ECE609 Spring07

FINAL EXAM

INSTRUCTIONS

- This examination is TWO HOURS in length. Open Notes and Books.
- Give your answers in the provided sheet, NO ADDITIONAL paper sheets will be graded
- YOU WILL MAKE USE of the following numerical values, also if it is not specified in the text you will consider $T = 300K$:

  Fundamental constant:

$$
\begin{aligned}
&m_e = 9.1 * 10^{-31} kg \\
&h = 6.63 * 10^{-34} J.s \\
&\hbar = 1.054 * 10^{-34} J.s \\
&c = 3.0 * 10^{8} m/s \\
&q = 1.6 * 10^{-19} C \\
&\epsilon_0 = 8.85 * 10^{-12} F/m \quad \text{with } 1F = 1A^2/kg/m^2 \\
&1/(q\beta) = (k_B T/q) \equiv 0.026 eV \quad \text{at } T = 300K \\
&\epsilon_{ox} = 3.9\epsilon_0
\end{aligned}
$$

Semiconductor properties at $T = 300K$:

	$E_g(eV)$	$n_i(cm^{-3})$	$N_c(cm^{-3})$	$N_v(cm^{-3})$	m_e^*	m_p^*	ϵ_r
Si	1.11	$1.5 * 10^{10}$	$2.8 * 10^{19}$	$1.0 * 10^{19}$	$1.1 m_e$	$0.56 m_e$	11.8
$GaAs$	1.43	$1.8 * 10^{6}$	$4.7 * 10^{17}$	$7.0 * 10^{18}$	$0.067 m_e$	$0.48 m_e$	13.2

Good Luck !!

1 Review of Two and Three Terminal Devices [50pts]

P-N junctions(10pts)

- What mechanisms cause current in a p-n junction?
- How does one calculate the current in a p-n junction?
- What is the full depletion approximation? Why do we need the full depletion approximation?

Metal-Semiconductors contacts(10pts)

- Complete the following table either using $qV_M > qV_{sc}$ or $qV_M < qV_{sc}$ (qV_M and qV_{sc} being respectively the work functions of the metal and the semiconductor).

	Schottky barrier	Ohmic contact
N-type semiconductor		
P-type semiconductor		

- What is then happening if the concentration of doping increases significantly ?
- What are the expressions of the built-in potential for a Schottky contact with a N-type and P-type semiconductors (give expressions in function of qV_M, E_g, n_i, β, N_d or N_a) ?

MOS capacitors(10pts)

- What are the basic assumptions regarding the charge in the accumulation and inversion layers (in terms of majority and minority carriers)?
- Let us consider the MOS transistor configurations given in Fig. 1 and Fig. 2. Complete the following table indicating what it is happening in the different cross sections.

	at $x = d$ or $x = -d$	$x = 0$
Figure 1 (with $V_G > V_T$ $V_T > 0$)		
Figure 2 (with $V_G < V_T$ $V_T < 0$)		

- What is happening if the oxide thickness decreases significantly ?

Heterojunctions(10pts)

Let us consider a III-V semiconductor composed by following layers (from gate to substrate) GaAs, n-AlGAAs, AlGaAs, GaAs. Describe what is then happening in Figure 3 if we look at the potential below the surface. Comments. Also what is the point of studying such devices ?

MOS transistors(10pts)

- What is the difference between the linear and the quadratic MOSFET model?
- Complete the following "step-by-step' table going from top to bottom.

step by step events ($V_G = 0$, $V_{DS} = 0$)	What happens for I_{DS}?	Comments
$0 < V_G < V_T$		
$0 < V_{DS} < V_G - V_T$		
$V_G > V_T$		
$V_{DS} > V_G - V_T$		

- List the three main limitations of the quadratic model.
- What is happening for the threshold voltage V_T if the channel length decreases significantly ?

2 Exercises (20pts)

- A light laser radiation is shining on a GaAs semiconductor with a 900nm wavelength, can we assume the creation of an electron-hole pair ? What is happening with a Silicon semiconductor ?

- Using the quadratic model and for the biases VDS = 2.5V and then V_{DS} = 3.0V , calculate the drain current of a silicon n-MOSFET with $V_T = 0.26V$, W = $10\mu m$, L = $1\mu m$, t_{ox} = 30nm, $V_{GS} = 3V$. Use a surface mobility of $300 cm^2/Vs$ (we also consider that the source and substrate are grounded). What is now happening if one considers the velocity saturation, calculate the new drain current for the two different values of VDS (We consider for the critical value of the electric field $E_c = V_{DS}/L$). Comments.

3 Problem - Introduction to PIN Diode (30pts)

The pin (p-i-n) diode, generally made of silicon, is derived from the p-n junction adding an undoped silicon zone (i for intrinsic) of width $\Delta \sim 100\mu m$ between the p and n regions. Lacking an analytical solution of the band bending in an i region, we attempt to understand it intuitively in the next questions.

1. Let us first consider a abrupt P-N junction first. Using the full depletion approximation, give the expression of the charge Q per unit of area stored as fixed charges in the band-bending regions on either side of the metallurgical junction in function of $V_i - V_A$ (built-in minus applied potential) and the doping N_a and N_d.

2. A simple approximation to a p-i junction is a p-n junction with $N_d << N_a$ (a $p - n^-$ junction). Using the above result, what can you say about the width of the space charge region zone in the n^- region compared to the width of the corresponding zone in the p region ? In which spatial region does most of the variation of the electrostatic potential occur (justify your answer)?

3. Consider next the behavior of the space charge zone in an intrinsic semiconductor. Write down the Poisson equation and the equation expressing the charge density as a function of the electrostatic potential (we consider a potential difference V with respect to the intrinsic region), and deduce the differential equation satisfied by the potential (in function of n_i).

4. The above equation has no analytical solution. Nonetheless, one can estimate the order of magnitude of the characteristics distance over which the bands curve. To this end, rewrite this equation in an approximate form in the limit of very small potential variation and give a general solution in terms of a characteristic distance λ (you may need also to include the constant coefficients α, β). Express the characteristics distance λ (the "screening length" or Debye length) as a function of the intrinsic equilibrium carrier density $2n_i$. Calculate λ for silicon at room temperature.

5. Suppose that the above results remain valid for $V > k_B T/q$, sketch and explain the energy band diagram for a pin diode at equilibrium.

1) Review

I) P-N junctions

3 (1) Drift and Diffusion - Diffusion if forward Bias
Drift if Reverse Bias.

2 (2) one can solve the Drift-Diffusion set of equation.

3 (3) • one does not consider the charge due to the free carriers in the depletion region and we suppose that the electric field $\vec{E}$ vanishes outside the depletion region.

2 • the Depletion approximation needs to be used if one seeks an analytical solution of the Poisson equation inside the depletion region.

II) Metal Semiconductors contacts

	Schottky	Ohmic
N-type	$qV_m > qV_{sc}$	$qV_m < qV_{sc}$
4 P-type	$qV_m < qV_{sc}$	$qV_m > qV_{sc}$

• if N_D or $N_A \gg 1$,

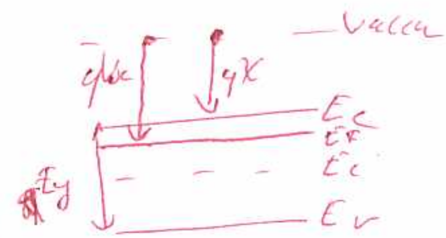
2 the depletion region width $W \ll 1$

tunneling probability becomes important for Schottky barrier

$\Longrightarrow$ can be assimilated as low-resistance Ohmic contact.

• N-type

$qV_i = qV_M - qV_{sc}$.



2 $qV_{sc} = q\chi + q\frac{E_g}{2} - (E_F - E_i)$

$E_F - E_i = +\frac{1}{\beta q}\ln\left(\frac{N_d}{n_i}\right)$

$$\left[qV_i = q\left(V_M - \chi - \frac{E_g}{2} + \frac{1}{\beta q}\ln\frac{N_d}{n_i}\right)\right.$$

2° P-type $V_{sc} = \chi + \frac{E_g}{2} + (E_i - E_F)$ $\quad (E_i - E_F) = \frac{1}{\beta q}\ln\left(\frac{N_A}{n_i}\right)$

$$\left[qV_i = q\left(V_M - \chi - \frac{E_g}{2} - \frac{1}{\beta q}\ln\left(\frac{N_A}{n_i}\right)\right)\right.$$

I MOS capacitors

• an accumulation layer is formed by concentration of majority carriers at the surface.

• an inversion layer is formed by concentration of minority carriers at the surface.

6	$x = d$ or $-d$	$x = 0$
Fig 1 $V_G > V_T$ $V_T > 0$	Accumulation (h^+)	Inversion. (e^-)
Fig 2. $V_G < V_T$ $V_T < 0$	Accumulation (e^-)	Inversion (h^+).

- if $t_{ox} \ll 1$, tunneling effect becomes important from semiconductor to the gate ⇒ gate leakage. (physical limitation of scaling).

IV Heterojunctions

Gate
GaAs
n-AlGaAs
AlGaAs
GaAs (substrate)

- Below the surface where there's no gate, a 2DEG is formed at the heterojunction AlGaAs / GaAs.
- Below the surface where there are gates, no electron gas is formed at the [illegible] $V_G < 0$

- if $V_G < 0$ the e^- are going to leave the region below the gate, to concentrate below the region where there is n-gate. The 2DEG at the hetero-junction will "reproduce" the geometry ~~of~~ drawn by the gate on the top. (4)
- we expect from such device to reproduce the basic properties of electromagnetism since now interference effects becomes important in the 2DEG. Also the mobility of this electron gas is very high since 2D.

I MOS Transistors (15) +5

- the linear model does not consider any space charge variation along the channel. While the quadratic model considers the variation of the inversion layer charge with the V_{DS} bias.

- step by step $V_G = 0$, $V_{DS} = 0$.

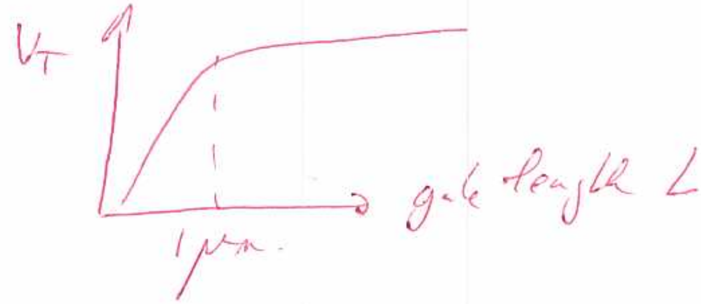
	I_{DS}	Comment
$0 < V_G < V_T$	$I_{DS} = 0$	[no inversion layer is formed, only depletion layer.
$0 < V_{DS} < V_G - V_T$	subthreshold current $I_{DS} \neq 0$	[the subthreshold current is independent of V_{DS}.

	I_{DS}	Comments
$V_G > V_T$	I_{DS} follows the quadratic model I-V characteristics	inversion layer is formed.
$V_{DS} \geq V_G - V_T$	I_{DS} saturates	the current is independent of V_{DS}.

Main limitations of the quadratic model

- Not valid after saturation $V_{DS} > V_G - V_T$
- Does not explain the subthreshold characteristics $V_G < V_T$
- Does not account for the variation of the depletion layer along the channel.

- if $V_T \ll 1$ this means that $L \ll 1$



$\Rightarrow$ short device.
$\Rightarrow$ short channel effect.

2) Exercises (20 pts)

6

I) $\lambda = 900\,nm.$

4 $E_h = h\nu = \frac{hc}{\lambda} = 1.38\,eV.$

$E_h < E_{g\,GaAs} \Rightarrow$ ~~[illegible]~~ e^--hole pair cannot be created

and $E_h > E_{g\,Si} \Rightarrow$ creation of e^--hole pairs 2 /2

II) $V_{DS} = 2.5\,V$; $V_{DS} = 3.0\,V$ $V_T = 0.26\,V$ $V_G = 0.3\,V$

16 First what is V_{Dsat}?

$\boxed{V_{Dsat} = V_G - V_T = 2.74\,V}$ /2

For $V_{DD} = 2.5\,V$

no saturation $I_D = \mu C_{ox} \frac{W}{L}\left[(V_G - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$C_{ox} = 0.000115\,F/m^2.$

$\boxed{I_D = 1.285\,mA}$ /3

• For $V_{DS} = 3.0V$

Saturation regime. $V_{DS} > V_{DSat}$

$$I_{D,sat} = \mu C_{ox} \frac{W}{L} \frac{(V_G - V_T)^2}{2}$$

$$\boxed{I_{D,sat} = 1.295\,mA}$$

/3

with velocity saturation

$E_c = V_{DS}/L$

new Drain saturation voltage.

$$V'_{Dsat} = L E_c \left[\sqrt{1 + \frac{2(V_G - V_T)}{L E_c}} - 1\right] = V_{DS}\left[\sqrt{1 + \frac{2(V_G - V_T)}{V_{DS}}} - 1\right]$$

$$\boxed{= 1.96V}$$

/3

Since $V_{DS} > V'_{Dsat} \Rightarrow$ saturation regime for both $V_{DS} = 2.5V$, $V_{DS} = 3.0V$.

$$\boxed{I'_{Dsat} = \frac{W}{L}\left(\frac{\mu_n}{1 + \frac{V'_{Dsat}}{V_{DS}}}\right) C_{ox}\left[(V_G - V_T)\, V'_{Dsat} - \frac{V'^2_{Dsat}}{2}\right] = 0.667\,mA}$$

/3

• If one accounts for velocity saturation, the drain saturation current is reduced. /2

Problem PIN Diode 30/30. (8)

(1) abrupt P-N junction first

| P | N |

$$|Q| = q\ell_p N_A = q\ell_n N_D = q\left[\frac{2\epsilon(V_i - V_A)}{q}\frac{N_A N_D}{N_A + N_D}\right]^{1/2}$$ /5

(2) $N_D << N_A$

$\Rightarrow \frac{N_A}{N_D} >> 1 \Rightarrow \frac{\ell_n}{\ell_p} >> 1$

so the space charge region is much larger on the weakly doped side N. /5

$$V(x) = \begin{cases} -\frac{qN_D}{2\epsilon_s}(x-\ell_n)^2 + V_{no} & \text{on the N side} \\ \frac{qN_A}{2\epsilon_s}(x+\ell_p)^2 + V_{po} & \text{on the P side} \end{cases}$$

so total variation of potential

$+\frac{qN_D}{2\epsilon_s}\ell_n^2$ on the N side $>>$ $\frac{qN_A}{2\epsilon_s}\ell_p^2$ on the P side $\Rightarrow$ Most of the potential variation occurs on the low-doped side N.

/5

3 Intrinsic regime

5

$$-\frac{d^2V}{dx^2} = \frac{\rho}{\epsilon} \quad \text{where } \rho = q(-n+p).$$

Boltzmann relationship $\Rightarrow$ $n = n_i \exp(q\beta V)$

$p = n_i \exp(-q\beta V)$

$$\Rightarrow \quad \frac{d^2V(x)}{dx^2} = q\frac{n_i}{\epsilon}\left(+\exp(q\beta V) - \exp(-q\beta V)\right)$$

$$\boxed{\frac{d^2V(x)}{dx^2} = \frac{2n_i q}{\epsilon} sh(q\beta V)}$$

non-linear PDE

4 if $V \ll k_BT$ $\left(V \ll \frac{1}{\beta}\right)$.

5

it comes

$$\boxed{\frac{d^2V}{dx^2} = \frac{2n_i q^2\beta V}{\epsilon}}$$

solution

$$V = \alpha \exp\left(\frac{x}{\lambda}\right) + \beta \exp\left(-\frac{x}{\lambda}\right) \quad \text{with } \lambda = \left(\sqrt{\frac{2n_i q^2\beta}{\epsilon}}\right)^{-1}$$

distance unit

$\lambda \simeq 24\,\mu m$ (screening distance)

5 10