Minimum Spanning Tree

University of Massachusetts Amherst
ECE 242 – Data Structures and Algorithms
Lecture 25

Graph operations

• Typical operations on graph
  – Traversal (done)
    • Depth first
    • Breadth first
  – Removing redundant edges (today)
    • Minimum spanning tree
  – Shortest paths on weighted graph (Wednesday)
    • Shortest path algorithms
Minimum Spanning Tree

• How to find minimum set of edges to connect all nodes?

  – How many edges in graph with n vertices?
Minimum Spanning Tree

• Traversal algorithms visit every node
  – Go only via valid edges

• Idea: record edges traversed in DFS
  – How can we record edges conveniently?

Minimum Spanning Tree

• Recursive implementation
  – Depth-first search traversal
  – Create new graph (minimum spanning tree) during traversal
    • Add edge during every recursive call
  – Return graph after traversal is completed
Code

• Method to set up MST computation:

```java
public Graph minimumSpanningTree() {
    Graph mst = new Graph(maxVertices); // create new graph
    for (int i=0; i<activeVertices; i++) {
        // copy nodes to graph
        mst.addVertex(vertices[i]);
    }
    clearVisited();
    recursiveMST(vertices[0], mst); // recursive call
    return mst;
}
```

• Recursive MST computation

```java
private void recursiveMST(Vertex v, Graph mst) {
    v.visited = true; // mark vertex as visited
    for (int i=0; i<activeVertices; i++) {
        if (edges[v.graphIndex][i]==1) {
            // iterate over all existing edges
            if (!vertices[i].visited){
                // check if edge leads to unvisited node
                mst.addEdge(v, vertices[i]);
                // add edge to spanning tree
                recursiveMST(vertices[i], mst); // recursively follow edge
            }
        }
    }
}
```
Minimum spanning tree example

Graph:

- Which edges are part of MST?

Minimum spanning tree example

- MST output:
Minimum spanning tree example

• As “tree”

Next Steps

• Class on Wednesday
• HW do on Thursday
• Midterm II on 11/12