Midterm Exam #1
ECE 314, Spring 2006
(16 March 2006, 6:00 p.m. – 8:00 p.m., Marston 132)

General Instructions: Write your solution to each of the following problems in the exam booklet. You must show your work in order to receive full credit. Before you hand in your exam, write and sign the ECE Honor Pledge on the back page of the exam booklet:

“On my honor, I have neither received nor given aid on this exam.”

Please fold the exam question pages and put them inside your exam booklet when you hand it in.

Problem 1 (10 points):
The company QComp builds pocket calculators. Each calculator is built in one of QComp’s two production lines, A and B. Production line A produces 70% of QComp’s calculators, production line B produces 30%. 20% of the calculators produced in line A do not function properly. 10% of the calculators produced in line B do not function properly. What is the overall percentage of QComp’s calculators that function properly?

Problem 2 (20 points):
Consider the random variable \( X(s) = x \), where \( x \) is the square of the number of dots on the upward facing side of a fair, six-sided die.

(a, 4 points) Determine the sample space \( S_X \);
(b, 3 points) determine the PMF and present the PMF in the form of a table;
(c, 2 points) plot the PMF;
(d, 3 points) determine the CDF and present it in the form of a table;
(e, 2 points) plot the CDF;
(f, 3 points) calculate the expected value of \( X \);
(g, 3 points) calculate the standard deviation of \( X \).

Problem 3 (10 points):
How many 5-letter words (ordered sequences of letters) can you form
(a, 5 points) with zeros and ones?
(b, 5 points) with the letters of a 26-letter alphabet?

Problem 4 (10 points):
Consider a group of three people picked at random. What is the probability
(a, 5 points) that none of them was born on a Sunday?
(b, 5 points) that each of them was born on a different day of the week (Monday through Sunday)?
Problem 5 (10 points):

The PMF of a discrete R.V. $X$ is nonzero at $x = 0$, $x = 1$, and $x = 2$ and zero otherwise. Is it possible that $X$ is

(a, 2 points) a Bernoulli R.V.?
(b, 2 points) a binomial R.V.?
(c, 2 points) a geometric R.V.?
(d, 2 points) a discrete uniform R.V.?
(e, 2 points) a Poisson R.V.?

Problem 6 (10 points):

A Positron Emission Tomography (PET) medical imaging system is used to determine the condition of the tissue being examined. Suppose that tissue may be either healthy ($H$) or damaged ($D$). If the tissue is healthy, the PET sensor output is a Poisson ($\alpha$) random variable with $\alpha = 4$. If the tissue is damaged, the sensor output is a Poisson ($\alpha$) random variable with $\alpha = 1$. Suppose that for the tissue we are examining, we know that $P(D) = 0.8$. Let $X$ denote the sensor output.

(a, 4 points) Find $E[X]$.
(b, 3 points) Find $P[X = 3]$.
(c, 3 points) Say that we run a PET scan and observe $X = 3$. Find the probability that the tissue is damaged, given this observation.

Problem 7 (20 points):

The experiment consists of flipping a fair coin two times. Let the R.V. $X$ be the number of heads facing up.

(a, 5 points) Calculate the expected value of $X$;
(b, 5 points) plot the CDF of $X$;
(c, 5 points) calculate the variance of $X$;
(d, 5 points) calculate the variance of the derived R.V. $Y = 3X + 12$.

Problem 8 (10 points):

(a, 5 points) How many different 3-member teams can you form out of 11 people?

(b, 5 points) A tree has 5 branches, 2 big ones and 3 smaller ones. The big branches have 8 twigs each, and the small branches have 2 twigs each. The twigs of the big branches have 10 leaves each, and the twigs of the small branches have 2 leaves each. How many leaves has the tree?