Weighted Graphs I

University of Massachusetts Amherst
ECE 242 – Data Structures and Algorithms
Lecture 26

Weighted graphs

• Edges have “weights”
  – Cost metric
• Example interpretations
  – Distance
  – Dollar cost of traversal
  – Delay
• Cost is assumed to be additive
Weighted tree example

- Example: cost of flying between airports
  - “0” means no edge

![Diagram of a weighted tree example](image)

Algorithms for weighted graphs

- Weights make algorithms more interesting

- Example: minimum spanning tree
  - Find cheapest set of flights to connect all airports

- Problem
  - Why does our previous algorithm not work?
  - Can you provide an example where it fails?
Minimum spanning tree

• How to find set of edges with minimum total weight to connect all nodes?
  – Not easy to “see” correct solution

Ideas

• Add edges that are cheap
  – If node still needs to be connected, might as well use a cheap edge
• Don’t add edges between nodes that have already been connected
  – Redundant edge not necessary
Kruskal’s algorithm

• Assign each vertex to its own set
• Add all edges to priority queue (descending)
• For each edge in priority queue
  – If vertices are in different sets
    • Add edge to MST
    • Merge sets

Prim’s algorithm

• Alternative implementation of MST
  – Used in textbook
• Pick one vertex from graph and assign to set
• Repeat until set includes all nodes
  – Find least-cost edge connecting new vertex to set
    • Add edge
    • Add vertex to set
Kruskal’s algorithm example

• Try Kruskal’s algorithm yourself!

Kruskal’s algorithm example

• Initial setup: all nodes in separate sets
Kruskal’s algorithm example

• Find least-cost edge: SWF-LGA $49

Kruskal’s algorithm example

• Next least-cost edge: JFK-HPN $55
Kruskal’s algorithm example

• Next least-cost edge: ALB-SWF $59

• Next least-cost edge: EWR-LGA $63
Kruskal’s algorithm example

• Next least-cost edge: ALB-LGA $75
  – Skipped (future skips not shown)

Kruskal’s algorithm example

• Next least-cost edge: BOS-BDL $82
Kruskal’s algorithm example

- Next least-cost edge: JFK-PVD $95

- Next least-cost edge: BOS-MHT $99
Kruskal’s algorithm example

• Next least-cost edge: PVD-MHT $110
  – Merging of two sets

Kruskal’s algorithm example

• Next least-cost edge: ALB-JFK $123
  – Merging of two sets
Kruskal’s algorithm example

- Next least-cost edge: BTV-MHT $139

Result:

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Code

```java
public Graph minimumSpanningTree() {
    Graph mst = new Graph(maxVertices); // create new graph
    int[] set = new int[activeVertices];
    for (int i=0; i<activeVertices; i++) { // copy nodes to graph
        mst.addVertex(vertices[i]);
        set[i]=i; // assign each node to its own set
    }
    PriorityQueue q = new PriorityQueue(maxVertices*maxVertices); // create priority queue
    for (int i=0; i<activeVertices; i++) { // copy edges to queue
        for (int j=0; j<activeVertices; j++) {
            if (edges[i][j]!=0) {
                q.enqueue(new Edge(vertices[i], vertices[j], edges[i][j]));
            }
        }
    }
   ...

    while (!q.isEmpty()) { // iterate over all edges in priority order
        Edge e = q.dequeue(); // consider next edge
        if (set[e.source.graphIndex]!=set[e.destination.graphIndex]) { // skip edges not connecting different sets
            mst.addEdge(e.source, e.destination, e.weight); // add edge to MST
            System.out.println("adding "+e);
            int setToMerge=set[e.destination.graphIndex]; // rename nodes from "other" set
            for (int i=0; i<activeVertices; i++) {
                if (set[i]==setToMerge) { // find nodes from "other" set
                    set[i]=set[e.source.graphIndex]; // reassign nodes
                }
            }
        }
    }
    return mst;
}
```
Next Steps

• HW due on Thursday
• No class on Friday