ECE 584 Homework 6 solution

1. Use a Smith Chart to calculate the magnitude of the reflection coefficient in dB for a quarter-wave transformer designed to match a 300 ohm load to a 50 ohm transmission line at three frequencies:

   a. The design frequency.
   b. 10% higher than the design frequency
   c. 20% higher than the design frequency

Measuring Gamma off the Smith chart plots above (measure from center of chart to red square at left end of red line). $RL(dB) = 20 \log_{10} |\Gamma|$. 

Design frequency RL is –infinity dB.
10% higher, RL=–16.0 dB;
20% higher, RL=–10.4 dB.
2. Repeat problem 1 for a two stage transformer, using two quarter-wave sections. The first quarter-wave section should match 300 ohms to 122.5 ohms (the geometric mean of 50 and 300) and the second section should match 122.5 ohms to 50 ohms. Comment on the change in performance of the two stage transformer as compared to the single stage transformer.

**First step:** (see upper left Smith Chart). First transformer stage matches 300 ohms to 122.5 ohms. The characteristic impedance is of the first transformer is 
\[ Z_0 = \sqrt{300 \times 122.5} = 191.7 \text{ ohms}. \]
Plot 300 ohms normalized to 191.7 ohms =1.56 ohms (green box). Rotate through a quarter wavelength (using outer scale, not shown) to yellow box (blue contour).
Second step: (see upper right hand Smith Chart). Read off normalized impedance (.64, green square). Find true impedance \((191.7 \times .64) = 122.5\) ohms. Renormalize to next transformer impedance: \(Z_0 = \sqrt{122.5 \times 50} = 78.25\) ohms. \(122.5/78.25 = 1.56\) ohms (yellow square).

Third Step: (see lower left hand Smith Chart). Rotate through lambda/4 (green contour) to yellow square. Read off normalized impedance (.64). Find true impedance \((78.25 \times .64) = 50\) ohms. Renormalize to 50 ohm transmission line (plot red square in center).

Return Loss is –infinity dB.

Move frequency up 10%:

Return loss is –32 dB.
20% increase above design frequency:

Measure Gamma. Return loss is –20 dB.

The two stage transformer maintains good impedance match over a wider bandwidth than the single stage transformer. On the plot above, notice that the blue trace overshoots the real axis, which makes the load impedance at the input of the second transformer stage slightly inductive. The second transformer stage, which now rotates through more than lambda/4 (due to increased frequency) compensates for this inductance and ends very close to the real axis.