c) The variable depletion layer model

includes now the variation of the charge in the depletion layer between Source and Drain.

$$Q_{inv} = -C_{ox}(V_G - V_T) \quad V_{GS} > V_T$$

however now we include the ~~explicit~~ implicit dependence of V_T on the charge in the depletion layer.

$$V_T = V_{FB} + V_S + \frac{\sqrt{2\epsilon q N_A V_S}}{C_{ox}} \qquad \boxed{V_S = 2V_F + V_C}$$

$$\Rightarrow \boxed{V_T = V_{FB} + 2V_F + V_C + \frac{\sqrt{2\epsilon q N_A (2V_F + V_C)}}{C_{ox}}}$$

the voltage V_C is the channel voltage $V_C(y)$

$$\Rightarrow I_D = \mu C_{ox} \frac{W}{dy}\left(V_G - V_{FB} - 2V_F - V_C(y) - \frac{\sqrt{2\epsilon q N_A (2V_F + V_C(y))}}{C_{ox}}\right) dV$$

y varies from 0 to L. $V_C(y)$ varies from 0 to V_{DS}

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_G - V_{FB} - 2V_F - \frac{V_{DS}}{2}\right) V_{DS}$$
$$- \frac{2}{3} \mu \frac{W}{L} \sqrt{2\epsilon q N_a}\left[(2V_F + V_{DS})^{3/2} - (2V_F)^{3/2}\right]$$

on integration from V_S to V_D

$$\boxed{I_D = \mu C_{ox} \frac{W}{L} \left\{ \left(V_G - V_{FB} - 2V_F - V_S - \frac{V_{DS}}{2} \right) V_{DS} - \frac{2}{3} \gamma \left[(2V_F + V_D)^{3/2} - (2V_F + V_S)^{3/2} \right] \right\}}$$

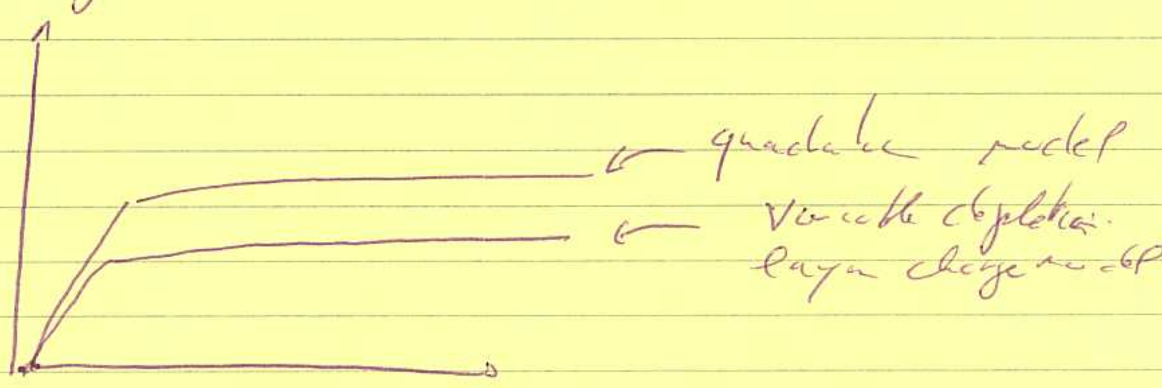
$$\gamma = \frac{\sqrt{2 q \epsilon_s N_a}}{C_{ox}}$$

Again this expression is not valid after the pinch-off point for describing the saturation regime.

The drain voltage at which saturation occurs can be ~~$V_{D,sat} = V_G$~~ found setting $\frac{dI_D}{dV_D} = 0$.

$$\Rightarrow \boxed{V_{Dsat} = V_G - V_{FB} - 2V_F + \frac{\gamma^2}{2} - \gamma \sqrt{V_G - V_{FB} + \frac{\gamma^2}{4}}}$$

Actually the quadratic model yields to larger current compared to this more accurate derivation.



it comes for the transconductance.

$$\cancel{g_{m,sat} = \mu_n C_{ox} \frac{W}{L} \left[V_G - V_{FB} - 2V_F \right.}$$

$$g_{m,\cancel{sat}} = \mu C_{ox} \frac{W}{L} V_{DD} \quad \text{(still)}$$

and $\boxed{g_{m,sat} = \mu C_{ox} \frac{W}{L} V_{Dsat}}$ ⇒ almost linear with V_G

So it can be also written

$$\boxed{g_{m,sat} = \mu^* C_{ox} \frac{W}{L} (V_G - V_T)}$$ with modified μ^*

$$\mu^* = \mu \left[1 - \frac{1}{\sqrt{1 + \frac{2(2V_F + V_S) C_{ox}^2}{q N_a \epsilon_{si}}}} \right]$$

(d) Influence of substrate bias on threshold voltage

we saw that $\boxed{V_T = V_{FB} + 2V_F + \gamma\sqrt{2V_F}}$

$$\gamma = \frac{\sqrt{2 q \epsilon_{si} N_a}}{C_{ox}}$$

where we supposed that both the source and the substrate were grounded. $V_s - V_{sub} = 0$.

In many applications, the source and the substrate may be at different potential.

the depletion charge under the channel is obtained by.

$$Q_d(y) = -\sqrt{2 q \epsilon_{si} N_a (2V_F + V_c(y) - V_{sub})} \quad \underline{V_{sub} < 0}$$

$$\boxed{V_T = V_{FB} + 2V_F + V_c(y) + \gamma\sqrt{2V_F + V_c(y) - V_{sub}}}$$

at $y = 0 \Rightarrow V_c(y) = V_s$

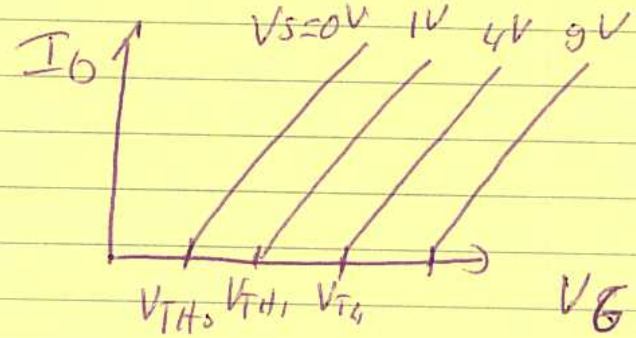
if (i) $V_{sub} = 0$; $V_s > 0 \Rightarrow$

$$V_{TH} = V_s + V_{FB} + 2V_F + \gamma\sqrt{2V_F + V_s}$$

(ii) $V_{sub} < 0$; $V_s = 0$

$$V_{TH} = V_{FB} + 2V_F + \gamma\sqrt{2V_F - V_{sub}}$$

In conclusion, V_T increases as a function of the potential difference between source and substrate.



V_{DS} is small
and $V_{sub} = 0$ (grounded).

(3) Surface mobility

The mobility μ_n that we used is the one associated with the "surface mobility". Surface mobility is lower than ~~the~~ bulk mobility because of scattering at the silicon-oxide interface.

The surface mobility depends on how much e^- interact at the interface, and therefore it depends on the vertical electrical field which 'pushes' e^- against the interface.

~~without~~ If $\vec{E} = 0$ we denote $\mu_n \equiv \mu_{n0}$.

The current in the transistor is given by

$$\boxed{I_D = \frac{W}{L} \int_{V_S}^{V_D} \mu_n \left(-Q_{inv}(y)\right) d(V(y))}$$

There exists an empirical relationship that describes the dependence of surface mobility on vertical electric field in the channel, $\mathcal{E}_x$.

$$\mu_n(y) = \frac{\mu_{no}}{1 + \theta\,\mathcal{E}_x(y)}$$ θ = mobility reduction factor.

The average electric field in the channel is given by

$$\mathcal{E}_x(y) = \frac{\mathcal{E}_{so}(y) + \mathcal{E}_{si}(y)}{2}$$

$\mathcal{E}_{so}$ = electric field at the silicon-oxide interface

$\mathcal{E}_{si}$ = electric field at the boundary between inversion layer and depletion region

According to Gauss' Law

(i) $\epsilon_{si}\,\mathcal{E}_{so}(y) = \epsilon_{ox}\,\mathcal{E}_{ox}(y)$ $\quad \mathcal{E}_{ox} = \dfrac{V_G - V_{FB} - V_{s}(y)}{d_{ox}}$ $\quad$ (surface) $= 2V_F + V_{CB}$

$$\Rightarrow \epsilon_{si}\,\mathcal{E}_{so}(y) = C_{ox}\left[V_G - V_{FB} - V_{s}(y)\right] = -\left[Q_{inv}(y) + Q_d(y)\right]$$

$= Q_G(y)$

(ii) $\epsilon_{si}\,\mathcal{E}_{si}(y) = -Q_d(y) = \gamma\, C_{ox}\sqrt{V_{surf}(y)}$

$$\Rightarrow \boxed{\mathcal{E}_x(y) = \frac{C_{ox}}{2\epsilon_{si}}\left(V_G - V_{FB} - V_{surf}(y) + \gamma\sqrt{V_{surf}(y)}\right)}$$

$$\Rightarrow \mu_n(y) = \frac{\mu_{n0}}{1 + \theta \frac{C_{ox}}{2\epsilon_{si}} \left(V_G - V_{FB} - V_{surf}(y) + \gamma \sqrt{V_{surf}(y)}\right)}$$

We want to simplify the calculation of the Drain current.

$$\boxed{I_D \text{ calculated with } \mu_n(y) \text{ depending on } E_x(y) \equiv I_D \text{ calculated with a cte } \mu_{eff}}$$

- Let us consider a small element dy.

$$\Rightarrow I_D\, dy = W \mu_{eff} \left(-Q_{in}(y)\right) dV(y)$$

$$\Rightarrow I_D \cdot dy = \frac{W \mu_{n0} \left(-Q_{in}(y)\right) dV(y)}{1 + \theta \frac{C_{ox}}{2\epsilon_{si}} \left(V_G - V_{FB} - V_{surf}(y) + \gamma \sqrt{V_{surf}(y)}\right)}$$

we substitute and integrate from source to drain.

$$\left[\int_0^L \left\{ 1 + \theta \frac{C_{ox}}{2\epsilon_{si}} \left[V_G - V_{FB} - V_{surf}(y) + \gamma \sqrt{V_{surf}(y)}\right] \right\} dy = L \frac{\mu_{n0}}{\mu_{eff}} \right] (*)$$

<u>Remark</u> $V_{surf}(y) = 2V_F + V_c(y)$

$$\frac{dV}{dy} \simeq \frac{V_D - V_S}{L} \quad \text{(linear approximation)}.$$

we can write the following identity:

$$\int_0^L \left[V_c(y) - \gamma\sqrt{2V_F + V_c(y)}\right] dy = \frac{L}{V_D - V_S}\int_{V_S}^{V_D}\left[V_c - \gamma\sqrt{2V_F + V_c}\right] dV_c$$

$$= \frac{L}{V_D - V_S}\left[\frac{V_D^2}{2} - \frac{V_S^2}{2} - \frac{2}{3}\gamma\left\{(2V_F + V_D)^{3/2} - (2V_F + V_S)^{3/2}\right\}\right]$$

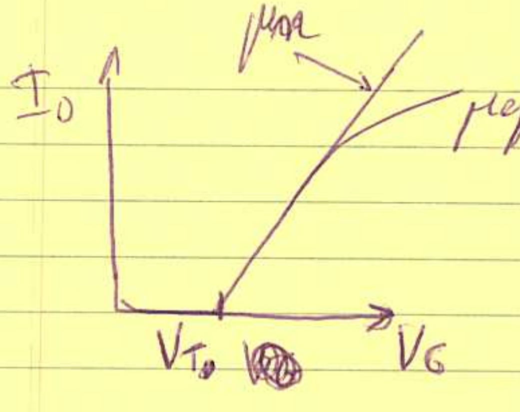
introducing in (*), we get (after calculation)

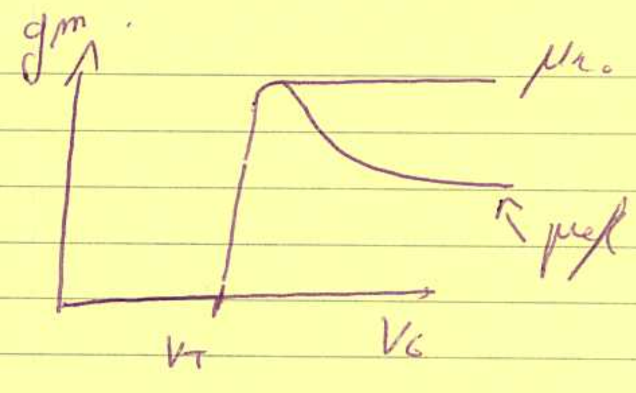
$$\frac{\mu_{no}}{\mu_{eff}} = 1 + \frac{\theta C_{ox}}{2\epsilon_{si}}\left[V_G - V_T + \gamma\sqrt{2V_F} - \frac{V_S + V_D}{2} + \frac{2}{3}\gamma\frac{(2V_F + V_D)^{3/2} - (2V_F + V_S)^{3/2}}{V_{DS}}\right]$$

$$\Rightarrow \boxed{\mu_{eff} = \frac{\mu_{no}}{1 + \frac{\theta C_{ox}}{2\epsilon_{si}}\left(V_G - V_T - V_S + 2\gamma\sqrt{2V_F + V_S}\right)}}$$

~~[illegible]~~ Neglecting the influence of depletion charge near the source; after simplification and also with $\alpha = \frac{\theta C_{ox}}{2\epsilon_{si}}$

$$\Rightarrow \boxed{\mu_{eff} = \frac{\mu_{no}}{1 + \theta(V_G - V_{TH} - V_S)}}$$





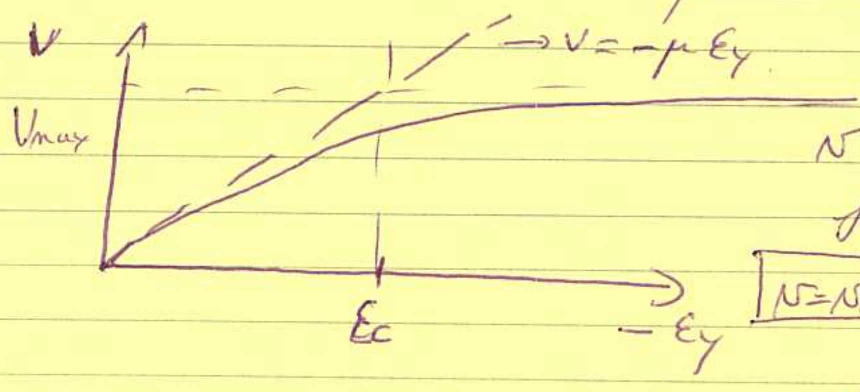
④ Carrier velocity saturation

So far we consider a linear dependency of the drift current on the lateral electric field. ——→ y

$$\left[q\mu_n n E_y = -q v n\right]$$ v = velocity of carrier in the inversion layer

Actually, this is working for low electric field only. At high electric field, and above a critical value, the carrier velocity saturates.

ex: e^- in silicon, max velocity is 10^7 cm/s



if $E_y \to +\infty$

$$\boxed{E_c = \frac{v_{max}}{\mu_n}}$$

v can be expressed as follows

$$\boxed{v = v_{max}\,\frac{E_y/E_c}{E_y/E_c - 1}}$$

What is the impact of the velocity saturation effect on the expression of the drain current of a MOSFET.

if $V_D \geq V_{Dsat}$ $\mathcal{E}_y$ reaches high value near the drain junction.

the drain current is:

$$\boxed{I_D = -W\, Q_{inv}(y)\, \mu_n \frac{dV(y)}{dy} = W\, Q_{inv}\, \mu_n\, \mathcal{E}_y}$$

if $\mathcal{E}_y \to +\infty$ then $Q_{inv}(y) \to 0$.

from $v = v_{max} \dfrac{\mathcal{E}_y/\mathcal{E}_c}{\mathcal{E}_y/\mathcal{E}_c - 1}$ and using $\left[\begin{array}{l} \mathcal{E}_c = v_{max}/\mu \\ \mathcal{E}_y = -dV(y)/dy \end{array}\right.$

$$\Rightarrow \quad v(y) = v_{max} \frac{-\dfrac{1}{\mathcal{E}_c}\dfrac{dV(y)}{dy}}{-\dfrac{1}{\mathcal{E}_c}\dfrac{dV(y)}{dy} - 1} = \mu_n \frac{\dfrac{dV(y)}{dy}}{1 + \dfrac{1}{\mathcal{E}_c}\dfrac{dV(y)}{dy}}$$

the expression of the current becomes.

$$\boxed{I_D = -W\left(\frac{\mu_n}{1+\dfrac{1}{\mathcal{E}_c}\dfrac{dV(y)}{dy}}\right) Q_{inv}(y)\, \frac{dV(y)}{dy}}$$

just replacing $\mu_n \to \dfrac{\mu_n}{1+\dfrac{1}{\mathcal{E}_c}\dfrac{dV(y)}{dy}}$ to account for velocity saturation integrating from Source to Drain yields.

$$I_D = -\frac{W}{L}\left(\frac{\mu_n}{1+\frac{V_{DS}}{L\mathcal{E}_c}}\right)\int_{V_S}^{V_D} Q_{inv}(y)\,dV(y).$$

$$\Rightarrow \boxed{I_D = \frac{W}{L}\left(\frac{\mu_n}{1+\frac{V_{DS}}{L\mathcal{E}_c}}\right) C_{ox}\left[(V_G - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]}$$

using quadratic model

by imposing $\frac{dI_D}{dV_{DS}} = 0 \Rightarrow$ drain saturation voltage is

$$\boxed{V_D'\,sat = L\mathcal{E}_c\left[\sqrt{1+\frac{2(V_G-V_T)}{L\mathcal{E}_c}} - 1\right]}$$

when velocity saturation is taken into account this is equivalent to making the channel longer L is multiply

by $\left(1+\frac{V_{DS}}{L\mathcal{E}_c}\right)$.

therefore $\Rightarrow$ the Drain saturation voltage and the drain saturation current are reduced.

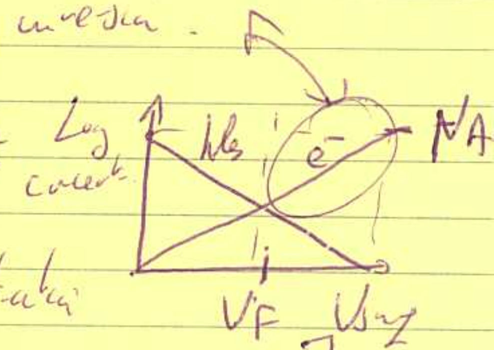
5 Subthreshold ~~current / Subthreshold slope~~ Characteristics.

(a) Subthreshold current

So far we assumed that $I_D = 0$ if $V_G \leq V_T$

Actually there is a significant amount of e^- near the surface if the device operates ~~at~~ below strong inversion.

$V_F < V_{surf} < 2V_F$ Reminder



[the actual dependence of the e^- concentration at the surface is an exponential function of V_{surf}]

⇒ Experimentally we observe that the Drain current below threshold "subthreshold current" is independent of the drain voltage. ⇒ this means that the subthreshold current is caused by diffusion rather than drift mechanism.

current density from source to Drain can be written

$$J_{ny} = -qD\frac{dn}{dy} \Rightarrow \boxed{I_D = qAD_n\frac{n(0)-n(L)}{L}}$$

A is the cross-sectional area of the channel region.

e^- density at the ~~[illegible]~~ edge of source and drain =

$$n(0) = n_{p0} \exp\left(\frac{q V_{surf}}{k_B T}\right) \qquad n(L) = n_{p0} \exp\left(\frac{q (V_{surf} - V_D)}{k_B T}\right)$$

$$n_{p0} = \frac{n_i^2}{N_a}$$

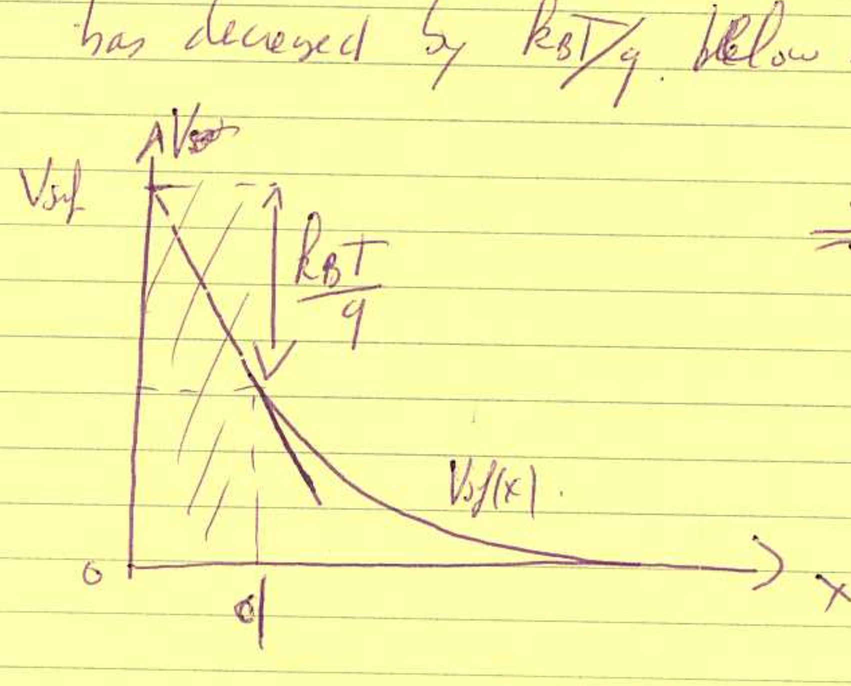
($\rightarrow$ Source is considered here at ground).

what is the area A?

we know that the e^- density varies as $\exp\left(\frac{q V_{surf}}{k_B T}\right)$ near the surface. we however going to

approximate the exp-profile by a constant e^- density extending to a depth d below the surface.

the depth d is defined as the depth at which the potential has decreased by k_BT/q below the surface potential value



$\Rightarrow$ so $A = W \times d$

W (width of the transistor)

$d = \frac{k_BT/q}{\mathcal{E}_s}$ $\quad \mathcal{E}_s = -\frac{dV_{surf}}{dx}\Big|_0$

Using the ~~[illegible]~~ Einstein relationship $\mu_n \frac{k_B T}{q} = D_n$

we get the subthreshold current

$$\boxed{I_D = \mu_n \frac{W}{L} q \left(\frac{k_B T}{q}\right)^2 \frac{n_i^2}{N_A} \left[1 - \exp(-qV_D/k_BT)\right] * \frac{\exp(qV_{surf}/k_BT)}{-\frac{dV_{surf}}{dx}}}$$

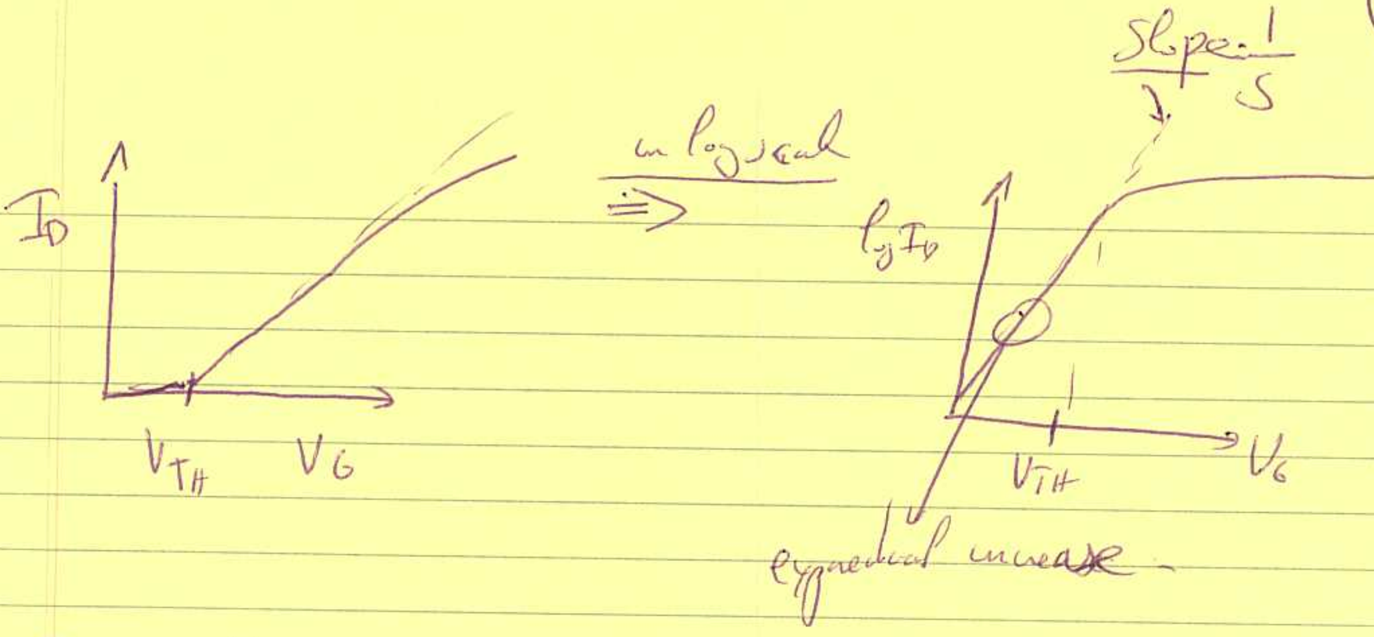
the electric field at the surface $\mathcal{E}_s$ is given by

(MOS capacitor section) $-\left.\frac{dV_{surf}}{dx}\right|_{x=0} = \mathcal{E}_s = \sqrt{\frac{2qN_A V_{surf}}{\epsilon_{Si}}}$

$$\Rightarrow \boxed{\mathcal{E}_s = q\frac{N_A}{C_D}}$$

$\Rightarrow I_D$ is independent of V_D as long as V_D is larger than a few $\frac{k_B T}{q}$. (exponential term $\to 0$)

$\Rightarrow I_D$ also increases exp. with the surface potential.



(b) Subthreshold slope

it is $\frac{1}{S}$ (inverse of the slope) $\Longrightarrow$ subthreshold slope $\boxed{S = \frac{dV_G}{d\log(I_D)}}$

$\Rightarrow$ represents how many millivolts per decade

$\Rightarrow$ "how many millivolts V_G be increased to increase I_D by a factor 10"

if $S \downarrow$ efficiency and switching speed of the device from off-state to on-state $\uparrow$

$$S = \frac{\ln(10)}{\frac{d(\ln(I_D))}{dV_G}}$$

$\log_{10} \rightarrow \log_e \equiv \ln$.

Also
$$\frac{d\ln(I_D)}{dV_G} = \frac{d(\ln I_D)}{dV_{surf}} \frac{dV_{surf}}{dV_G}$$

$$\frac{d\ln(I_D)}{dV_{surf}} = \frac{d}{dV_{surf}}\left[\ln\left(e^{+\frac{qV_{surf}}{k_BT}}\right) - \ln\left(\frac{dV_{surf}}{dx}\right)\right]$$

$$= \left[\frac{q}{k_BT} - \frac{\frac{d}{dV_{surf}}\left(-\frac{dV_{surf}}{dx}\right)}{-\frac{dV_{surf}}{dx}}\right] \quad (*)$$

$$\frac{d}{dV_{surf}}\left(-\frac{dV_{surf}}{dx}\right) = \frac{C_D}{\epsilon_{si}} \leftarrow \text{(from Poisson)}$$

C_D is the depletion capacitance $\boxed{C_D = -\frac{dQ_d}{dV_{surf}}} = \frac{\epsilon_{si}}{x_d}$

$$\Rightarrow \frac{\frac{d}{dV_{surf}}\left(-\frac{dV_{surf}}{dx}\right)}{-\frac{dV_{surf}}{dx}} = \frac{C_D^2}{qN_a\epsilon_{si}} = \frac{1}{2V_{surf}}$$

in weak inversion regime $V_F < V_{surf} < 2V_F$,

$\frac{1}{2V_{surf}}$ is small compared to $\frac{q}{k_BT}$ $\Rightarrow$ it causes (second term in $*$ is neglected).

$$\boxed{\frac{d(\ln(I_D))}{dV_G} \simeq \frac{q}{k_BT}\frac{dV_{surf}}{dV_G}}$$

Also $\left[V_G = V_{FB} + V_{surf} - \frac{Q_d}{C_{ox}} \right]$

$$\Rightarrow \frac{dV_{surf}}{dV_G} = \left(1 + \frac{C_D}{C_{ox}}\right)^{-1} \quad \left[\text{do } \frac{dV_G}{dV_{surf}} \text{ first}\right]$$

$\Rightarrow$ finally

$$\boxed{S = \frac{k_B T}{q} \ln(10) \left(1 + \frac{C_D}{C_{ox}}\right)} \qquad \boxed{\left(1 + \frac{C_D}{C_{ox}}\right) = \underbrace{m}_{\text{body factor}}}$$

Also since $\frac{d(\ln I_D)}{dV_G} = \frac{q}{k_B T m}$

$$\Rightarrow \boxed{I_D \sim \exp\left(\frac{q V_G}{m k_B T}\right)}$$

the subthreshold current varies exponentially as a function of the gate voltage.

* In case where we consider interface states (or traps) in the Si energy bandgap at the silicon/oxide interface. One can ~~define~~ associate a charge of these interfaces state Q_{it}.

$$\Rightarrow C_{it} = \frac{-dQ_{it}}{dV_{surf}}$$

we get after calculations.

$$\boxed{S = \frac{k_B T}{q} \ln(10) \left(1 + \frac{C_D + C_{it}}{C_{ox}}\right)}$$

$$= 60 \left(1 + \frac{C_D + C_{it}}{C_{ox}}\right) \text{ mV/decade}$$ at room temperature

(6) Continuous model

The model developed so far for $V_G > V_T$ or for $V_G < V_T$ do not connect well around $V_G = V_T$ (discontinuity). Since often V_G is close to V_T, this is a problem.

A model which is valid everywhere can be derived.

Indeed, $I_D = \mu_n C_{ox} \frac{W}{L} \int_{V_S}^{V_D} -\frac{Q_{inv}(y)}{C_{ox}} \, dV(y)$.

can be rewritten

$$I_D = \mu_n C_{ox} \frac{W}{L} \left\{ \int_{V_S}^{V_P} -\frac{Q_{inv}(y)}{C_{ox}} \, dV(y) - \int_{V_D}^{V_P} -\frac{Q_{in}(y)}{C_{ox}} \, dV(y) \right\}$$

$V_P = (V_G - V_T) \equiv$ drain saturation voltage if source is grounded and $V_G > V_T$

$I_D = I_F - I_R$

forward current — Reverse current.

It is possible to find a mathematical function which describes the evolution of the current as a function of gate and drain voltage for all regime of operation (depletion, weak and strong inversion)

$\Rightarrow$ Enz, Krummenacher, Vittoz

$\Rightarrow$ EKV Model

$$I_D = 2\mu_n C_{ox} \frac{W}{L} \left(\frac{k_B T}{q}\right)^2 \left\{ \left[\ln\left(1 + \exp\left(\frac{V_P - V_S}{2 k_B T/q}\right)\right)\right]^2 - \left[\ln\left(1 + \exp\left(\frac{V_P - V_D}{2 k_B T/q}\right)\right)\right]^2 \right\}$$

For example:

if $V_S = 0$, $V_D < V_P$ $V_G > V_T$ ⇒ the transistor operates in non-saturated regime.

⇒ exponential terms $\gg 1$

$$I_D = 2\mu_n C_{ox}\frac{W}{L}\left(\frac{k_BT}{q}\right)^2\left[\left(\frac{V_P}{2k_BT/q}\right)^2 - \left(\frac{V_P - V_D}{2k_BT/q}\right)^2\right]$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2V_DV_P - V_D^2\right].$$

$$\Rightarrow \boxed{I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]}$$

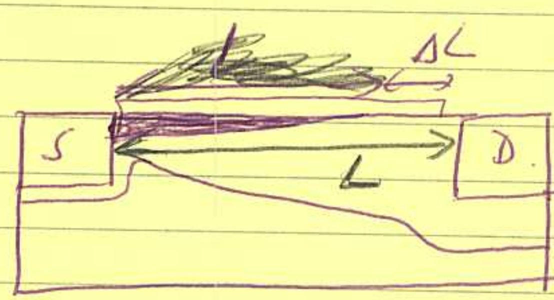
quadratic model

(7) Channel length modulation

Previously we assumed that when $V_D > V_{Dsat}$, the Drain current of a MOSFET is constant and equal to I_{Dsat}.

Actually if $V_D \nearrow$ beyond V_{Dsat}, the depletion ~~dep.~~ region and local threshold voltage near the Drain are increased the effective length of the channel is $L - \Delta L$

(~~[illegible]~~



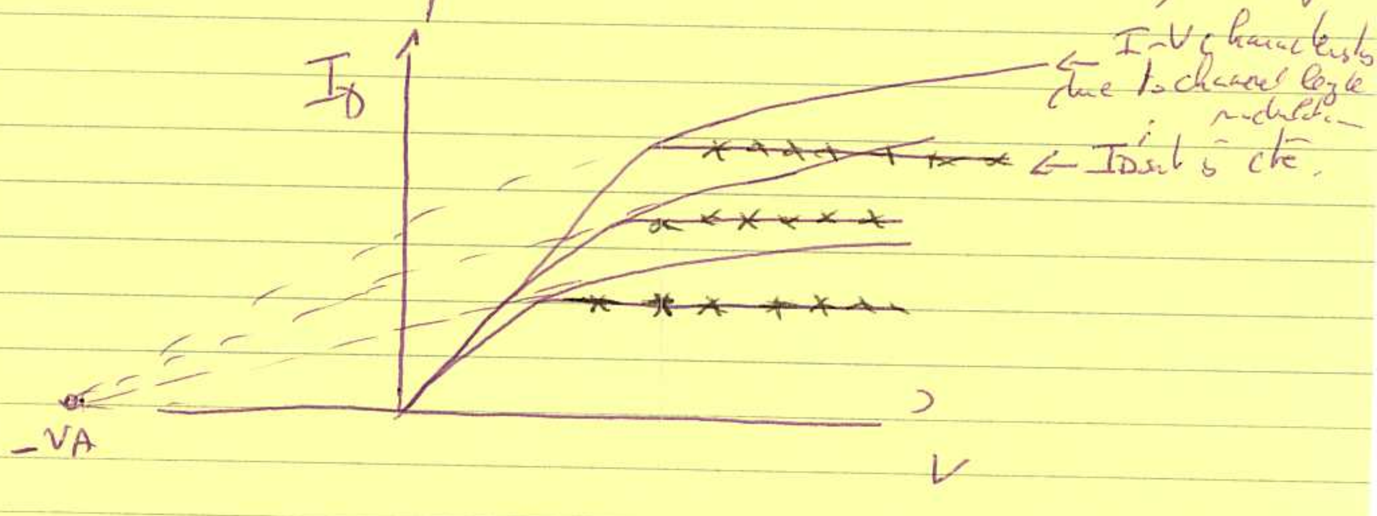
and shorter channel length ⇒ increase in Drain current.

if I_{Dsat} ~~is [illegible]~~ is obtained if ($V_D = V_{Dsat}$)

if $V_D > V_{Dsat}$

$$I'_{Dsat} = I_{Dsat}\left(1 + \frac{V_D - V_{Dsat}}{V_A + V_{Dsat}}\right)$$

where V_A is a positive voltage value that can be obtained experimentally. V_A is called "early voltage"



in term of channel length modulation, we get

$$\frac{I'_{Dsat}}{L} = \frac{I_{Dsat}}{L - \Delta L}$$

$$\Rightarrow \Delta L = L\left[1 - \frac{1}{1 + \dfrac{V_D - V_{Dsat}}{V_A + V_{Dsat}}}\right]$$

the saturation output ~~current~~ conductance, which was previously considered equal to zero, is now given by:

$$g_{Dsat} = \frac{dI'_{Dsat}}{dV_D} = \frac{I_{Dsat}}{V_A + V_{Dsat}} \simeq \frac{I_{Dsat}}{V_A}$$

* the channel length modulation effect typically increases in small devices with low-doped substrates.
* Proper scaling can reduce the channel length modulation by increasing the doping density (as the gate length is reduced)

8 Short-channel effect

evolution of semiconductor processing technology enables scaling of transistors.

Ex gate length of MOSFET used in 256K DRAM in 1984 was $\sim 1.2\,\mu m$.

in 1994, 64 M DRAM were produced using $0.4\,\mu m$

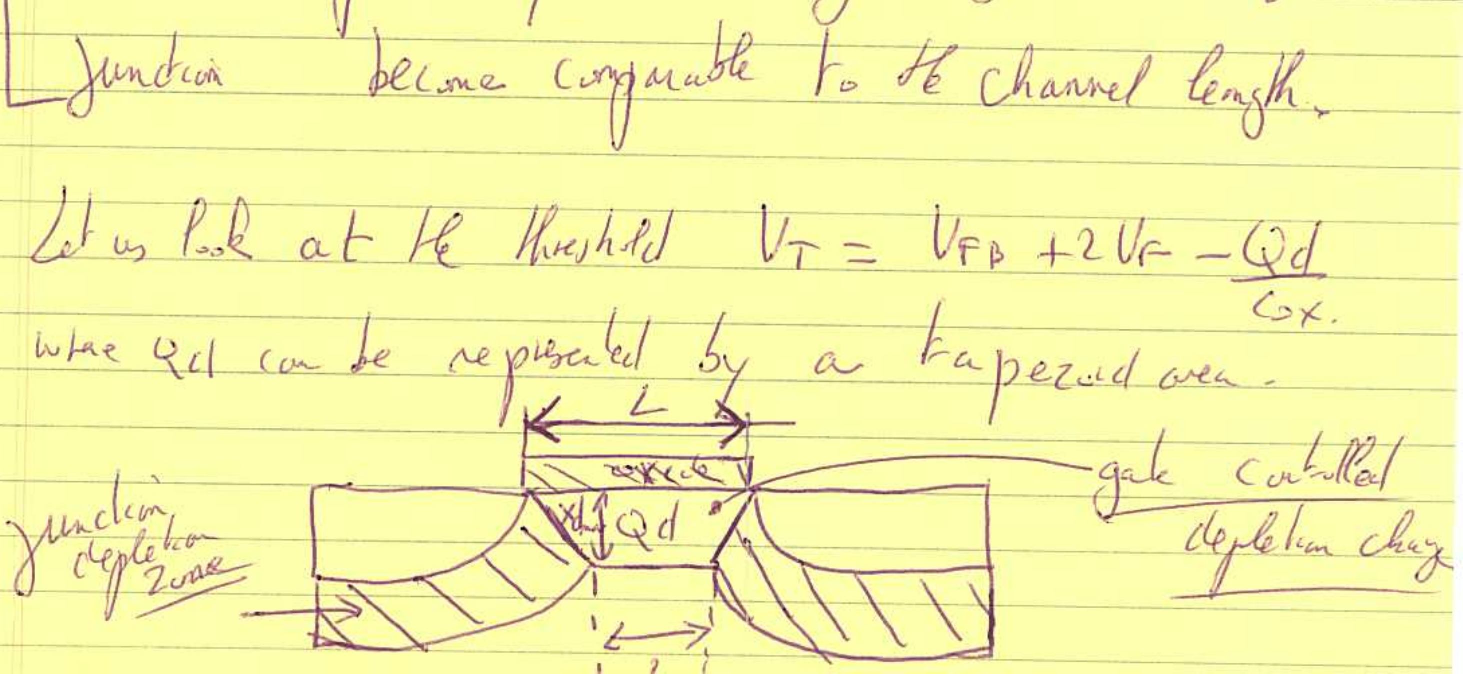
prediction 50 nm in 2009
35 nm in 2012.

⇒ aggressive scaling ⇒ undesirable effects including the short-channel effect

here the depletion layer width of the source and drain junction become comparable to the channel length.

Let us look at the threshold $V_T = V_{FB} + 2V_F - \frac{Q_d}{C_{ox}}$

where Q_d can be represented by a trapezoid area.



Area of the trapezoid is $(L+L_1)\dfrac{x_{dmax}}{2}$.

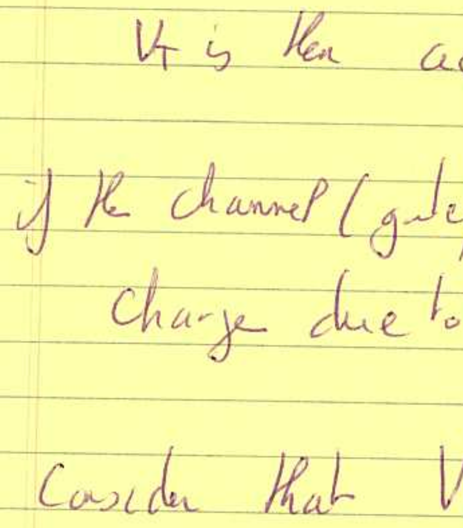
• if the channel is long $L \simeq L_1 \Rightarrow$ area $\approx L\,x_{dmax}$ (rectangular area)

V_T is then accurately described by the equation derived previously.

• if the channel (gate) is short $L_1 < L$, then the depletion charge due to V_G under the gate electrode is reduced.

Consider that $V_G = V_T$ and $V_{DS} \simeq 0$.

Based on geometrical consideration we get:



$$x_{dmax}^2 + (x+r_j)^2 = (r_j + X_{dmax})^2$$

Here we suppose also that the built-in potential of the Source and Drain junction is approximately equal to $2V_F$ $\Rightarrow$ width of the depletion region around source and drain is then $\doteq X_{dmax}$

$$\Rightarrow x^2 + 2r_j x - 2r_j X_{dmax} = 0$$

where $x = -r_j \pm \sqrt{r_j^2 + 2r_j X_{dmax}} = r_j\left(-1 \pm \sqrt{1+\dfrac{2X_{dmax}}{r_j}}\right)$

$\Rightarrow$ x must be positive. So

L_1 then is given by

$$L_1 = L - 2x = L - 2r_j\left(\sqrt{1+\frac{2X_{dmax}}{r_j}} - 1\right)$$

the depletion charge controlled by the gate voltage is

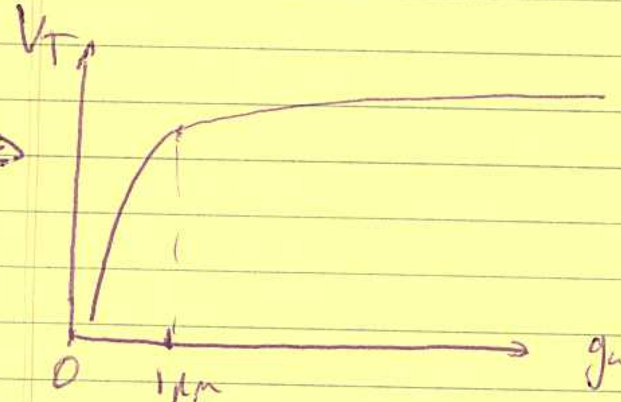
$$Q_d = -q\,X_{dmax}\,N_A\left(\frac{L+L_1}{2}\right)$$

$$= \underbrace{-q\,X_{dmax}\,N_A\,L}_{Q_{d0}}\left(1 - \frac{r_j}{L}\left(\sqrt{1+\frac{2X_{dmax}}{r_j}} - 1\right)\right)$$

Q_{d0} which is the depletion charge if the depletion region would be considered rectangular.

$$V_T = V_{FB} + 2V_F - \frac{Q_d}{C_{ox}} = V_{FB} + 2V_F + q\frac{X_{dmax}\,N_A}{C_{ox}} \times \left(1 - \frac{r_j}{L}\left(\sqrt{1+\frac{2X_{dmax}}{r_j}} - 1\right)\right)$$

$\Rightarrow$



in short devices. Small statistical variations in gate length give rise to large statistical variations of V_T (problem of reproducibility in integrated circuits)

Remarks

- the short channel effect can be reduced by using shallower junctions and higher substrate doping ⇒ to reduce the extension of the source & Drain depletion regions in the channel.
- if the gate length is very small and V_{DS} is high enough, the source and Drain depletion region can touch one another $V_1 < 0$. ⇒ the potential in the channel region is no longer controlled by the gate and a large current can flows from source to Drain.

⇒ this is called "punchthrough"