## ECE609 Spring06 <br> Mid-Term Exam

## 1 Review of Modern Physics (20pts)

- Calculate the typical de Broglie wavelengths for (i) 0.1 kg bullet at $100 \mathrm{~m} / \mathrm{s}$ (ii) electron at $10^{6} \mathrm{~m} / \mathrm{s}$. Comment your results in terms of classical and quantum mechanical expectations (we give $h=$ $6.626 * 10^{-34} \mathrm{~J} . \mathrm{s}$ and $m_{e^{-}}=9.109 * 10^{-31} \mathrm{~kg}$ )
- Where is the most probable location to find an electron in the second energy state in an infinite square well ? (Hint: the wave function inside the well are givien by $\Psi_{n}(x)=A \sin (n \pi x / L), L$ being the size of the well)
- We consider a potential barrier $U$ (Fig 1), and we consider one electron injected from the left to the right. For $E>U$ and then for $0<E<U$, sketch what is happening for this electron in both classical and quantum pictures (you will sketch 4 independent graphs). Extract from the course two physical examples of potential barrier.


Figure 1: Potential barrier

- Calculate the energy relative to the Fermi energy for which the Fermi function equals $5 \%$. Write the answer in units of $k_{B} T$.


## 2 Energy Band Theory (15pts)

- We consider a crystal with a periodic potential. The first and second form of the Bloch theorem are given by (i) $\Psi_{k}(\mathbf{r}+\mathbf{R})=e^{\mathrm{ikR}} \Psi_{k}(\mathbf{r})$, where $\mathbf{R}$ has the periodicity of the crystal, and (ii) $\Psi_{k}(\mathbf{r})=$ $e^{\mathbf{i k r}} u_{k}(\mathbf{r})$ with $u_{k}(\mathbf{r}+\mathbf{R})=u_{k}(\mathbf{r})$. From (ii) you will show (i). From (i) you will show (ii).
- The Schrödinger equation is given by:

$$
-\frac{\hbar^{2}}{2 m} \Delta_{r} \Psi(r)+U(r) \Psi(r)=E \Psi(r)
$$

where $U(r)$ is periodic. Using the second form of the Bloch theorem, derive the partial differential equation for $u_{k}$.

## 3 Semiconductor Fundamentals (20pts)

- Jeopardy: Find the questions of the following answers:

1. It is a pure semiconductor
2. It is the probability that an allowed energy E, will be occupied by a hole.
3. Electron can then jump from the valence band to the conduction band
4. It has intentionally added dopants to control the number of charge carriers
5. It is an insulator with a very narrow bandgap
6. At finite temperature, it is a conductor with very large resistivity
7. Because we want to increase the electron concentration
8. Because we want to increase the hole concentration
9. The band-gap will be reduced

- Calculate the temperature at which there is a $1 \%$ probability that a state 0.30 eV below the Fermi level will be empty of one electron (we give $k_{B} T=25.9 \mathrm{meV}$ for $T=300 \mathrm{~K}$ ).
- Evaluate the approximation errors if one uses the Maxwell-Boltzmann statistics instead of the FermiDirac statistics, for the following energies: $E-E_{F}=2.2 k_{B} T, E-E_{F}=2.9 k_{B} T, E-E_{F}=4.6 k_{B} T$


## 4 Theory of Electrical Conduction (35pts)

1. We consider a N-type doped semiconductor going from $x=0$ to $x=W$. Holes minority carriers are injected at $x=0$ with the density $p(0)$. All the excess minority carriers are extracted at $x=W$ such that $p(W)=p_{0}$.

- Using the continuity equation and the definition of the hole current density, derive an equation for $p(x)$. We will consider SRH processes for intrinsic generation recombination.
- If we assume (i) steady state condition, (ii) no electric field, (iii) no generation due to an external source of energy, write the new equation for $p(x)$ (we will introduce $L_{p}$ the diffusion length)
- We consider the following general solution for the above equation:

$$
p(x)-p_{0}=A \exp \left(-x / L_{p}\right)+B \exp \left(x / L_{p}\right)
$$

Find the values of A and B. Write the new expression for $p(x)$. What is happening if $W \ll L_{p}$ and $W \gg L_{p}$. Comment.

- Give the expression of the hole current density at $x=W$. What is happening if $W \ll L_{p}$ and $W \gg L_{p}$. Comment.

2. Assume that a GaAs semiconductor is completly pure, what is the resistivity at $\mathrm{T}=300 \mathrm{~K}$ ? We give the band gap $E c-E v=1.4 \mathrm{eV}$, effective mass of the electron $m_{e}=7 * 10^{-2} \mathrm{~m}$, effective mass of hole $m_{h}=0.5 m$ (where $m$ is the mass of the electron), electron mobility $\mu_{e}=8500 \mathrm{~cm}^{2} V^{-1} \mathrm{~s}^{-1}$, hole mobility $\mu_{h}=400 \mathrm{~cm}^{2} V^{-1} \mathrm{~s}^{-1}$. Also, you will make use of the following numerical value:

$$
N_{0}=\frac{2}{\hbar}\left(2 \pi m k_{B} T\right)^{3 / 2}=2.5 * 10^{25} m^{-3}
$$

## $5 \quad$ P-N junctions (10pts)

- Sketch the variation of the conduction band, valence band, Fermi-level, Intrinsic Fermi-level of the P-N junction. Sketch the direction of the drift and diffusion currents both for the electrons and holes.
- We set $N_{d}=10^{19} \mathrm{~cm}^{-3}, l_{p}=1.03 \mu \mathrm{~m}$ (size of the depletion region in the P-region), what is the value of $l_{n}$ if we consider $N_{a}=10^{15}$ ? Comment.

