Infinite Arrays - Some Preliminary Concepts

Why study infinite arrays?

1) Good approximation to central elements of a large array. Recall

\[ Y_{mn} = \frac{I_{mn}}{V_{mn}} = \sum \sum Y_{mn, pq} \frac{V_{pq}}{V_{mn}} \]

*Usually decreases with distance*

2) Floquet analysis \( \Rightarrow \) can limit analysis to ONE UNIT CELL.

\[ V_n = V_0 e^{-j2\pi nd \sin \theta} \]

\[ \int E \otimes \Omega^3 = \int E \otimes \Omega^3 x e^{-j2\pi nd \sin \theta} \]
The total field of an infinite, periodic, uniform array of identical elements is (by superposition)

\[ \bar{E}(\vec{r}) = K |q_{00}| \bar{F}_{00}(u, v) \]

\[ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j \frac{\lambda}{2} |\vec{r} - \vec{r}_{mn}|}}{1 - \bar{r}_{mn} |\vec{r} - \vec{r}_{mn}|} e^{-j \frac{\lambda}{2} (u_0 m \lambda_x + v_0 n \lambda_y)} \]

Note: exact form

steering phase from \( a_{mn} \) (rectangular grid)

\[ \bar{E}(\vec{r}) = K |q_{00}| \bar{F}_{00}(u, v) \sum_{m} \sum_{n} e^{-j \frac{\lambda}{2} (u_0 m \lambda_x + v_0 n \lambda_y)} \]

\[ \frac{e^{-j \frac{\lambda}{2} \sqrt{(x-x_0-m \lambda_x)^2 + (y-y_0-n \lambda_y)^2 + z^2}}}{\sqrt{(x-x_0-m \lambda_x)^2 + (y-y_0-n \lambda_y)^2 + z^2}} \]

By the Poisson Sum Formula, we get

\[ \bar{E}(\vec{r}) = K |q_{00}| \bar{F}_{00}(u, v) \frac{\sqrt{2 \pi}}{dx \, dy} \]

\[ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} e^{-j \frac{\lambda}{2} \left[ u_p (x-x_0) + v_q (y-y_0) \right]} e^{-j \frac{\lambda}{2} \frac{\gamma_{pq} z^2}{\bar{R}_{pq}}} \]

where

\[ u_p = u_0 + p \frac{\lambda}{dx} \]

\[ v_q = v_0 + q \frac{\lambda}{dy} \]

\[ \gamma_{pq} = \frac{\lambda}{2} \sqrt{1 - u_p^2 - v_q^2} \]
Notes:

1. Poisson Sum Formula - 1D

\[ \sum_{n=-\infty}^{\infty} f(n \Delta x) = \frac{1}{\Delta x} \sum_{m=-\infty}^{\infty} F \left( \frac{m \omega x}{\Delta x} \right) \]

where \( F(h) = \int_{-\infty}^{\infty} f(x) e^{-i \omega x} \, dx \)

The derivation of \( \Theta \) is a spherical wave to plane wave transformation.

2. Eqn (x) is often referred to as a grating lobe series. It expresses the field in terms of discrete plane waves associated with the points of the grating lobe diagram.

3. "Radiation Pattern" is not well defined for an infinite array. The far-field of an array is a single plane wave (if not radiating grating lobes). The "beamwidth" is zero and the pattern is a delta function.

\[ \bigcirc \rightarrow \ldots \wedge \wedge \wedge \wedge \wedge \wedge \wedge \ldots \]
Infinite Array of Parallel Plate Waveguides


Consider a 2-D slot array fed by parallel-plate waveguides

Infinite, periodic array. Slots of width $d$ are centered in WG.
Assumptions:

1. \( b < 0.5 \lambda \) \hspace{1cm} (Only TEM mode in \( W/6 \))

2. \( d/\lambda \ll 1 \) \hspace{1cm} (\( E \) constant in slot)

3. Matched generator excites \( W/6 \); we want to find \( M \) for \( z \) sufficiently less than zero so that only TEM mode reflection is observed.

4. PEC (lossless) structure

Solve using modal expansion and continuity of power at aperture.

In the waveguide, \( z < 0 \), TM\(_2n\) modes:

\[
A_m = \sum \psi_m \tag{1}
\]

\[
\psi_m = \begin{cases} 
\text{cos} \frac{m \pi x}{b} e^{\pm j \sqrt{2 \mu \epsilon} m z} , & m = 0 \\
\text{sin} \frac{m \pi x}{b} e^{\pm j \sqrt{2 \mu \epsilon} m z} , & m = 2, 4, 6, \ldots \tag{2}
\end{cases}
\]
\[ l_{f} = \omega \sqrt{\frac{\mu_{f}}{\epsilon_{f}}} \]

\[ l_{f m} = \sqrt{l_{f}^2 - \left(\frac{m \pi}{b}\right)^2}, \quad \text{Im} \, \sqrt{l_{f m}^2} \leq 0 \]

\[ E_{x m} = \begin{cases} 
\frac{l_{f}}{\omega \epsilon_{f}} e^{\mp j l_{f} z} & , \ m = 0 \\
\frac{m \pi l_{f m}}{\omega \epsilon_{f} b} \sin \frac{m \pi x}{b} e^{\mp j l_{f} m} & , \ m = 1, 3, 5, \ldots \\
\frac{m \pi l_{f m}}{\omega \epsilon_{f} b} \cos \frac{m \pi x}{b} e^{\mp j l_{f} m} & , \ m = 2, 4, \ldots 
\end{cases} \]

Using normalized mode functions (see Harrington, sec 8.1), the transverse part of \( \mathbf{E} \) is

\[ E_{f m}^{\pm} = \sqrt{\frac{\mu_{f}}{\epsilon_{f}}} E_{f m} e^{\mp j l_{f m} z} \]

\[ \mathbf{E}_{f m} = \begin{cases} 
\frac{1}{\sqrt{b}} & , \ m = 0 \\
\sqrt{\frac{2}{b}} \sin \frac{m \pi x}{b} & , \ m = 1, 3, \ldots \\
\sqrt{\frac{2}{b}} \cos \frac{m \pi x}{b} & , \ m = 2, 4, \ldots 
\end{cases} \]
\[ \overline{H}_{fm} = I_{fm}^{\dagger} \overline{h}_{fm} e^{j\sigma_f k_{fm} z} \]  

(7)

\[ \overline{h}_{fm} = \hat{z} \times \overline{e}_{fm} \]  

(8)

In the region above the aperture, use Floquet mode functions:

\[ \overline{E}_{un} = Y_{un}^{+} \overline{e}_{un} e^{-j\omega_{2}un z} \]  

(9)

\[ \overline{e}_{un} = \hat{x} \frac{1}{\sqrt{B}} \ e^{-j\omega_{2}x_{un} x} \]  

(10)

\[ \omega_{2x_{n}} = \omega_{2o} \sin \theta_o + \frac{\eta \omega_{2o}}{B} \]  

(11)

\[ \omega_{2un} = \sqrt{\omega_{2o}^2 - \omega_{2x_{n}}^2}, \ \text{Im} \omega_{2un} \geq 0 \]  

(12)

Notes:

1. \[ \overline{E}_{un} (x+B) = Y_{un}^{+} \frac{1}{\sqrt{B}} e^{-j\omega_{un} (x+B)} e^{-j\omega_{un} z} \]  

\[ = \hat{e} \frac{\omega_{2o}}{B \sin \theta_o} \overline{E}_{un} (x) \]  

(13)
2.  \( I^\pm_m = \pm V^\pm_m Y_m \)  
where 
\[ Y_m = \frac{\omega}{k_{zm}} \]  

In the aperture plane, \( z = 0 \),

\[ \vec{E}_{ap} = \begin{cases} \chi \omega \ , & 1 \times 1 \leq \frac{d}{2} \\ \chi \omega \ , & \frac{d}{2} < 1 \times 1 \leq \frac{B}{2} \end{cases} \]  

To solve the problem, (1) find fields in \( W/G \) and upper region in terms of \( \vec{E}_{ap} \), and (2) use continuity of power to find the aperture admittance \( Y_a \).

In the feed waveguide, at \( z = 0^- \),

\[ \vec{E}_{ap} = V^+_f (1 + \delta) \vec{e}_{f_0} + \sum_{m=1}^{8} Y^{-}_f \vec{e}_{f_m} \]  

Use mode orthogonality to get
\[ V_{f_0}^{+} (1+\eta) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \bar{E}_{q_p} \cdot \bar{E}_{f_0}^* \, dx = \int_{-\frac{d}{2}}^{\frac{d}{2}} \frac{a}{\sqrt{b'}} \, dx = \frac{a d}{\sqrt{b'}} \tag{18} \]

\[ V_{f_m}^{-} = \int_{-\frac{a}{2}}^{\frac{a}{2}} \bar{E}_{q_p} \cdot \bar{E}_{f_m}^* \, dx = \begin{cases} 0 & , \text{m odd} \\ \frac{a b c}{\pi b^2} \sin \frac{m \pi d}{a b} & , \text{m even} \end{cases} \tag{19} \]

In the upper region at \( z = 0^+ \),

\[ \bar{E}_{q_p} = \sum_{n=-\infty}^{\infty} V_{u_n}^{+} \bar{E}_{u_n} \tag{20} \]

\[ V_{u_n}^{+} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \bar{E}_{q_p} \cdot \bar{E}_{u_n}^* \, dx = \frac{c d}{\sqrt{b'}} \frac{\sin \delta_{x_n} d}{\delta_{x_n} d} \tag{21} \]

Note that modal powers are orthogonal and the power flowing in \( z \) direction for mode "m" is
\[ P_m = \int_{-b/2}^{b/2} (\bar{E}_m \times \bar{H}_m^*) \cdot \bar{Z} \, dx \]

\[ = \int_{-b/2}^{b/2} V_m I_m^* (\bar{e}_{f_m} \times \bar{h}_{f_m}^*) \cdot \bar{Z} \, dx \]

\[ = V_m I_m^* \int_{-b/2}^{b/2} \{ \bar{e}_{f_m} \times (\bar{Z} \times \bar{e}_{f_m}^*) \} \cdot \bar{Z} \, dx \]

\[ = V_m I_m^* \int_{-b/2}^{b/2} (\bar{e}_{f_m} \cdot \bar{e}_{f_m}^*) \bar{Z} \cdot \bar{Z} \, dx \]

\[ = V_m I_m^* \]

(22)

Continuity of the total power at \( z=0 \):

\[ V_f^T I_f^T = \sum_{m=1}^{\infty} V_{f_m}^T I_{f_m}^T = \sum_{n=-\infty}^{\infty} V_{u_n}^T I_{u_n}^T \quad (23) \]

where

\[ V_{f_0}^T = V_{f_0}^T + V_{f_0}^- = V_{f_0}^T (1 + \Gamma) \quad (24) \]

\[ I_{f_0}^T = I_{f_0}^T (1 - \Gamma) \quad (25) \]
Recall

\[ Y_a = \frac{I_{f_0}^T}{V_{f_0}^T} \quad (26) \]

\[ \frac{I_{f_m}^-}{V_{f_m}^-} = -Y_{f_m} = -\frac{w \xi_f}{\beta_{2f_m}} \quad (27) \]

\[ \frac{I_{u_n}^+}{V_{u_n}^+} = Y_{u_n} = \frac{w \xi_o}{\beta_{2u_n}} \quad (28) \]

So, (23) becomes

\[ \left| V_{f_0}^T \right|^2 Y_a^* - \sum_{m=1}^{\infty} \left| V_{f_m}^T \right|^2 Y_{f_m}^* = \sum_{n=-\infty}^{\infty} \left| V_{u_n}^T \right|^2 Y_{u_n}^* \quad (29) \]

That is,

\[ Y_a = \sum_{n=-\infty}^{\infty} \frac{\left| V_{u_n}^+ \right|^2}{\left| V_{f_0}^T \right|^2} Y_{u_n} + \sum_{m=1}^{\infty} \frac{\left| V_{f_m}^- \right|^2}{\left| V_{f_0}^T \right|^2} Y_{f_m} \quad (30) \]
Discussion of Result

1. \( Y_a = Y_{\text{ext}} + Y_{\text{int}} \)

\[
= \sum_{n=0}^{\infty} T_n^2 Y_{\text{un}} + jB_{\text{int}} \quad \text{if only } m=0 \text{ propagates in waveguide}
\]

\[
T_n = \left| \frac{V_{\text{un}}^+}{V_{\text{fo}}^-} \right| = \sqrt{\frac{b^2 \sin \delta x_n}{B}} \frac{d}{k x_n \epsilon_r}
\]

2. \( Y_a \) is a parallel combination of \( Y_{\text{un}} \) seen through a transformer, so \( Y_a \to \infty \) if any \( Y_{\text{un}} \to \infty \).

\[
Y_{\text{un}} = \frac{w_{60}}{k x_n} = \frac{w_{60}}{\sqrt{k_x^2 - \frac{b^2}{k_{x_n}^2}}} = \frac{w_{60}/k_0}{\sqrt{1 - (\sin \theta + n \frac{\lambda_0}{B})^2}}
\]
\[ Y_{00} = \frac{1}{\eta \cos \theta} \rightarrow \infty \] \( \theta = \frac{\pi}{2} \)

\[ Y_{nn} \rightarrow \infty \] \( (\sin \theta + n \frac{\lambda_0}{b}) = 1 \)

3. \( Y_{00} \) is real for all \( 0 \leq \theta \leq \frac{\pi}{2} \).

\( Y_{nn} \), \( n \eta > 0 \), may be real or imaginary; depends if grating lobe corresponding to \( n \) propagates.

4. Often we consider array that is "conjugate matched at broadside."

\[ z \quad \Rightarrow \quad \begin{array}{c}
         \text{\( jX_b \)} \\
      \end{array} \quad \begin{array}{c}
         \text{\( R_b \)} \\
      \end{array} \]

Use a source with \( R_b \) & add a tuner

\[ n = \frac{R + j(X - X_b) - R_b}{R + j(X - X_b) + R_b} \]
Narrow Slots Fed by Parallel Plate Waveguide

Reflection for Several Element Spacings
b=0.3, d=0.025

Magnitude vs. Theta for different values of B:
- B=0.3 (black line)
- B=0.5 (blue line)
- B=0.7 (green line)
- B=0.9 (red line)
Radiated Field of Infinite Array

Above the slot array,

\[ E_x(x, z) = \sum_{n=-\infty}^{\infty} \frac{V_{un}^+}{\sqrt{B^1}} \cdot e^{-j \frac{k_{un} x}{\lambda}} e^{-j \frac{h_{un} z}{\lambda}} \]  \hspace{1cm} (1)

where

\[ k_{un} = k_0 \sin \theta_0 + \frac{n\pi}{\lambda} \]  \hspace{1cm} (2)

\[ h_{un} = \sqrt{\frac{k_0^2 - \lambda^2}{k_{un}^2}}, \quad \text{and} \quad k_{un}^2 \lambda = 0 \]  \hspace{1cm} (3)

\[ V_{un}^+ = \frac{B}{2} \int_{-B/2}^{B/2} E_{ap} \cdot \frac{e^{j k_{un} x}}{\sqrt{B^1}} \, dx \]  \hspace{1cm} (4)

1) Evaluating (1) at \( z = 0 \),

\[ E_x(x, z=0) = \sum_{n=-\infty}^{\infty} \frac{V_{un}^+}{\sqrt{B^1}} \cdot e^{-j k_0 \sin \theta_0} e^{-i \frac{n\pi x}{\lambda}} \]

\[ = e^{-j k_0 \sin \theta_0} \sum_{n=-\infty}^{\infty} \frac{V_{un}^+}{\sqrt{B^1}} e^{-i \frac{n\pi x}{\lambda}} \]

\[ \text{Fourier series for } E_{ap} \cdot x \]

\[ \text{in unit cell} \]
2) Consider the field for $\varepsilon > 0$.

\[ h_{x n} = h_0 \sin \theta_0 + \frac{n \pi \varepsilon}{B} = h_0 \left( u_0 + n \frac{\lambda_0}{B} \right) \]

\[ h_{un} = \sqrt{h_0^2 - h_{xn}^2} = \left\{ \begin{array}{ll} h_0 \sqrt{1 - \left( u_0 + n \frac{\lambda_0}{B} \right)^2} \\ -j h_0 \frac{1}{\left( u_0 + n \frac{\lambda_0}{B} \right)^2 - 1} \end{array} \right. \]

\[ n = 0 \quad \Rightarrow \quad h_{x0} = h_0 \sin \theta_0 \quad \text{and} \quad h_{u0} = h_0 \cos \theta_0 \]

\[ n = \pm 1 \quad \Rightarrow \quad |h_{xn}| \text{ depends on } u_0 \pm \frac{\lambda_0}{B} \]
Element Pattern in Infinite Array

Consider an infinite, planar array with element spacing < λ/2 (no grating lobes).

\[ \begin{array}{cccc} \ldots & \hat{\ldots} & \hat{\ldots} & \hat{\ldots} \\ z_0 & z_0 & z_0 & z_0 \end{array} \]

Since all elements are identical, look at one element and consider power received as a function of scan angle.
The power density of incident plane wave is

\[ |\mathbf{E}_0| = |\mathbf{E}_0 \times \mathbf{H}_0^*| = \frac{\eta_0}{2} |\mathbf{E}_0|^2 \]

At the aperture plane, (lossless antennas)

\[ p_i = p_r + p_l \rightarrow \text{power delivered to load, } \mathcal{Z}_0 \]

Since the projected area of the element is proportional to \( \cos \theta \),

\[ p_i = P_0 \cos \theta = |\mathbf{E}_0| A_{\text{elem}} \cos \theta \]

Therefore, \( p_r + p_l \) is proportional to \( \cos \theta \).
Now, consider the antenna as a two-port network.

\[ P_L = P_i^r - P_r^r = P_i^r (1 - |S_{22}|^2) = P_i^r (1 - |S_{11}|^2) \]

\[ \therefore P_L = P_i^r (1 - |I_i|^2) \]

That is,

\[ P_L = |S_0| A_{elem} (1 - |I_i|^2) \cos \theta \]
Recall that (for a lossless antenna)

\[ P^L = \frac{A_e}{\sqrt{S_0}} = \frac{G \lambda^2}{4\pi} \frac{1}{\sqrt{S_0}} \]

Recall also that an element within an infinite array that is perfectly matched at broadside will receive

\[ P^L = |S_0| A_{\text{elem}} \]
\[ = |S_0| A_{\text{elem}} (1 - |\Gamma^a(\theta,\phi)|^2) \cos \theta \]
\[ = \frac{G(\theta,\phi) \lambda^2}{4\pi} |S_0| \quad \text{at} \ \theta = 0^\circ \]

Solving for \( G(\theta,\phi) \) & reintroducing angle dependence

\[ G(\theta,\phi) = \frac{4\pi A_{\text{elem}}}{\lambda^2} (1 - |\Gamma^a(\theta,\phi)|^2) \cos \theta \]
Notes:

1. Although this result has been obtained through heuristic development, it can be obtained more rigorously. See, for example,

2. \( F^o(\theta, \phi) \) is the transmitting ref. coefficient when all antennas are driven and phased to radiate a single plane wave. That is, the reciprocal of the receive case considered above.

3. Recall the discussion of element patterns and mutual coupling.

\[
E(\bar{E}) = K e^{-j \frac{2 \pi r}{\lambda}} \sum_{m} a_m e^{j \phi_i} \overline{F_m} \overline{\frac{1}{f_m}(\theta, \phi)}
\]

Obtained by exciting one element with others terminated in \( Z_0 \).
For the infinite array, the gain associated with \( f_m(\theta, \phi) \) must be the same as our expression for \( g(\theta, \phi) \).

Therefore, for the case of a lossless, reciprocal, infinite array the element gain and its active reflection coefficient are related. Furthermore, the element gain measured with one element driven and others terminated is related to the reflection coefficient when all elements are driven.

\[
G(\theta, \phi) = \frac{4\pi A_{elem}}{\lambda^2} (1 - |r|^2) \cos \theta
\]
4. This result in (3) is the "Realized Gain" for a lossless element. If the element has ohmic losses,

\[ G^R(\theta, \phi) = \frac{4\pi \text{ A}_{\text{elem}}}{\lambda} \cdot \epsilon \left[ 1 - |\Gamma^o(\theta, \phi)|^2 \right] \cos \theta \]

\[ \text{Efficiency} \]

Normalizing this to the peak gain for an array element that is perfectly matched at broadside ($\theta=0$) yields

\[ g^R(\theta, \phi) = \left[ 1 - |\Gamma^o(\theta, \phi)|^2 \right] \cos \theta \]

An "ideal" phased array element would be perfectly matched at all scan angles (this is not possible), so the "ideal element pattern" is

\[ g^{\text{ideal}}(\theta, \phi) = \cos \theta \]
Fig. 1. Geometry of a finite array of rectangular microstrip patches.

Fig. 3. Reflection coefficient magnitude versus scan angle ($E$- and $H$-plane) for a finite ($9 \times 9$, center element) patch array, compared with infinite array results. $\varepsilon_r = 12.8$, $d = 0.06 \lambda_0$, $a = b = 0.5 \lambda_0$, $L = 0.1074 \lambda_0$, $W = 0.15 \lambda_0$, $X_p = -L/2$, $Y_p = 0$. 

Fig. 5. (a) E-plane. (b) H-plane active center element gains for patch arrays of various sizes: \(s = 12.8\), \(d = 0.06\lambda_0\), \(a = b = 0.5\lambda_0\), \(L = 0.1074\lambda_0\), \(W = 0.15\lambda_0\), \(X_p = -L/2\), \(Y_p = 0\).

Theory and Analysis of Phased Array Antennas

By Noach Amitay, Victor Galindo and Chen Pang Wu
Wiggles due to finite array effect

Fig. 8.9. Normalized radiation patterns for the array of nineteen elements with excitation shown in (a), (b), and (c).
Asymmetry of edge elements.

Fig. 8.10. Normalized radiation patterns for the array of nineteen elements with excitation shown in (d), (e), and (f).
A "Proof" That $S_{ij} \neq 0$

Consider an infinite array as depicted below. The power radiated by each element is

$$P_{\text{rad}} = \int_0^L \int_0^L (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{z} \, dx \, dy$$

![Diagrams showing an infinite array and power radiated by each element](image)
If the antennas are lossless, conservation of power yields

\[ P_{rd} = P_{inc} (1 - |\Gamma_0^q(\theta, \phi)|^2) \]

where \( P_{inc} \) is the power incident from the generator onto the element.

Recall that

\[ \Gamma_0^q(\theta, \phi) = \sum m \sum n \sum_{S_{00}, mn} e^{j \delta(U_{0m}x + V_{0n}y)} \]

is independent of \( U_0, V_0 \)

Now, consider that the array is reasonably well designed so that

\[ |\Gamma_0^q(0, \phi)| = \left| \sum m \sum n \sum_{S_{00}, mn} \right| < 1 \]
That is, the array radiates some power in the broadside direction.

\[ P_{\text{rad}}(\theta=0, \phi) = P_{\text{inc}} \left( 1 - |\Gamma_{\infty}(0, \phi)|^2 \right) > 0 \]

If the element spacing is less than \( \lambda/2 \), then no grating lobes are present, even if \( \theta_0 = 90^\circ \).

But, for \( \theta_0 = 90^\circ \), \( \langle E \times H^* \rangle \cdot \vec{z} = 0 \). That is, the beam is scanned to end-fire and the "beam" propagates parallel to the array face.

In this case,

\[ P_{\text{rad}} = \int_\alpha^\beta \int_0^{\pi} \langle E \times H^* \rangle \cdot \vec{z} \, dx \, dy = 0 \]

That means

\[ P_{\text{rad}} = P_{\text{inc}} \left( 1 - |\Gamma_{\infty}(90^\circ, \phi)|^2 \right) = 0 \]
Since $\Pi_{\phi_0} > 0$, we must have
\[ |\Gamma^q(90^\circ, \phi)| = 1 \]

Now, suppose we claim
\[ S_{00, mn} = 0 \quad \text{for all } m, n \neq 0, 0 \]

\[ S_{00, 01} = 0 \quad S_{00, 02} = 0 \]

\[ S_{00, 00} = 0 \]

Then,
\[ \Gamma^q(0, \phi) = S_{00, 00} \]

and independent of $u_0, v_0$
So, $|\Gamma_{\infty}^0(\theta, \phi)|$ cannot be $< 1$ at $\theta = 0^\circ$ and $= 0$ at $\theta = 90^\circ$.

Therefore, $S_{\infty, mn}$ cannot be zero for all $m, n$. 