1. **Multi-commodity flow problem** (simplified version).
Nodes $z_1$ and $z_2$ represent factories which are capable of producing certain material at rates of 7 and 5 units, respectively. Nodes $z_3, z_4$ and $z_5$ represent depots which are capable of receiving the units at respective rates of 6, 4 and 3.

At what rate should $z_1$ and $z_2$ produce and over what routes should the material be shipped to *maximize product flow*? Each route has a maximum capacity for transporting the material, shown in the graph. Assume that the undirected edges have capacity in either direction.

Describe the model you use to solve this problem and show all your work.

![Graph](image)

2. **Assignment Problem**
The following matrix $M$ specifies the possible assignment (matching) of workers to jobs, where $M(i,j) = 1$ indicates that worker $i$ can perform job $j$.

$$
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & & & 1 & & \\
2 & 1 & 1 & & & 1 \\
3 & & & 1 & & \\
4 & & & 1 & 1 & \\
5 & 1 & & & 1 & \\
6 & & & & & 1 \\
\end{array}
$$

Find the solution that maximizes the number of workers with jobs. Each worker can perform at most one job and each job can be done by at most one worker.

Formulate the problem as a network flow problem. Which algorithm will you use to solve it? Show all the steps of your algorithm and prove that your solution is optimum.
3. Programming Assignment

Implement a computer program to solve the **Flight Scheduling** problem described in the textbook (problem C-7.9) same as given in the midterm exam. Explain exactly how you model the problem as graph algorithm.

Consider the following data. Assume that the connection times are equal to 1 hour for each airport.

(SFO, LAX) - (2:00, 3:00)  (SFO, DFW) - (1:30, 4:30)
(SFO, ORD) - (0.30, 4:30)  (LAX, DFW) - (4:00, 6:00)
(DFW, MIA) - (8:00, 10:00)  (DFW, ORD) - (6:15, 9:15)
(ORD, BOS) - (10:30, 13:30)  (ORD, JFK) - (10:15, 13:15)
(MIA, JFK) - (12:00, 15:00)  (JFK, BOS) - (14:50, 15:50)
(MIA, BOS) - (11:30, 14:30)  (SFO, BOS) - (0:45, 6:45)
(LAX, MIA) - (5:00, 9:00)  (LAX, SFO) - (17:00, 18:00)
(BOS, MIA) - (10:00, 13:00)  (BOS, JFK) - (11:00, 12:00)
(BOS, ORD) - (11:00, 14:00)  (BOS, SFO) - (7:30, 13:30)
(JFK, MIA) - (13:15, 16:15)  (JFK, ORD) - (8:00, 11:00)
(ORD, SFO) - (12:00, 16:00)  (ORD, DFW) - (16:15, 19:15)
(MIA, DFW) - (14:00, 16:00)  (MIA, LAX) - (11:00, 15:00)
(DFW, LAX) - (21:00, 23:00)  (DFW, SFO) - (18:00, 21:00).

Find schedules with the **earliest arrival time** for the following data. If more than one solution exists, your program should select the one with minimum number of connections.

- From SFO to JFK, starting at 0:30.
- From BOS to LAX, starting at 9:30.
- From MIA to SFO, starting at 12:00

Show the solution (give printouts of your running program. Include the program with your solution (give link) and clear instructions how to run it.