Side lobe blanking

This is not DBF and it is not an adaptive array, but it is a commonly used technique.

Small target in main beam

Large clutter in sidelobe at range bin p. (Clutter could be a desired target that is not in main beam.)

Looks like a target in direction of beam
We can blank out the sidelobe clutter by using an auxiliary antenna and receiver.
Notes:
1. Auxilliary antenna gain at all angles should exceed SLL at those angles, but be significantly less than main beam.

2. A large target in a sidelobe causes the receiver to blank that range bin for all angles. That is, a small target in main beam is "blanked" by side lobe clutter at the same range.

3. Side lobe blanking is most useful for low-duty-cycle clutter that blanks only a few range bins. High-duty-cycle clutter requires a sidelobe canceller.
Side Lobe Canceller

This is an example of an adaptive array. We will look at the operation of a single side lobe canceller now, as an introduction. Then we will formulate the general case of adaptive beam forming.

\[
\text{Received signal} = G_s S_s + G_j S_j
\]
A problem occurs when $G_s S_s$ is not significantly larger than $G_j S_j$.

- Beam Space Adaptation

- Beam $G_2(u_s)$
- Beam $G_5(u_j)$

- Diagram of circuit components
\[ V_T = V_2 - \alpha V_S \]
\[ = \sqrt{G_a(U_s)} V_S + \sqrt{G_a(U_j)} V_j \]
\[ - \alpha \sqrt{G_S(U_s)} V_S - \alpha \sqrt{G_S(U_j)} V_j \]

Adjust \( \alpha = \frac{\sqrt{G_a(U_j)}}{\sqrt{G_S(U_j)}} \). Then

\[ V_T = V_S \left( \sqrt{G_a(U_s)} - \sqrt{G_S(U_s)} \right) \left( \frac{\sqrt{G_a(U_j)}}{\sqrt{G_S(U_j)}} \right) \]

signal \( \gamma \)  
adaptation  
reduction due to nulling jammer

Note: \( G_S(U_s) \ll G_S(U_j) \)  
and \( G_a(U_j) \ll G_a(U_s) \)

The jammer signal is eliminated from \( V_T \) at the expense of some signal loss and added complexity.
Auxiliary Antenna Adaptation

Note: Voltages (complex phasors) are combined to form $V_T$.

Let

$$V_m = g_m(u_s) E_s + g_m(u_j) E_j$$

be the phasor voltage from the main antenna due to incident plane waves with $|E| = E_s$ and $E_j$. 
$V_m$ accounts for phase center locations of main & aux antennas and for angles of arrival.

Likewise,

\[ V_a = g_a(u_s) E_s + g_a(u_j) E_j \]

Choosing $\alpha = \frac{g_m(u_j)}{g_a(u_j)}$ results in

\[ V_T = E_s \left[ g_m(u_s) - g_a(u_s) \frac{g_m(u_j)}{g_a(u_j)} \right] \]

The resulting antenna pattern is:

The resulting antenna pattern of main + weighted aux is:

\[ g < g_m(u_s) \]

Places null at $u_j$
Adaptive Antenna Arrays

Examples of adaptive array architectures:

Figure 3.17 Adaptive array configurations: (a) fully adaptive array (element weighting); (b) fully adaptive array with multiple-beam feed (beam space adaptivity); (c) partially adaptive array; and (d) array with multiple canceler elements.
Two Common Algorithms:

- Widrow — minimize LMS difference between array output signal and a known reference signal.

- Howells-Applebaum — optimize S/N subject to constraint of a quiescent pattern.

Matrix Formulation of Howells-Applebaum Adaptivity

Consider a linear array

\[ u_s^* = g_n(u_c) E_s e^{j\omega nd_s} \]
\[ V_T^S = \sum_{n=1}^{N} w_n v_n^S = W^T V^S \]  

where

\[ W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \quad V^S = \begin{bmatrix} v_1^S \\ v_2^S \\ \vdots \\ v_N^S \end{bmatrix} \]

\( V_T^S \) is maximized by choosing the weight vector

\[ W^o = \begin{bmatrix} w_1^o \\ w_2^o \\ \vdots \\ w_N^o \end{bmatrix} \quad w_n^o = \left| w_n^o \right| e^{-j2\pi n u_s} \]

all ones for "uniform illumination" or weighted for low SLL.
The signal power is

\[ P_s = E \left\{ V_T^s * V_T^s \right\} \]

\[ = E \left\{ (W^T V_s)^* (W^T V_s) \right\} \]  \hspace{1cm} (4)

\[ = E \left\{ \sum_{m=1}^{N} \sum_{n=1}^{N} w_m^* V_m^s V_n^s w_n \right\} \]

\[ = E \{ W^+ M_s^s W^* \} \]

where

\[ W^+ = [w_1^* \ w_2^* \ \ldots \ \ w_N^*] \]

\[ M_s^s = \overline{V_s^s * V_s^s} = \begin{bmatrix}
\overline{v_1^s * v_1^s} & \overline{v_1^s * v_2^s} & \ldots \\
\overline{v_2^s * v_1^s} & \overline{v_2^s * v_2^s} & \ldots \\
\vdots & \vdots & \ddots \\
\overline{v_N^s * v_1^s} & \overline{v_N^s * v_2^s} & \ldots 
\end{bmatrix} \]  \hspace{1cm} (5)

\[ M_{m,n}^s = \overline{v_m^s * v_n^s} \]

\[ = \frac{1}{T} \int_{t_0}^{t_0+T} v_m^s(t) v_n^s(t) \, dt \]
Now consider that there is uncorrelated noise in the antenna channels and one or more uncorrelated (narrow-band) interfering signals from directions $u_i$.

The total signal at the $n^{th}$ antenna port is

\[ v_n = g_n(u_S) E_S e^{j2\pi d u_S} + n_n \]
\[ + \sum_{i=1}^{I} g_n(u_i) E_i e^{j2\pi d u_i} \]

\[ \equiv v_{n-S} + v_{n-I} \]

After combining all antenna signals with weights $w_n$,

\[ v_T = W^T v = W^T v_S + W^T v_I \]
The total noise power + interference power at the output of the combiner is

\[
P_{IN} = \mathbb{E}\{(W^T V i)\} = \mathbb{E}\{W^T W\}
\]

where

\[
M_{mn} = \mathbb{E}\{[ n_m^* + \sum_{i=1}^I g_m^*(u_{ij}) E_j e^{-j 2 \pi d \Delta x u_{ij}}] [ n_n + \sum_{k=1}^J g_n^*(u_{nk}) E_k e^{j 2 \pi d \Delta x u_{nk}}]\}
\]

\[
= N_n S_{mn} + \sum_{j=1}^I g_m^*(u_{ij}) g_n(u_{ij}) |E_j|^2 e^{j 2 \pi d (n-m) \Delta x u_{ij}}
\]
Using (4) & (8),

$$\text{SINR} = \frac{W^T M^s W}{W^T M W} \quad (9)$$

The adaptive array adjusts the weights $\hat{w}_n$ to maximize SINR. Eqn (9) is the ratio of quadratic forms, and it is maximized for a signal from direction $\mathbf{u}_s$ by choosing,

$$W = M^{-1} W^0 \quad (10)$$

**Notes:**

1. $M$ is obtained from $M = E\{V_1^T V_1^T\}$ where $V_1$ is formed from the antenna voltages $v_1$ due to noise and interference without the signal. Typically, we do not have access to $v_n$. 
However, we do have $v_n$

$$v_n = v^n_s + v^n_n + v^n_I$$

desired signal noise sum of all interference

$$= v^n_s + v^n_I$$

If $v^n_s \geq v^n_I$, the formation of a beam with main lobe in direction $UL$ suppresses $v^n_I$ relative to $v^n_s$, and there is no need for pattern adaptation.

If $v^n_s \ll v^n_I$, then

$$v_n \approx v^n_I$$

and we form $M$ using

$$M_{mn} = E \sum \left\{ v^*_m v^n_n \right\}$$

$$= \frac{1}{P} \sum_{p=1}^{P} v^*_m(t_p) v^n_n(t_p)$$
2. Sample antenna patterns

Quiescent Pattern
10 elements

Adapted Pattern

3. Recalling the side lobe canceller example, adaptive nulling an interference signal might be thought of as follows:
Step 1: Determine the direction of arrival and signal strength of the interference.
Step 2: Form a (second) beam with the array that is steered toward the interference source and has signal strength to cancel interference contribution to quiescent pattern.
Step 3: Form a new antenna pattern by combining the cancelling beam with the quiescent pattern.

4. The interpretation above emphasizes the need to accurately locate the direction of the interference. DSP folks have nonlinear spectral estimation techniques that can determine DOA to better accuracy than the antenna beamwidth.
Results like those above can be obtained with the MUSIC algorithm (Multiple Signal Classification), ESPRIT (Estimation of Signal Parameters by Rotational Invariant Techniques), etc.

These methods usually require the signal of interest (the interference in this case) to be much stronger than other signals & noise. This is precisely the situation that is important for adaptive nulling.