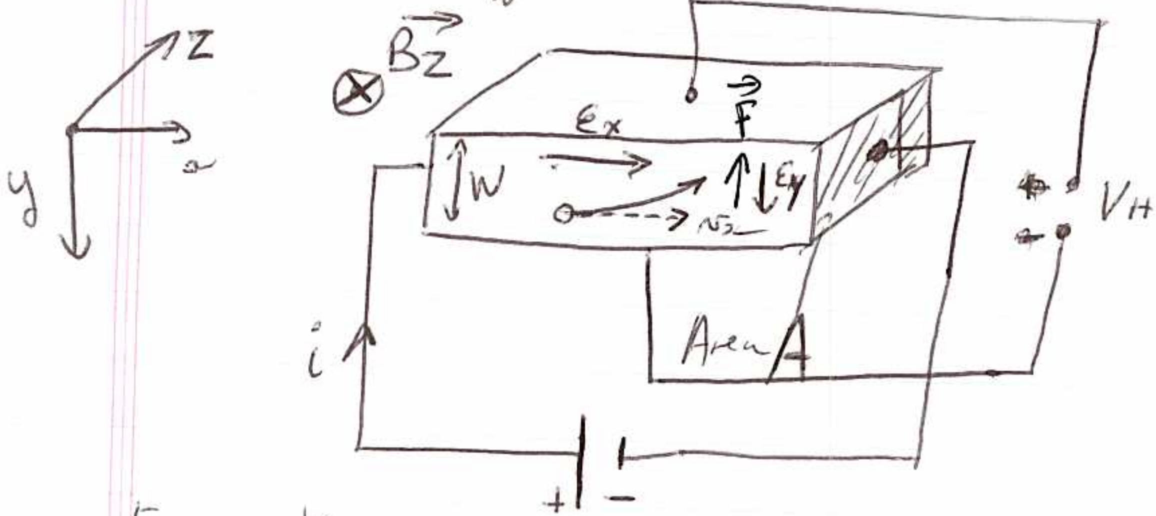
c) The Hall effect



p-type semiconductor

Experiment

in order to ~~measure~~ obtain the mobility of the carrier and carrier concentration.

- a magnetic field $\vec{B}$ is applied $\perp$ to the direction of the carrier flow $\Rightarrow$ a potential difference appears in the direction $\perp$ to both the current flow and $\vec{B}$. ~~(right hand rule)~~
- in the y-direction, no current can flow, it appears an Electric force which exactly counteracts the Lorentz force (obtained by right hand rule)

$\Rightarrow$ Hall field

it comes $\Rightarrow$ $\mathcal{E}_y = v_x B_z$

since $\boxed{\vec{J_p} = q p \mu_p \vec{\mathcal{E}} = q p \vec{v_p}}$

$$\Rightarrow \boxed{\mathcal{E}_y = \left[\frac{J_p}{qp}\right] B_z = R_H J_p B_z}$$

with $R_H = \dfrac{1}{qp}$ the Hall coefficient. $\left[\text{if } e^- \quad R_H = \dfrac{-1}{qn}\right]$

The conductivity for P-type semiconductor is

$$\sigma_p = q\,\mu_p\,p \quad \text{using Hall coefficient definition}$$

$$\Rightarrow \quad \mu_p = R_{H,p}\,\sigma_p \qquad \left[\text{or } \mu_n = -R_{H,n}\,\sigma_n \text{ for } n\text{-type device}\right]$$

The mobility of the carriers can be obtained using a conductivity ~~[illegible]~~ measurement and a Hall effect measurement.
(measure of E_y ($\equiv V_H$) and J_p)

Once μ_p is known we get the carrier concentration using (*) or $R_H = \frac{1}{qp}$

4) Carrier diffusion

Carriers diffuse from regions where the density is high to regions where the density is low (carrier density gradient)

Diffusion of carriers can be obtained:

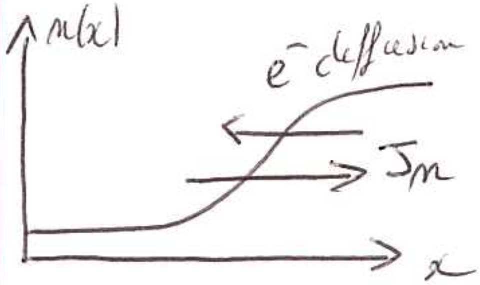
- by varying the doping density in a semiconductor
- by applying a thermal gradient.

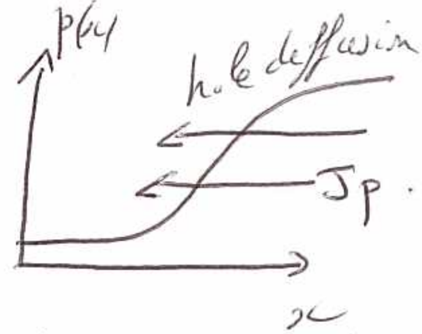
* The flux of electrons ϕ_n (or holes ϕ_p), resulting from the diffusion process is directly proportional to the e^- concentration gradient dn/dx.

$$\phi_p = -D_p \frac{dp}{dx} \qquad \phi_n = -D_n \frac{dn}{dx}$$

for e^- we get $\quad J_n = -q\phi_n = qD_n\frac{dn}{dx}$

for holes $\quad J_p = q\phi_p = -qD_p\frac{dp}{dx}$





$D_n, D_p \Rightarrow$ Diffusion constants for e^- and holes

(5) Drift – Diffusion equation

(a) Total current

Obtained by adding the current due to diffusion to the drift current, resulting in:

$$\boxed{\begin{aligned} J_n &= qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} \\ J_p &= qp\mu_p\mathcal{E} + qD_p\frac{dp}{dx} \end{aligned}}$$

Rq in 3D: $\frac{dn}{dx} \to \nabla n$, $J_n \to \vec{J}_n$, $\mathcal{E} \to \vec{\mathcal{E}}$

The total density of the current flowing at any point in the semiconductor is simply: $\boxed{\vec{J} = \vec{J}_n + \vec{J}_p}$

b) Einstein relationships

relation between μ and D ;

we assume :

- equilibrium conditions $\Rightarrow$ $J_n = qn\mu_n \mathcal{E} + qD_n \frac{\partial n}{\partial x} = 0$
- non - degenerate semiconductor $\Rightarrow$

$$n = N_c \exp\left[\frac{E_F - E_c}{k_B T}\right] \rightarrow n = n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]$$

we get

$$\frac{\partial n}{\partial x} = \left(\underbrace{\frac{\partial E_F}{\partial x}}_{0} - \underbrace{\frac{\partial E_i}{\partial x}}_{q\mathcal{E}}\right) \frac{1}{k_B T} \underbrace{n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]}_{n}$$

$\boxed{\text{Since}}$ $\mathcal{E} = -\nabla V$

$-qV = E_i$ (E_i potential energy)

$\mathcal{E} = \frac{1}{q} \nabla E_i$

$$\Rightarrow \boxed{\frac{\partial n}{\partial x} = -\frac{q}{k_B T} n \mathcal{E}}$$

Since (equilibrium condition) $qn\mu_n \mathcal{E} + qD_n \left[\frac{-q}{k_B T}\right] n \mathcal{E} = 0$

$$\Rightarrow \boxed{D_n = \frac{k_B T}{q} \mu_n} \quad \text{also} \quad \boxed{D_p = \frac{k_B T}{q} \mu_p}$$

6 Transport equations

In addition to the drift diffusion equation, the transport model is also defined by

the carrier density and Poisson equations, the continuity equation

a) Carrier density and Poisson equations

In order to calculate J_n and J_p, one needs to know n, p and $\mathcal{E}$. $\mathcal{E}$ is obtained from the electrostatics potential V, $\mathcal{E} = -\nabla V$.

From Maxwell equation $\Longrightarrow$ Poisson equation.

$$\boxed{-\Delta V_{(\vec{r})} = +q\left(-n_{(\vec{r})} + p_{(\vec{r})} + Nd_{(\vec{r})} - Na_{(\vec{r})}\right)}$$

Nd, Na are known.
V depends on n and p, however n and p depends also on $V \Longrightarrow$ self-consistent problem.

* For non-degenerate semiconductor

$$n(\vec{r}) = n_i \exp\left[\frac{E_f - E_i(\vec{r})}{k_B T}\right]$$

$$p(\vec{r}) = n_i \exp\left[\frac{-E_f + E_i(\vec{r})}{\ldots}\right]$$

Because of the electric field in the semiconductor.

$E_i \to \underbrace{E_{i0} - qV}$ potential energy due to the field

E_{i0} is taken as reference (see alignement of Fermi level case in previous section)

$$\Rightarrow \left[\begin{array}{l} n(\vec{r}) = n_i \exp\left[\dfrac{E_F - E_{i0}}{k_B T}\right] \exp\left(\dfrac{qV}{k_B T}\right) \\ p(\vec{r}) = n_i \exp\left[\dfrac{-E_F + E_{i0}}{k_B T}\right] \exp\left[-\dfrac{qV}{k_B T}\right] \end{array} \right.$$

Boltzmann relationship $\Rightarrow$ true under thermodynamic equilibrium conditions / $\Rightarrow$ no transport.

* Under non equilibrium conditions, E_F is not unique for e^- and holes $\Rightarrow$ due to the contacts) $\to$ $\boxed{\text{Carrier injections}}$
the expressions of n and p become very complicated.

Boltzmann relationship are however still valid if one introduces the notion of quasi-Fermi level $\left\{ \begin{array}{l} E_F \to E_{F_n}(\vec{r}) \text{ for } e^- \\ E_F \to E_{F_p}(\vec{r}) \text{ for holes} \end{array} \right.$

Quasi-Fermi levels are "imaginary reference"
$\Rightarrow$ imref (Remark spelled backwards $\to$ fermi) :-)

(b) continuity equations

So far, a steady state was assumed (no time dependence for transport)

→ continuity equations describe the evolution of carrier concentration with time.

The local carrier density may vary for the following reasons:

(i) external force can be applied to a region of the semiconductor ~~[illegible]~~ carrier ~~injection~~ are added or removed from this region (contact)

(ii) An external source of energy can increase the hole and e^- concentration. e^- can jump from the VB to CB a free e^--hole pair is then created. (example = solar cell)

generation process

→ the generation rate due to this external source of energy is called extrinsic generation rate.

(iii) In the absence of any outside influence, free e^- and holes can be created or annihilated within a region of the semiconductor.

→ if the width of the bandgap is small enough, e^- ~~[illegible]~~ can jump from CB to VB / (or VB to CB)

→ they can also jump into a permitted energy ~~[illegible]~~ located inside the bandgap (created by impurities or defects)

if free e^- and hole are created → generation process

if free e^- and hole are lost → recombination process

The net intrinsic generation/recombination rates are noted U_n for e^- and U_p for holes.

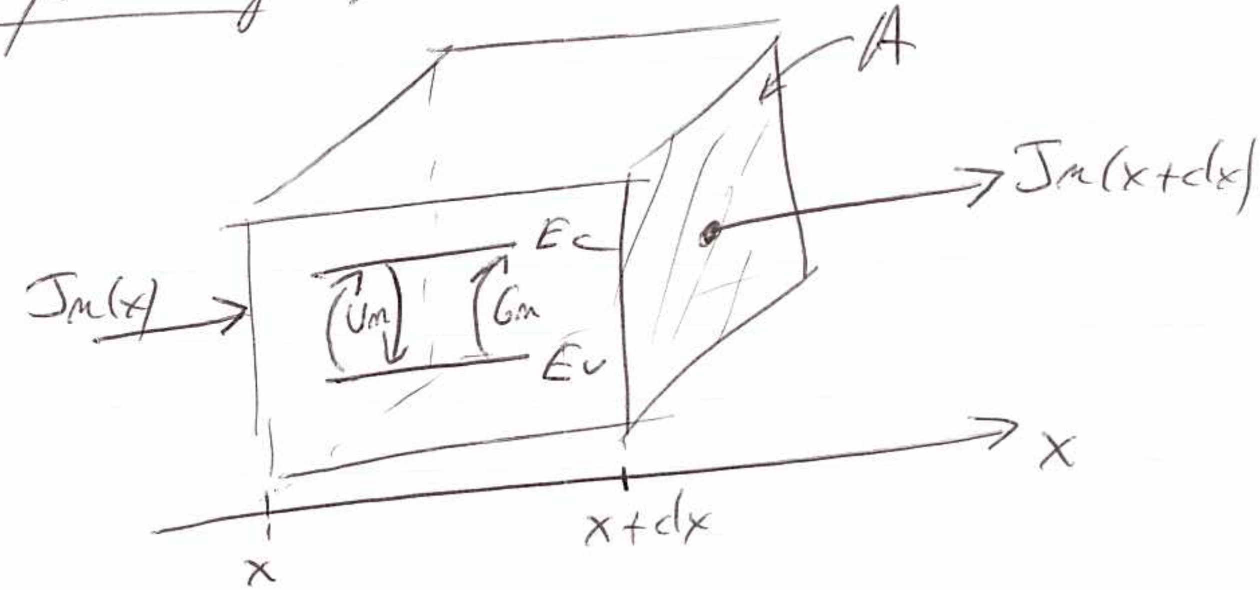
U_n and $U_p > 0$ if recombination dominates over generation.

U_n and $U_p < 0$ if generation process dominates over recombination.

U_n = (free e^- intrinsic recombination rate − free e^- intrinsic generation rate)

[These phenomena extrinsic generation and intrinsic generation/recombination process are analyzed in detail in the next section.

So finally we get:



for e^- in 1D

$$\underbrace{A\frac{\partial n}{\partial t}dx}_{\substack{\text{Variation of number}\\ \text{of free } e^- \text{ in the volume } A\,dx\\ \text{as a function of time.}}} = A\left(\underbrace{\frac{J_n(x)}{-q}}_{\substack{\text{number of } e^-\\ \text{entering the}\\ \text{volume}}} - \underbrace{\frac{J_n(x+dx)}{-q}}_{\substack{\text{number of } e^-\\ \text{flowing out}\\ \text{of the volume.}}}\right) + A\underbrace{(G_n - U_n)}_{\substack{\text{generation/}\\ \text{recombination}\\ \text{processes}}}dx$$

Using Taylor expansion $\Rightarrow$

$$J_n(x+dx) = J_n(x) + \frac{dJ_n(x)}{dx}dx$$

$$\Rightarrow \boxed{\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} + (G_n - U_n)} \text{ for } e^-$$

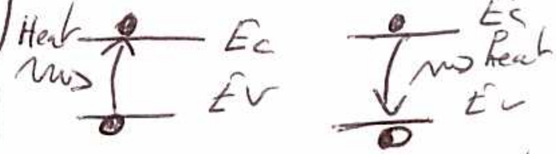
Similarly $$\boxed{\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x)}{\partial x} + (G_p - U_p)} \text{ for holes}$$

7 Carrier generation and recombination

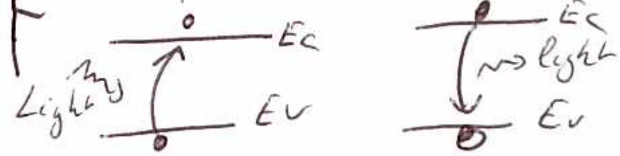
a) Classification

2 particles →

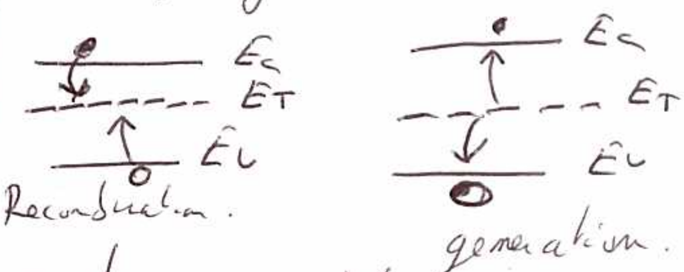
- **one step** (Direct) →
  - Direct thermal generation or recombination



  - Photogeneration or radiative recombination



- **two step** (indirect) →
  - Shockley-Read-Hall (SRH) mechanism
  - Surface generation - recombination



  ⇒ trap-assisted recombination, generation.

3 particles → Auger

- Recombination → (carriers near band edges)
- Generation → (energetic carrier involved)