ECE609 Spring07

MID-TERM EXAM

1 Review of Modern Physics and Semiconductor Fundamentals [50pts]

3pt List three different type of physical effects on particles, which can only be explained using quantum mechanics.

* tunnelling effect
* confinement effect
* interference effect

2pt What are the two primary assumptions of the Bohr model.

* e^- moves along a circular trajectory around the proton (orbit)
* orbits are quantized

2pt Why are the energies in solids not discret as in an atom. Why are they not continuous, as is the case for a free electron?

* the wavefunctions can overlap between atoms (tunnelling effects) so the spectrum (energy) cannot be discrete
* In contrast to a continuum spectrum, the energy bands appear because the potential in solids is periodic. (For high energy, the e^- will not "see" the periodic potential anymore) $\Rightarrow$ continuum

2pt What two parameters are linked by Poisson's equation?

* n electron density [in fact ρ = charge density]
* V potential

2pt Describe the modeling procedure needed to determine energy bands in solids.

* one needs to solve the wavefunctions Ψ solution of Schrodinger-type equation.
* one needs to compute the electron density
* one needs to solve the Poisson equation

self-consistent process

3pt How does the conductivity of a solid depend on whether the energy bands are completely filled, partially filled or empty?

* Empty bands do not contain e^-. They do not contribute to the electrical conductivity of material [redacted]

* Partially filled bands contain e^- or holes. They do contribute to the electrical conductivity since states are available for e^-.

* Completely filled band do contain plenty of e^-, but do not contribute to the electrical conductivity [no states are available

2pt What are holes? Carefully justify your definition.

holes are missing e^-. They can be considered as positively charged particle (+q) located inside the valence band.

The notion of holes is in fact due to the combined behavior of all electrons inside the valence band.

3pt How many states are there in 1 micron sized solid for which an electron has a kinetic energy less than 1 eV? Treat the electron as a free electron confined to a 1D well with infinite potential walls.

$$E = \frac{1}{q}\frac{\hbar^2 \pi^2 n^2}{2mL^2} (eV) \qquad L = 1\times10^{-6} m. \qquad m_e = 9.1\times10^{-31} kg.$$

$$E = 3.765\times10^{-7} n^2 < 1$$

$$1 \leq n \leq 1629$$

So 1629 states are available

2pt What is the physical meaning of the Fermi energy?

at T=0K It is the highest energy level that could be occupied by an e

2pt What is the basic assumption used in statistical thermodynamics when calculating the probability distribution functions?

we could use a Maxwell boltzmann statistics instead of the Fermi Dirac statistics if $(E-E_F) > 3k_BT$ for e^- ("Non-degenerate semiconductors") $(E_F-E) > 3k_BT$ for h^+

3pt List the assumptions made to obtain the following equation:

$$n_0 = N_C \exp((E_F - E_C)/(k_BT))$$

* Maxwell-boltzmann statistics (Non degenerate semiconductors)
* Effective mass approximation
* (Born-Van Karman) Periodic B.C for Ψ at the edge of the solid
* 3D devices.

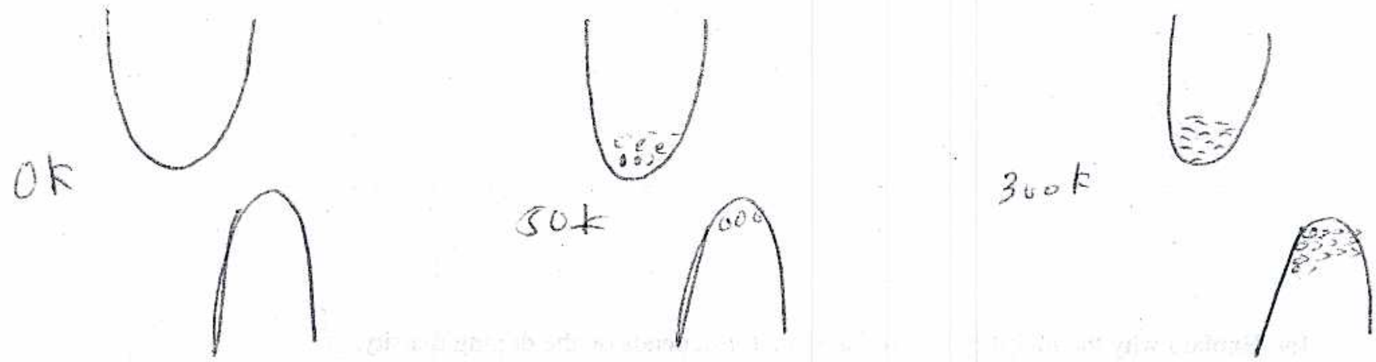
2pt What is the difference between an intrinsic semiconductor and an extrinsic semiconductor?

* Intrinsic SC = it is a pure SC - no impurities
* Extrinsic SC = it is a SC with Impurities or defects.

3pt What assumptions are made when deriving equation $n_0 \equiv N_d$?

* we have a N-type semiconductor $n_0 \gg p_0$ since $N_d \gg N_a$
* Complete Ionization since $N_d^+ \simeq N_d$
* neutrality condition

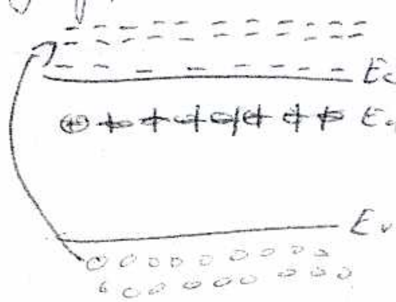
3pt Sketch the bandstructure (in k-space) of an semiconductor with an indirect bandgap near 0K, 50K and room temperature showing the location of any charge carriers.



4pt Describe and plot the temperature dependence of the carrier density in a N-type semiconductor ((n/N_D) in function of T) Identify the three regions and explain what happens by indicating the filled and empty states on an energy band diagram.

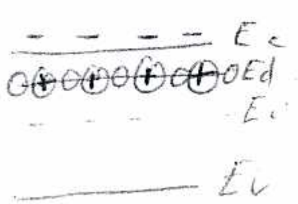
Intrinsic region

$n_i \sim N_D$ so semiconductor because intrinsic.
[e⁻ has ~~[illegible]~~ plenty of energy to jump from VB to CB]



Freeze out region

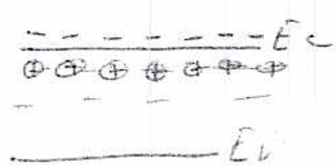
Not enough energy to ionize all the donors impurities



Extrinsic region

Complete ionization

$N_D^+ = N_D = n$



2pt Name and describe the two transport mechanisms in semiconductors.

* Drift - caused by electric field
* Diffusion - caused by concentration difference in the SC.

4pt Explain why the mobility in a semiconductor depends on the doping density. [sketch a graph].

μ of carriers

doping density

for high doping density $\mu \searrow$ due to ionized impurity scattering.

for low doping density μ is ~ cte and it is limited mainly by phonon scattering.

3pt List three recombination-generation mechanisms.

* Band to Band (1 step – direct transition)
* Trap-assisted (2 steps – indirect transition) $\Rightarrow$ SRH mechanism.
* Auger (3 particles)

3pt Describe the continuity equation in words.

The continuity equation describes the evolution of carrier concentration with time.
This change of carrier concentration with time is due to the difference between incoming and outgoing flux of carriers plus the generation and minus the recombination.

2 Practice [20pts]

6 Calculate in (eV) the energy bandgap E_G of GaAs at $T = 300K$ (You will NOT suppose that E_F is in the middle of the bandgap).

$$n = N_c \exp\left(-\beta\left(E_c - E_F\right)\right)$$

$$p = N_v \exp\left(+\beta\left(E_v - E_F\right)\right)$$

$$np = n_i^2$$

$$n_i^2 = N_c N_v \exp\left[-\beta\left[E_c - \cancel{E_F} - E_v + \cancel{E_F}\right]\right]$$

$$n_i = \sqrt{N_c N_v} \exp\left(-\beta \frac{E_g}{2}\right)$$

$$-\frac{2}{q\beta} \ln\left[\frac{n_i}{\sqrt{N_c N_v}}\right] = E_g\,(eV) \simeq \underline{1.437\ eV}$$

8 Consider a InAs where $E_G = 0.36eV$, electron effective mass is $0.023me$, ~~the hole effective mass is 0.4me~~. Find an ANALYTICAL expression of the intrinsic electron density of InAs in fonction of the intrinsic electron density of $GaAs$. Numerical Application.

$$\begin{cases} n_i \equiv n_{i\,GaAs} & N_C \equiv N_{C\,GaAs} \\ n_i' = n_{i\,InAs} & N_C' = N_{C\,InAs} \end{cases}$$

$$n_i = N_C \exp\left(-\beta \overbrace{(E_C - E_i)}^{E_G/2}\right)$$

$$n_i' = N_C' \exp\left(-\beta \frac{E_G'}{2}\right)$$

$$= 2\left[\frac{2\pi m^*_{InAs} k_B T}{h^2}\right]^{3/2} \exp\left[-\beta \frac{E_G'}{2}\right] \exp\left(-\beta\frac{E_G}{2}\right) \exp\left(\beta\frac{E_g}{2}\right)$$

$$n_i' = 2\left[\frac{2\pi m^*_{GaAs} k_B T}{h^2}\right]^{3/2} \left(\frac{m^*_{InAs}}{m^*_{GaAs}}\right)^{3/2} \exp\left(-\beta\left(\frac{E_G' - E_G}{2}\right)\right) \exp\left(+\beta\frac{E_G}{2}\right)$$

$$\boxed{n_i' = n_i \left(\frac{m^*_{InAs}}{m^*_{GaAs}}\right)^{3/2} \exp\left(-\beta\left(\frac{E_G' - E_G}{2}\right)\right)}$$

$$\boxed{n_i' = n_i \left(\frac{0.023}{0.067}\right)^{3/2} \exp\left(-\beta q\left(\frac{0.36 - 1.43}{2}\right)\right)}$$

$$= 1.73 \times 10^8\, n_i = \boxed{3.12 \times 10^{14}\ cm^{-3}}$$

6 Consider a GaAs P-N junction (ideal diode) with a reverse saturation current $J_s = 10^{-18} A$, calculate the applied bias potential required to obtain a current of $10mA$.

$$J = J_s \left[\exp(q\beta V_A) - 1\right]$$

$$\frac{J}{J_s} + 1 = \exp(q\beta V_A)$$

$$V_A = \frac{1}{q\beta} \ln\left[\frac{J}{J_s} + 1\right]$$

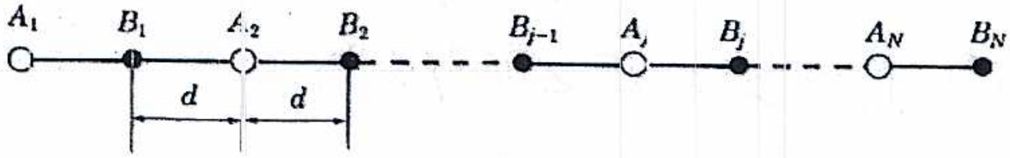
$J = 10 \times 10^{-3} A$

$V_A \simeq 0.958\ V$

$$\boxed{V_A \simeq 0.96 V.}$$

3 Problem [30pts]

20pt **1D A-B crystal:** The crystal is shown schematically below. The elementary cell has period $2d$. For a given atom of type A (resp. type B), only one energy state v_n^A (resp. v_n^B) is available. We would like to solve the Schrodinger equation: $H\Psi = E\Psi$. We propose to show that it appears two permitted energy band separated by an energy bandgap.



- Write $\Psi(x)$ a a linear combination of of basis functions v_n^A and. v_n^B respectively associated with the coefficients c_n and d_n.
- Show that c_n and d_n satisfy the following equations (we consider only the interactions between the first neighbors):

$$c_n E_0^A - d_n A - d_{n-1} A = E c_n$$

  and

$$d_n E_0^B - c_{n+1} A - c_n A = E d_n$$

  where you will give the expression of E_0^A (on site energy for atom A), E_0^B (on site energy for atom B) and A (coupling term).
- We consider solutions of this form: $c_n = \alpha \exp(ikn2d)$ and $d_n = \beta \exp(ikn2d)$, where k belongs to the first Brillouin zone. Derive the dispersion relation $E(k)$ in function of $\cancel{\alpha}$, β, E_0^A, E_0^B, A (Hint: Put the previous expressions into a matrix form and look at the determinant).
- Plot the dispersion relation considering $E_0^A < E_0^B$. What is the value the bandgap ?

$$* \quad \Psi(x) = \sum_{n=1}^{N} c_n v_n^A(x) + \sum_{n=1}^{N} d_n v_n^B(x)$$

Re: (v_n^A, v_{n-1}^B), (v_n^A, v_n^B) First neighbors, (v_n^B, v_{n+1}^A), (v_n^B, v_n^A)

* pre-multiply by v_n^A and Integrate

$$\langle v_n^A | H | \Psi \rangle = E \langle v_n^A | \Psi \rangle$$

$$c_n \langle v_n^A | H | v_n^A \rangle + d_n \langle v_n^A | H | v_n^B \rangle + d_{n-1} \langle v_n^A | H | v_{n-1}^B \rangle = E c_n \quad (\text{since } \langle v_n^A | v_n^A \rangle = 1)$$

$$\Rightarrow \boxed{c_n E_0^A - d_n A - d_{n-1} A = E c_n}$$

where
$$\left[\begin{array}{l} E_0^A = \langle \nu_n^A | H | \nu_n^A \rangle \\ A = -\langle \nu_n^A | H | \nu_n^B \rangle = -\langle \nu_n^A | H | \nu_{n-1}^B \rangle \end{array}\right.$$

(ii)
$$\langle \nu_n^B | H | \psi \rangle = E \langle \nu_n^B | \psi \rangle$$

$$d_n \langle \nu_n^B | H | \nu_n^B \rangle + c_{n+1} \langle \nu_n^B | H | \nu_{n+1}^A \rangle + c_n \langle \nu_n^B | H | \nu_n^A \rangle = E d_n .$$

$$\Rightarrow \boxed{d_n E_0^B - A c_{n+1} - A c_n = E d_n}$$

where
$$\left[\begin{array}{l} E_0^B = \langle \nu_n^B | H | \nu_n^B \rangle \\ A = -\langle \nu_n^B | H | \nu_{n+1}^A \rangle = -\langle \nu_n^B | H | \nu_n^A \rangle . \end{array}\right.$$

$$* \quad c_n = \alpha e^{ikn2d} \qquad d_n = \beta e^{ikn2d}$$

$$k \in \left[-\frac{\pi}{2d} ; \frac{\pi}{2d} \right[$$

<u>First equation</u>

$$\alpha e^{ik2nd} E_0^A - \beta e^{ikn2d} A - \beta e^{ik(n-1)2d} A = E \alpha e^{ikn2d}$$

$$\alpha E_0^A - \beta(1+e^{-ik2d})A = E\alpha.$$

$$\boxed{\alpha\begin{pmatrix}E_0^A \\ -E\end{pmatrix} - \beta e^{-ikd}(2\cos kd)A = 0}$$

Second equation

$$\beta e^{ikn2d}E_0^B - \alpha e^{ik(n+1)2d}A - \alpha e^{ikn2d}A = E e^{ikn2d}\beta$$

$$\Rightarrow \beta E_0^B - \alpha(1+e^{ik2d})A = E\beta$$

$$\Rightarrow \boxed{\beta(E_0^B - E) - \alpha e^{+ikd}(2\cos kd)A = 0.}$$

Matrix form

~~E_0^A ... $-(1+e^{ika}$~~

$$\begin{pmatrix} (E_0^A - E) & (-e^{-ikd}2\cos(kd)A) \\ (-\alpha e^{ikd}2\cos(kd)A) & (E_0^B - E) \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}0 \\ 0\end{pmatrix}$$

Non trivial Solution if determinant $= 0$

Determinant $=$ $\boxed{(E_0^A - E)(E_0^B - E) - 4A^2\cos^2 kd = 0}$

$$E^2 - E\left(E_0^B + E_0^A\right) + E_0^A E_0^B - 4A^2 \cos^2 kd = 0.$$

<u>Second order equation</u> $\quad ax^2+bx+c=0$

solutions $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$ $\quad \Delta = b^2 - 4ac$

$$\Delta = \left(E_0^B + E_0^A\right)^2 - 4\left(E_0^A E_0^B - 4A^2 \cos^2 kd\right) = 0.$$

$$\Delta = \left(E_0^B - E_0^A\right)^2 + 16 A^2 \cos^2 kd$$

$$E(k) = \frac{\left(E_0^A + E_0^B\right) \pm \sqrt{\left(E_0^B - E_0^A\right)^2 + 16A^2\cos^2 kd}}{2}.$$

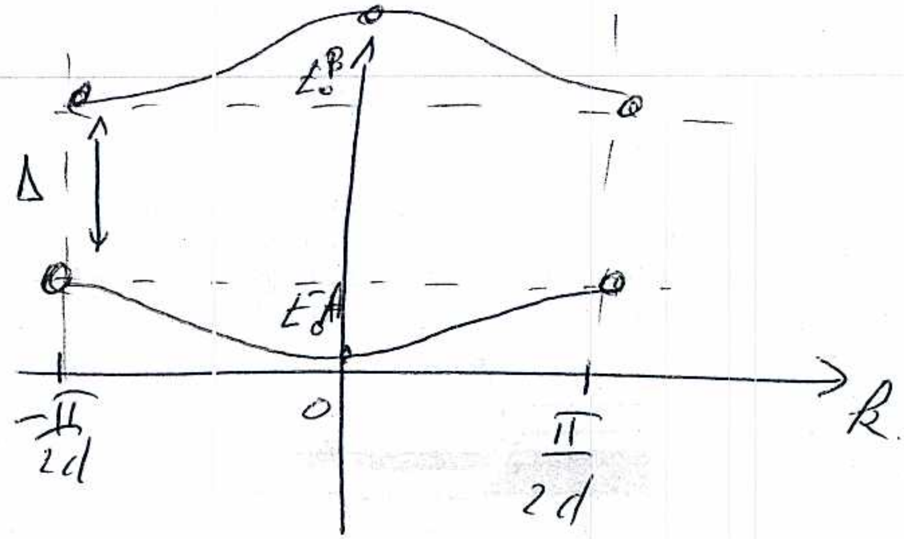
$E_0^B - E_0^A > 0$

* Let us plot in the first Brillouin zone.

if $k = \pm\dfrac{\pi}{2d}$ $\quad E_+ = E_0^B$

$E_- = E_0^A$

if $k = 0$ $\quad E_\pm = \dfrac{\left(E_0^A + E_0^B\right) \pm \sqrt{\left(E_0^B - E_0^A\right)^2 + 16A^2}}{2}$



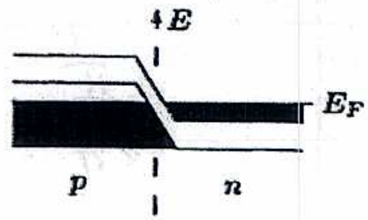
gap?

$$\Delta = \min_k |E^+(k) - E^-(k)| = \min_k \sqrt{(E_0^B - E_0^A)^2 + 16A^2\cos^2 kd}$$

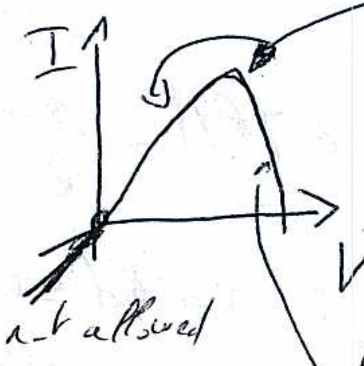
minimum if $\cos kd = 0$ $\quad k = \pm \frac{\pi}{2d}$.

$$\Rightarrow \boxed{\Delta = E_0^B - E_0^A}$$

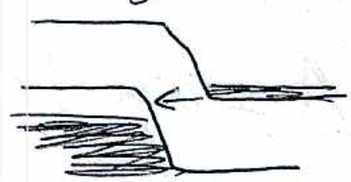
10pt When semiconductors are heavily doped, the depletion region at the junction can become quite narrow. In such situation (see Figure below), tunneling can occur when possible. Without any calculations, plot the I-V curves for such device (You will first pictorially describe what is happening if a small then large forward bias is applied).



tunneling current



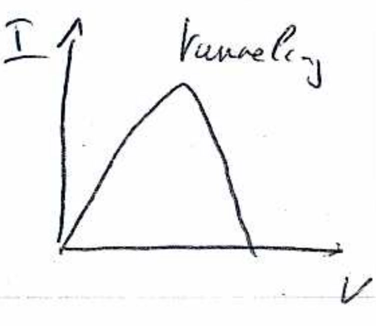
Small bias voltage, tunneling is allowed



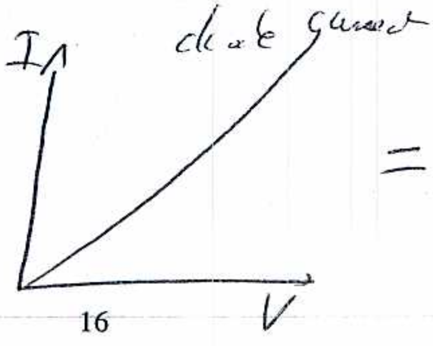
tunneling is not allowed at V=0

Large bias voltage, tunneling is not allowed anymore and device now acts as a regular diode.

The I-V characteristics of the tunnel diode ~~are~~ made up of a tunneling part and a conventional diode part



+



=