-curve constructed using a load-displacement curve of a trouser tear specimen.

straight crack growth had occurred along the initial crack direction as intended. The point $S$ on the loading curve was marked during the test when the actual crack growth in the specimen reached 10 mm. With $C_S = 10$ mm and $(X_S - X_Q) = 4.4$ mm, one can find based on (9) that $C_Q = 7.8$ mm. Therefore, with $X_Q' = X_Q - X_0 = 22.6$ mm, the crack growth $C$ can be written based on (10) as $C = 0.345 X'$. The R-curve constructed using this loading curve is shown in Fig. 11. The scale of the $R$ axis has been adjusted based on (8), and the scale of the $C$ axis has been adjusted based on (10). As can be seen in the figure, the crack resistance increases steadily after the crack initiation until it reaches the plateau resistance value $10.7 \text{ kJ m}^{-2}$.

6. Conclusion

It has been shown that the load-displacement record of a trouser tear specimen of a flexible material can be interpreted as an R-curve which describes the crack resistance $R$ as a function of crack growth $C$. The load axis can be converted to the resistance axis by adjusting the scale using $R = 2P/B$, and the displacement axis can be converted to the crack growth axis by adjusting the scale using $C = [C(Q)/X_Q']X'$. Although slight approximations are involved in deriving these linear conversions, this method of determining R-curves directly from load-displacement curves is a quick and easy way of investigating the fracture behavior of flexible materials as a function of crack growth. The effectiveness of this method has been illustrated through an experimental study using a thermoplastic rubber.

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References

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Résumé. On montre que l’on peut déterminer la courbe $R$ relative à des matériaux flexibles en recourant à un essai charges-déplacements obtenus sur des éprouvettes d’arrachement en se basant sur la méthode des lieux, qui permet de déterminer la valeur critique $J_c$ de l’intégrale $J$ en utilisant la ligne des lieux des points d’amorçage relevés sur des courbes charges-déplacement relatives à des éprouvettes qui ne diffèrent que par la longueur initiale de la fissure.

En se basant sur cette analyse, on montre que la résistance à la fissuration au cours de la phase d’amorçage et de croissance stable de la fissure, peut être exprimée en fonction de quantités directement liées à la relation charges-déplacements, et que $R$ peut être ramené de manière simplifiée à une fonction linéaire de la charge. On montre également que la croissance $C$ de la fissure peut être exprimée comme une fonction linéaire du déplacement du point de mise en charge.

L’enregistrement charges-déplacement d’un essai d’arrachement se trouve être convertible en une courbe $R$ par un simple changement d’échelle des axes.

On illustre l’utilité de cette méthode en construisant expérimentalement la courbe $R$ directement au départ d’un enregistrement charges-déplacements d’une éprouvette d’arrachement réalisée dans une feuille de caoutchouc thermoplastique.