1. **Problem.** Find the resistivity $\rho$ (in ohm-cm) for a piece of Si doped with both acceptors ($N_A = 10^{19} \text{ cm}^{-3}$) and donors ($N_D = 10^{16} \text{ cm}^{-3}$). Since the electron and hole mobilities depend on the concentration of the dopants, use the following empirical expressions to evaluate them:

\[
\mu_n = 232 + \frac{1180}{1 + \left(\frac{N_A + N_D}{8 \times 10^{16}}\right)^{0.9}},
\]

\[
\mu_p = 48 + \frac{447}{1 + \left(\frac{N_A + N_D}{1.3 \times 10^{16}}\right)^{0.76}},
\]

where the mobilities are expressed in cm$^2$/V s and $N_A$ and $N_D$ in cm$^{-3}$.

**Solution.** Substituting for the values of $N_A$ and $N_D$ in the given equations,

\[
\mu_n = 247.06 \text{ cm}^2/\text{Vs},
\]

and

\[
\mu_p = 50.84 \text{ cm}^2/\text{Vs}.
\]

Since $\sigma = eN_D \mu_n + eN_A \mu_p$ and $\rho = 1/\sigma$, we have

\[
\rho = 1.22 \times 10^{-2} \text{ } \Omega\text{cm}.
\]

2. **Problem.** Consider a sample of $p$-type Si doped with $N_A = 10^{18} \text{ cm}^{-3}$ and $N_D = 0$. Over a length of 1 $\mu$m the electron concentration drops from $10^{16} \text{ cm}^{-3}$ to $10^{13} \text{ cm}^{-3}$. Using Eq. (1) above, calculate the current density due to diffusion alone.
Solution. The electron current due to diffusion alone is:

\[ J_{diff} = eD_n \frac{dn}{dx}. \]

Since from Einstein relation \( eD_n = k_B T \mu_n \), assuming \( dn/dx \approx 10^{16} \text{ cm}^{-3}/10^{-4} \text{ cm} \) and using \( \mu_n \) from the equation given in the first problem, we have \( \mu_n = 342.18 \text{ cm}^2/\text{Vs} \) and \( J_{diff} = 141.65 \text{ A/cm}^2 \).

3. Problem. A \( p-n \) Si junction is formed between a \( n \)-type region with \( N_D = 2 \times 10^{18} \text{ cm}^{-3} \) and a \( p \)-type region with \( N_A = 5 \times 10^{16} \text{ cm}^{-3} \). Find:
   (a) the width of the depletion region on the \( n \) side, \( x_{n0} \) (in \( \mu \text{m} \)),
   (b) the width of the depletion region on the \( p \) side, \( x_{p0} \) (in \( \mu \text{m} \)),
   (c) the built-in potential \( V_{bi} \) (in eV),
   (d) the fraction of \( V_{bi} \) which drops over the \( n \)-side of the junction,
   (e) the fraction of \( V_{bi} \) which drops over the \( p \)-side of the junction.

Finally, using the results just obtained, plot (to scale as accurately as you can) the band diagram of the junction at equilibrium.

Solution. (a) From

\[ W = \left[ \frac{2\epsilon k_B T}{e^2} \ln \left( \frac{N_A N_D}{n_i^2} \right) \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2}, \]

we get \( W = 0.154 \text{ \mu m} \) and

\[ x_{n0} = \frac{WN_A}{N_A + N_D} = 0.00375 \text{ \mu m}. \]

(b) Similarly,

\[ x_{p0} = \frac{WN_D}{N_A + N_D} = 0.15 \text{ \mu m}. \]
(c) The built-in potential is given by:

\[ V_{bi} = \frac{k_B T}{e} \ln \left( \frac{N_A N_D}{n_i^2} \right) = 0.892 \text{ V} . \]

(d) The potential drop on the \( n \) side of the junction is:

\[ V_{bi,n} = \frac{V_{bi} x_n 0}{W} = 0.021 \text{ V} . \]

(d) On the other end, the potential drop on the \( p \) side of the junction is:

\[ V_{bi,p} = \frac{V_{bi} x_p 0}{W} = 0.871 \text{ V} . \]

4. **Problem.** A Si \( p-n \) junction has a saturated reverse current \( I_s = 10^{-14} \text{ A} \) at 300 K. Determine the forward bias required to get a current of (a) 50 \( \mu \text{A} \) and (b) 2.50 mA.

   **Solution.** (a) From the expression for the diode current

   \[ I = I_s \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right] \]

   and from the fact that \( I_s = 10^{-14} \text{ A} \) we have \( V = 0.5784 \text{ V} \) when \( I = 50 \mu \text{A} \). (b) Similarly, for \( I = 2.5 \text{ mA} \) we have \( V = 0.6797 \text{ V} \).

5. **Problem.** The saturated reverse current of a GaAs \( p-n \) junction is \( 5 \times 10^{-21} \text{ A} \). Calculate the current under a forward bias of (a) 0.8 V and (b) 1.2 V.

   **Solution.** (a) As in the previous problem, we have \( I = 0.129 \mu \text{A} \) for \( V = 0.8 \text{ V} \) and (b) \( I = 0.66 \text{ A} \) for \( V = 1.2 \text{ V} \).
6. **Problem.** Derive Eq. (419) of the Lecture Notes, Part 2.

**Solution.** We require:

\[
J(x = l_n) = J = J \left\{ \frac{1}{M_n} + \int_{-l_p}^{l_n} dx \alpha_p \exp \left[ - \int_{-l_p}^{x} d\alpha (\alpha_n - \alpha_p) \right] \right\} \exp \left[ \int_{-l_p}^{l_n} d\alpha' (\alpha_n - \alpha_p) \right]
\]

(3)

Now let's multiply both sides of this equation by:

\[
\frac{1}{J} \exp \left[ - \int_{-l_p}^{l_n} d\alpha' (\alpha_n - \alpha_p) \right],
\]

so that Eq. (3) becomes:

\[
\exp \left[ - \int_{-l_p}^{l_n} d\alpha' (\alpha_n - \alpha_p) \right] = \frac{1}{M_n} + \int_{-l_p}^{l_n} dx \alpha_p \exp \left[ - \int_{-l_p}^{x} d\alpha' (\alpha_n - \alpha_p) \right],
\]

(4)

or

\[
\frac{1}{M_n} = \exp \left[ - \int_{-l_p}^{l_n} d\alpha' (\alpha_n - \alpha_p) \right] - \int_{-l_p}^{l_n} dx \alpha_p \exp \left[ - \int_{-l_p}^{x} d\alpha' (\alpha_n - \alpha_p) \right].
\]

(5)

Now let's add and subtract \(\alpha_n\) in the integrand of the last term:

\[
\frac{1}{M_n} = \exp \left[ - \int_{-l_p}^{l_n} d\alpha' (\alpha_n - \alpha_p) \right] + \int_{-l_p}^{l_n} dx (\alpha_n - \alpha_p) \exp \left[ - \int_{-l_p}^{x} d\alpha' (\alpha_n - \alpha_p) \right] - \\
- \int_{-l_p}^{l_n} dx \alpha_n \exp \left[ - \int_{-l_p}^{x} d\alpha' (\alpha_n - \alpha_p) \right].
\]

(6)
The second term on the right-hand side can be integrated immediately, since in general:

\[
\int_a^b dx f(x) \exp \left( - \int_a^x dx' f(x') \right) = - \exp \left( - \int_a^x dx' f(x') \right) \bigg|_a^b = 1 - \exp \left( - \int_a^b dx' f(x') \right).
\]

So, Eq. (6) becomes:

\[
\frac{1}{M_n} = \exp \left[ - \int_{-l_p}^{l_n} dx' (\alpha_n - \alpha_p) \right] + 1 - \exp \left[ - \int_{-l_p}^{l_n} dx' (\alpha_n - \alpha_p) \right] - \int_{-l_p}^{l_n} dx \alpha_n \exp \left[ - \int_{-l_p}^x dx' (\alpha_n - \alpha_p) \right] = 1 - \int_{-l_p}^{l_n} dx \alpha_n \exp \left[ - \int_{-l_p}^x dx' (\alpha_n - \alpha_p) \right],
\]

so that

\[
1 - \frac{1}{M_n} = \int_{-l_p}^{l_n} dx \alpha_n \exp \left[ - \int_{-l_p}^x dx' (\alpha_n - \alpha_p) \right],
\]

which is Eq. (419) of the Lecture Notes, Part 2.

Note that the result above is completely general. We have not made use of the assumption \( \alpha_n = \alpha_p \), nor of any arbitrary assumption of the type

\[
\exp \left[ \int_{-l_p}^{x} dx (\alpha_n - \alpha_p) \right] = \exp \left[ (\alpha_n - \alpha_p)(x + l_p) \right],
\]

which is only valid if \( \alpha_n \) and \( \alpha_p \) do not depend on \( x \), and, finally, it is valid also far from breakdown, so it does not require any condition of the sort \( \int_{-l_p}^{l_n} dx \alpha = 1 \).

7. **Problem. (a).** Plot the Zener tunneling current \( J_{\text{Zener}} \) given by Eq. (425) of the Lecture Notes as a function of applied bias \( V_a \) using \( m^* = 0.32 \ m_{el} = 0.32 \times 9.1 \times 10^{-31} \) kg and \( E_G = 1.1 \) eV (the gap of Si). Assume
for the field $F$ the approximate value $F \approx F_{\text{max}}$, where the maximum field in the junction, $F_{\text{max}}$, is given by Eq. (364). Assume $N_A = 10^{17}\text{cm}^{-3}$ and $N_D = 10^{19}\text{cm}^{-3}$ to estimate the width $l_p$ and $l_n$ of the depletion regions.

(b). Repeat the calculation, but now using $E_G = 0.64\text{ eV}$ (the gap of Ge) and $m^* = 0.22\text{ }m_{el}$.

(c). Repeat the calculations in (a) and (b), but now assuming higher doping, $N_A = 10^{19}\text{cm}^{-3}$ and $N_D = 10^{21}\text{cm}^{-3}$. What happens? Why?

(d). Considering that the current VLSI technology requires increasingly higher doping, can you foresee any problems with the possible use of Ge (instead of Si) when the width of the depletion regions shrinks?

**Solution.** Higher doping enhances tunneling. Ge exhibits a much larger Zener tunneling current than Si because of its smaller band-gap. This drawback will become more severe as the doping concentrations increase. The figure below shows the result:
Note that the current is nonzero only under reverse bias such that $V_a < -(E_G - V_{bi})$ or under forward bias $V_a > E_G + V_{bi}$. For a bias of smaller magnitude, $|V_a|$, tunneling cannot occur because there are no final states available. The plot shows only the current in reverse bias, since this matters in devices. (Under forward bias the diode current is very large anyway and tunneling is not a concern.)

**NOTE:** The most important dependence of the Zener tunneling current on the applied reverse bias $V_a$ is via the depletion width $l_p$ appearing in the expression for the field $F_{max} = eN_A l_p / \varepsilon_s$. Many of you have forgotten this crucial dependence.

8. **Problem.** The current trends in VLSI technology also demand reduced applied biases (i.e., a smaller $V_a$). Without doing any calculation, do you think that the strictest breakdown limitations will be due to impact ionization or to Zener tunneling?
Solution. Zener tunneling will dominate, since the energy threshold for impact-ionization is $\sim E_G$, while for Zener tunneling is $E_G - eV_{bi}$, where $V_{bi}$ is the built-in potential of the drain/body junction in a MOSFET.

9. Problem. Using Eqns. (450) and (451), plot the total charge at the GaAs-side of the GaAs-Al$_x$Ga$_{1-x}$As heterojunction as a function of the interface potential $\psi_i = e\psi_i/(k_BT)$. Indicate clearly the accumulation, depletion, inversion, and strong inversion regions. Refer to the discussion on page 135 of the Notes for help.
Solution. The equations you were told to use account only for the charge of the majority carriers (electrons) and donors. They do not account for minority carriers (holes), so that it is impossible to obtain the charge in inversion (weak or strong), unless the hole charge-density is added. The charge density of the minority carriers is included in the equations for the charge at the interface of an MOS capacitor. So, one could use a similar equation, remembering that in that case we considered a $p$-type substrate, while here we are dealing with an $n$-type substrate, so the plot should be ‘flipped’ (as in the ‘mirror image’ $\psi_i \to -\psi_i$).

Solution. The potential energy is

$$V_{im}(z) = -\frac{e^2}{16\pi\epsilon_s z} - eF_z z.$$  \hspace{1cm} (9)

The maximum of the potential energy is given by

$$0 = \frac{dV_{im}(z)}{dz} = \frac{e^2}{16\pi\epsilon_s z^2} - eF_z.$$  \hspace{1cm} (10)

The value of $z$ at which this occurs is thus:

$$z_0 = \left(\frac{e}{16\pi\epsilon_s F_z}\right)^{1/2}.$$  \hspace{1cm} (11)
Inserting this into Eq. (9) we get:

$$V_{im}(z_0) = -\frac{e^2}{16\pi\epsilon_s z_0} - eF_z z_0 = - e \left( \frac{eF_z}{4\pi\epsilon_s} \right)^{1/2},$$

so that the barrier-lowering (in V) will be $\Delta \Phi_B = [eF_z/(4\pi\epsilon_s)]^{1/2}$.

11. **Problem.** Using the derivation of Eq. (480) as a guide, derive Eq. (482).

**Solution.** The expression given in class, Eq. (475) of the Notes, Part 2, for the tunneling current from the metal to the semiconductor is:

$$j_{MS} = \frac{em^*_M}{2\pi^2\hbar^3} \int_0^\infty dE E f_M(E) [1 - f_S(E)] \exp \left\{ -2 \int_0^{z_t} dz \kappa(z) \right\}.$$  \hspace{1cm} (13)

Approximating the potential barrier with $\phi(z) \approx \phi'_B - F zz$, we have for the tunneling distance $z_t = (e\phi'_B - E)/(eF_z)$ and:

$$\int_0^{z_t} dz \kappa(z) = \frac{(2m_S^*)^{1/2}}{\hbar} \int_0^{zt} dz (e\phi'_B - eF_z - E)^{1/2}.$$  \hspace{1cm} (14)

Using the new integration variable $y = e\phi'_B - eF_z - E$ we get:

$$\int_0^{z_t} dz \kappa(z) = \frac{(2m_S^*)^{1/2}}{e\hbar F_z} \int_0^{e\phi'_B-E} dy y^{1/2} = \frac{2(2m_S^*)^{1/2}}{3e\hbar F_z} (e\phi'_B - E)^{3/2},$$  \hspace{1cm} (15)

so that

$$j_{MS} = \frac{em^*_M}{2\pi^2\hbar^3} \int_0^\infty dE E f_M(E) [1 - f_S(E)] \exp \left\{ -\frac{4(2m_S^*)^{1/2}}{3e\hbar F_z} (e\phi'_B - E)^{3/2} \right\},$$  \hspace{1cm} (16)

which is Eq. (482).