For an e^- in the CB, potential energy is $-q\dfrac{q^2 N_d (W-x)^2}{2\epsilon}$ (31)

To simplify the expression, we suppose that the electric field in the depletion region is $c^{te} == \mathcal{E}_{max}$ [$E == \mathcal{E}$...

potential energy for the e^- $\Rightarrow$ $-q\,\mathcal{E}_{max}\,x$ (linear).

So total potential energies (from the mirror charge and the depletion region).

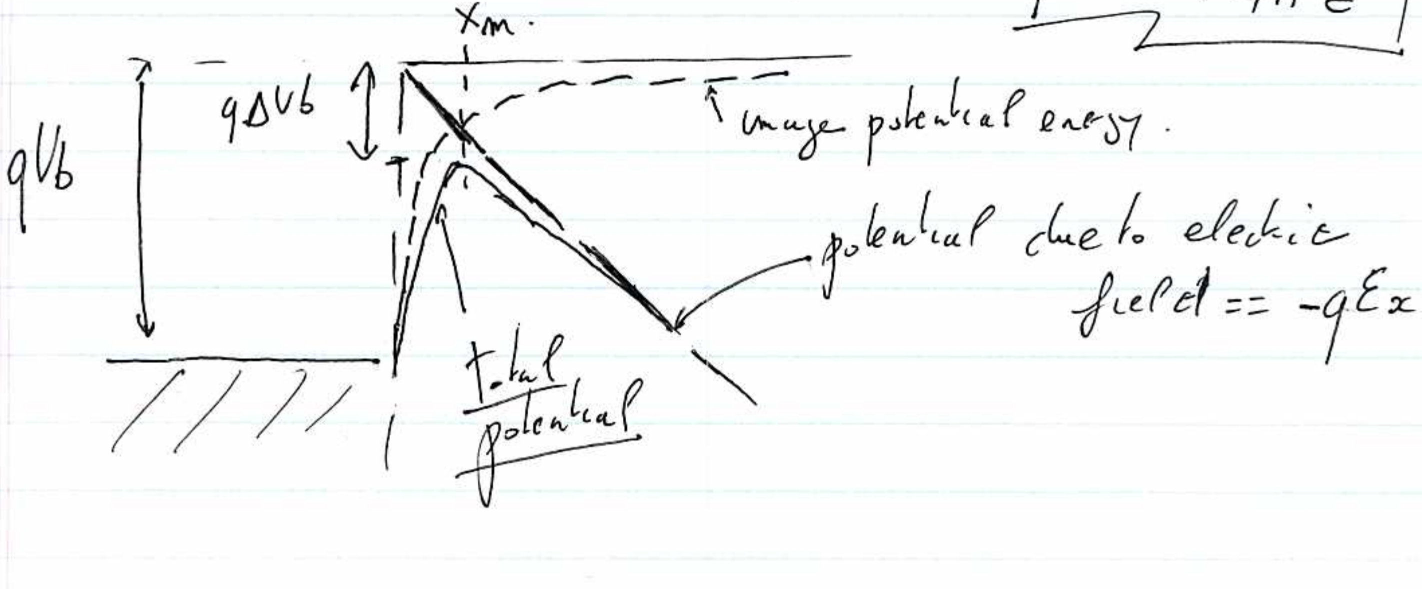
$$U(x) = -q\,\mathcal{E}_{max}\,x - \frac{q^2}{16\pi\epsilon x}$$

the maximum potential at $x = x_m$ $\Rightarrow \left.\dfrac{dU}{dx}\right| = 0$

$$\Rightarrow x_m = \sqrt{\frac{q}{16\pi\epsilon\,\mathcal{E}_{max}}}$$

potential due to image charge $-q\,\Delta V_B = -q\sqrt{\dfrac{q\,\mathcal{E}_{max}}{4\pi\epsilon_{sc}}} < 0.$

$\Rightarrow$ ΔV_B is the barrier height reduction $\boxed{\Delta V_B = \sqrt{\dfrac{q\,\mathcal{E}_{max}}{4\pi\epsilon}}}$



2) I-V characteristics of Schottky diode

The current across a M-SC junction is mainly due to majority carriers.

3 different mechanisms exist:

(i) diffusion of carriers from the SC → M
"assumes that the driving force is distributed over the length of the depletion region".

(ii) Thermionic emission of carriers across the Schottky barrier
"only energetic carriers, which energy ≥ CB energy at the interface, contribute to the current"

(iii) Quantum mechanical tunneling through the barrier.
"wave nature of the e^-".

- The analysis reveals that the diffusion and thermionic currents can be written in the following form:

$$\boxed{J_n = q\, \underbrace{v}_{\text{velocity}}\, \underbrace{N_c \exp(-q\beta V_B)\left[\exp(q\beta V_a) - 1\right]}_{\text{density of available carriers, located next to the interface}}}$$

$v = \mu \mathcal{E}$ for the diffusion current

$v = v_R$ Richardson velocity for thermionic current

⇒ if no applied voltage $J_n = 0$.

- for the tunneling current $\boxed{J_n = q v_R n \Theta}$

Θ is the tunneling probability term.

a) Diffusion current

we assume that the depletion layer W is large compared to the mean free path $\Rightarrow$ concept of drift-diffusion are valid.

$$J_n = q\left(\mu_n n \mathcal{E} + D_n \frac{dn}{dx}\right)$$

also equal to

$$J_n \exp(-V \times q \times \beta) = q D_n \left(-q\beta n \frac{dV}{dx} + \frac{dn}{dx}\right) \exp(-q\beta V).$$

$$= q D_n \frac{d}{dx}\left[n \exp(-V q \beta)\right]$$

in steady state [illegible]

integration over W $\Rightarrow$ J_n does not depend on x

$$\rightarrow \quad J_n = \frac{q D_n \left[n \exp(-q\beta V)\right]_0^W}{\int_0^W \exp(-q\beta V)\, dx}$$

Using the following B.C

x	$n(x)$	$V(x)$
0	$N_c \exp(-q\beta V)$	$-(V_i - V_A)$
W	$N_d = N_c \exp(-q\beta V_B) \exp(+q\beta V_i)$	0

$$\Rightarrow \quad J_n = \frac{q D_n N_c \exp(-q\beta V_B)\left[\exp(q\beta V_A) - 1\right]}{\int_0^W \exp(-q\beta V^*)\, dx}.$$

where $\boxed{\left(V^* = V + V_i - V_A\right)}$

the denominator can be solved using the potential obtained from the full depletion approximation.

$$V = \frac{-qN_d}{2\epsilon}(x-W)^2$$

so $$V^* = V - V(0) = -\frac{qN_d}{2\epsilon}\left[x^2 + W^2 - 2xW - W^2\right] = \frac{qN_d}{\epsilon}x\left(W - \frac{x}{2}\right)$$

$V^* \simeq \frac{qN_d}{\epsilon} x W$ since the linear term is dominant if $x \ll W$.

$$\boxed{V^* \simeq (V_i - V_A)\frac{2x}{W}}$$

$$\Rightarrow \int_0^W \exp(-V^* q\beta)\,dx = -\frac{W}{2(V_i - V_A)q\beta}\left[\exp(-q\beta V^*)\right]_0^W$$

$$= -\frac{W}{2(V_i-V_A)q\beta}\left[\exp\left[-2\left(q\beta(V_i-V_A)\right)\right] - 1\right]$$

$\to 0$ $(V_i - V_A) \gg k_BT$

$$\simeq \frac{W}{2(V_i-V_A)q\beta} = \frac{1}{q\beta}\sqrt{\frac{\epsilon}{2qN_d(V_i-V_A)}}$$

$$\Rightarrow \boxed{J_n = q^2\beta D_n N_c \underbrace{\sqrt{\frac{2qN_d(V_i-V_A)}{\epsilon}}}_{|E_{max}|} \exp(-V_B q\beta)\left[\exp(q\beta V_A) - 1\right]}$$

we can also write "physically"

$$J_n = \underbrace{q\mu_n |E_{max}| N_c \exp(-q\beta V_B)}\left[\exp(q\beta V_A) - 1\right]$$

Drift current at the interface, if $V_A = 0$ balance the diffusion current

(b) Thermionic current

the thermionic emission theory assumes that e^- which have an energy larger than the top of the barrier will cross it if they move toward ~~the barrier~~.
the actual shape of the barrier is ignored.

$$J = \int_{E_c(x=\infty)+qV_i} q V_x \frac{dn}{dE} dE \qquad \boxed{\frac{dn}{dE} = n(E)}$$

For non-degenerate semiconductor, the density of e^- between E and $E+dE$ is given by:

$$\frac{dn}{dE} = g_c(E) F(E) = \frac{4\pi(2m^*)^{3/2}}{h^3}(E-E_c)^{1/2} \exp-\left[\frac{E-E_{F_n}}{k_BT}\right]$$

if parabolic CB with m^* constant

$$\Rightarrow \quad E - E_c = \frac{m^* v^2}{2} \qquad dE = m^* v\, dv$$

$$\sqrt{E-E_c} = v\sqrt{\frac{m^*}{2}}$$

$$\frac{dn}{dE} dE = 2\left(\frac{m^*}{h}\right)^3 \exp\left[-\left(\frac{E_c(x=\infty) - E_{F,N}}{k_BT}\right)\right] \exp\left(-\frac{m^* v^2}{2k_BT}\right) 4\pi v^2 dv$$

if we replace v^2 by $v_x^2+v_y^2+v_z^2$ and $4\pi v^2 dv$ by $dv_x\, dv_y\, dv_z$

$$\Rightarrow J = 2\left(\frac{m^*}{h}\right)^3 \int_{-\infty}^{\infty} \exp\left(-\frac{m^* v_y^2}{2k_BT}\right) dv_y \int_{-\infty}^{+\infty} \exp\left(-\frac{m^* v_z^2}{2k_BT}\right) dv_z$$

$$\int_{v_{0x}} q v_x \exp\left(-\frac{m^* v_x^2}{2k_BT}\right) dv_x \quad \exp\left[-\frac{(E_c - E_{F,N})}{k_BT}\right]$$

Setting V_{ox} = the minimal velocity in the quasi-neutral region that is needed to cross the barrier

Using $\int_{-\infty}^{+\infty} \exp\left(-\frac{m^* v_y^2}{2k_BT}\right) dv_y = \int_{-\infty}^{+\infty} \exp\left(\frac{-m^* V_z^2}{2k_BT}\right) dV_z = \sqrt{\frac{2\pi k_BT}{m^*}}$

$\Rightarrow$

$$J = 2q\left(\frac{m^*}{h}\right)^3 \frac{2\pi k_BT}{m^*} \exp\left[-\frac{(E_C - E_{F,N})}{k_BT}\right] \exp\left[-\frac{m^* v_{ox}^2}{2k_BT}\right] \frac{k_BT}{m^*}$$

V_{ox} is obtained by setting the kinetic energy == potential across the n-type region.

$$\frac{m^* V_{ox}^2}{2} = q V_i$$

Using $V_i = V_B - V_A - \frac{1}{q}[E_C - E_{F,N}]$

$$\Rightarrow J_{MS} = A^* T^2 \exp[-(V_B q\beta)][\exp(q\beta V_A)]$$

for the total current.

- 1 term is added to account for the current flowing from right to left; when $V_A = 0 \Rightarrow J_{MS} = -J_{SM}$.

$$\Rightarrow J_n = A^* T^2 \exp\left[\frac{-qV_b}{k_BT}\right]\left[\exp(q\beta V_A) - 1\right]$$

A^* is the Richardson constant $\boxed{A^* = \frac{4\pi q m^* k_B^2}{h^3}}$

one can also define an average velocity for which the e^- at the interface approach the ...

(37)

This velocity is referred as the Richardson velocity

$$v_R = \sqrt{\frac{k_B T}{2\pi m}}$$

$$\Rightarrow \boxed{J_n = q v_R N_c \exp(-q\beta V_B)\left[\exp(q\beta V_A) - 1\right]}$$

Ⓒ Tunneling current

e^- in the metal can tunnel across the Schottky barrier and enter in the semiconductor. Similarly, e^- can tunnel from the SC → M.

we start from the Schrodinger equation

$$-\frac{\hbar^2}{2m^*}\frac{d^2\Psi}{dx^2} + V(x)\Psi = E\Psi$$

$$\Rightarrow \frac{d^2\Psi}{dx^2} = \frac{2m^*(V - E)}{\hbar^2}\Psi$$

Assuming that V is independent of position in a section between x, $x + dx$ (~~slowly varying function~~)

we can solve it yielding:

$$\Psi(x+dx) = \Psi(x)\exp(-k\,dx) \qquad k = \frac{\sqrt{2m^*(V(x) - E)}}{\hbar}$$

For a slowly varying function potential, the wave function at $x = L$ can be related to ψ at $x = 0$

$$\psi(L) = \psi(0) \exp\left[-\int_0^L \frac{\sqrt{2m^*(V(x)-E)}}{\hbar}\,dx\right]$$

$\Rightarrow$ this equation is referred as the

WKB approximation (Wentzel, Kramers, Brillouin) 1926

we suppose a triangular barrier $\boxed{(V(x)-E) = qV_B\left(1-\frac{x}{L}\right)}$

tunneling probability $\Theta = \frac{|\psi(L)|^2}{|\psi(0)|^2} = \exp\left[-2\int_0^L \frac{\sqrt{2m^*}}{\hbar}\sqrt{qV_B\left(1-\frac{x}{L}\right)}\,dx\right]$

we replace L by

$$\Rightarrow \boxed{\Theta = \exp\left(-\frac{4}{3}\frac{\sqrt{2qm^*}}{\hbar}\frac{V_B^{3/2}}{\mathcal{E}}\right)}$$ where $\boxed{\mathcal{E} = \frac{V_B}{L}}$ electric field

tunneling current is then defined by:

$$J_n = q\underbrace{v_R}_{\text{Richardson velocity}}\underbrace{n\Theta}_{}$$

Richardson velocity

carrier density = density of available e^- multiplied with the tunneling probability.

③ Additional Comment for the Schottky contact

(1) ● lowering barrier effect (already described) due to mirror charge previously.

ⓐ Influence of interface States

So far, we considered "an ideal junction".

In practice, the periodic nature of the SC crystal is disturbed at the interface, which gives rise to a large number of permitted states in the bandgap of the SC near the interface.

⟹ interface States or interface traps.

with Energy ranging from E_V to E_C. and occupied by e^- if they are below the Fermi-level.

when we put the M and SC together the Fermi level is 'pinned' at some particular energy in the gap.

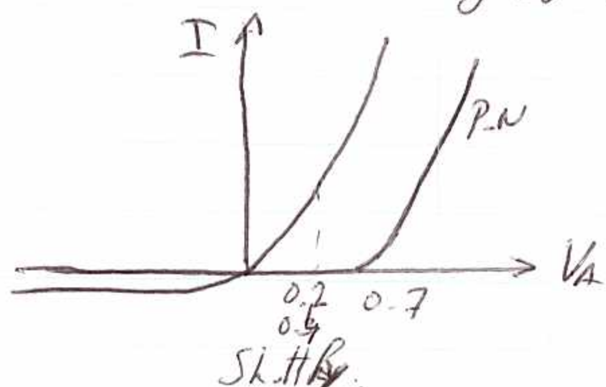
We do not know what pins the Fermi level (complicated physics at the interface)

⟹ Schottky barrier height as to be determined experimentally

(b) Comparison with the P-N junction.

* **Reverse bias**: the saturation current of a Schottky diode is 100 to 1000 times larger than that of PN junction (junction accounts for a larger leakage current)

* **In forward bias**: I-V characteristics show strong conduction at 0.2/0.4V for Schottky compared to 0.7V for PN

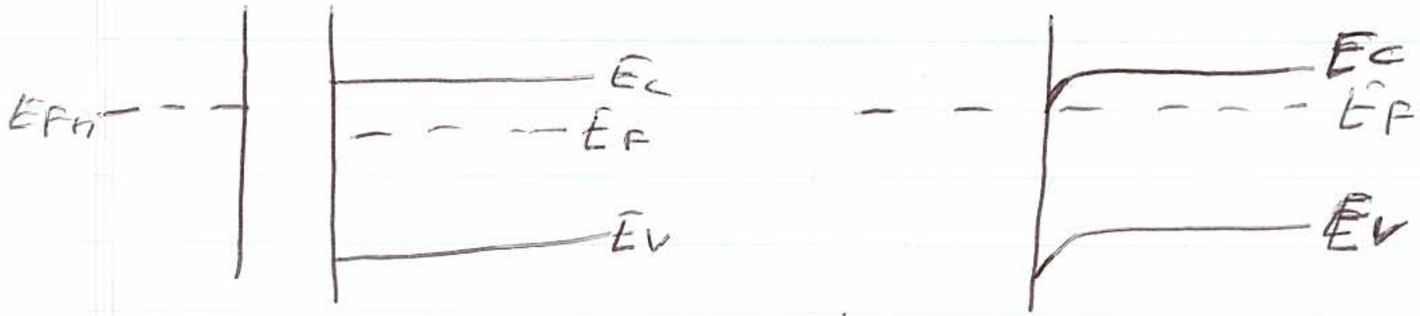


⇒ Schottky diodes are capable of very fast switching
⇒ due to majority carrier.
⇒ for P-N junction, device operation is slowed by recombination of excess minority carriers.

4 Ohmic contact

it is a non-rectifying contact.
the I-V should obey Ohm's law $U = RI$,
and R should be as low as possible.

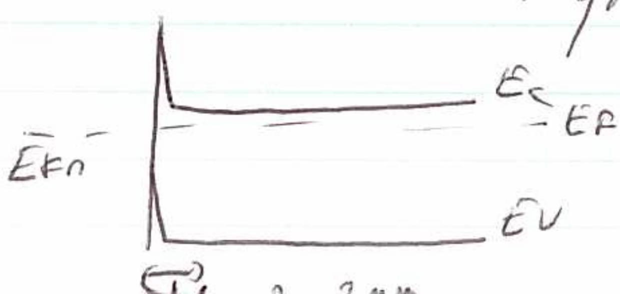
we consider M-SC contact with $E_{Fm} > E_F$ [n-type semiconductor]



the magnitude of the band bending and its extension to the semiconductor are very small.
⇒ there is virtually no potential barrier and e^- can flow freely through this Ohmic contact.

* it is also possible to obtain an Ohmic contact from a Schottky diode $E_{Fm} < E_F$ if the impurity concentration is high enough.
($N_d = 10^{20}\ cm^{-3}$)

since $W(V_A) = \sqrt{\dfrac{2\varepsilon (V_i - V_A)}{q N_d}}$ if $N_d \nearrow$ $W \searrow$



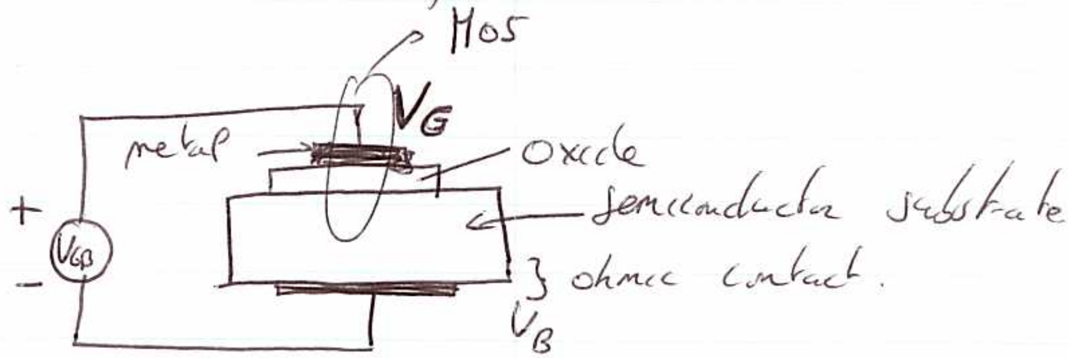
⇒ tunnel contact ~~with~~ low-resistance
⇒ Ohmic contact

III The MOS capacitors

MOS = Metal - Oxide - Semiconductor.
(or MIS = Metal - Insulator - Semiconductor)

Most used device in VLSI technology [Si/SiO_2]
oxide thickness typically from 5-50nm.
oxide is an amorphous material

(1) Structure and principle of operation



- for a P-type substrate we talk about nMOS capacitor. (since inversion layer contains e^-).
- for a N-type substrate we talk about pMOS capacitor.

. we consider in the following nMOS capacitor (p-type semiconductor)

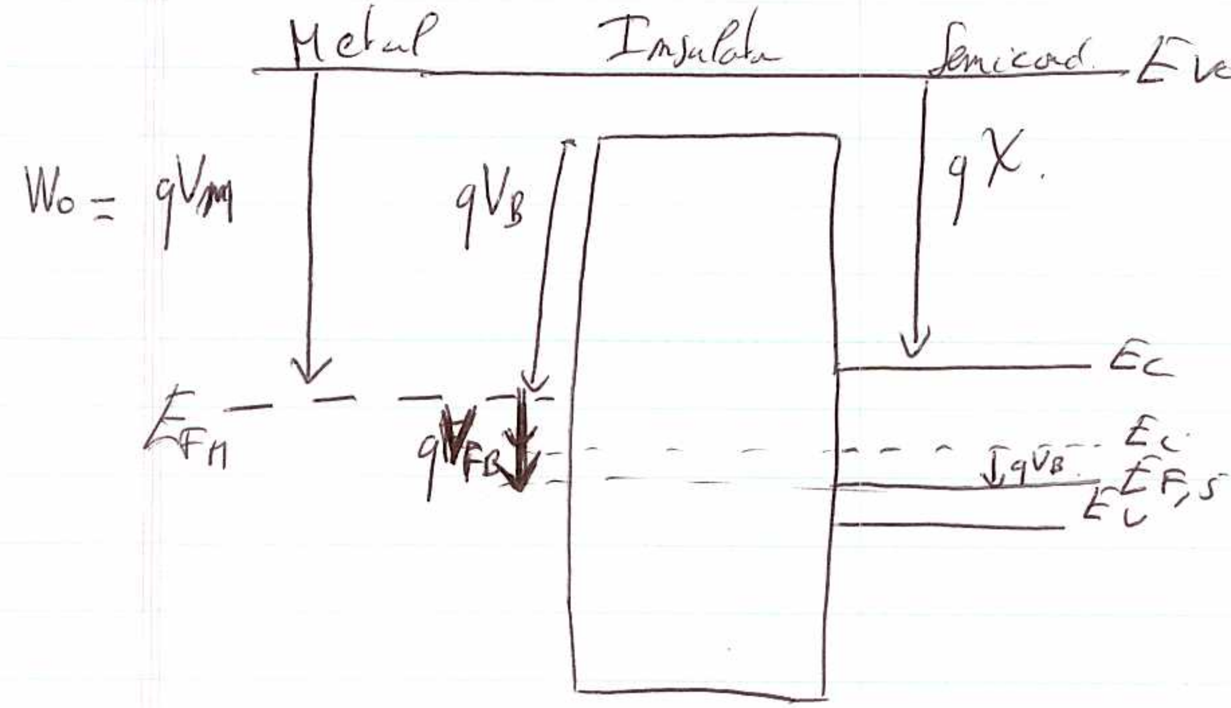
* 4 modes of operation,

flat-band, accumulation, depletion, inversion

(a) 'Unbiased' junction

~~(a) Unbiased junction flat band diagram~~

The energy band diagram of the semiconductor is flat.



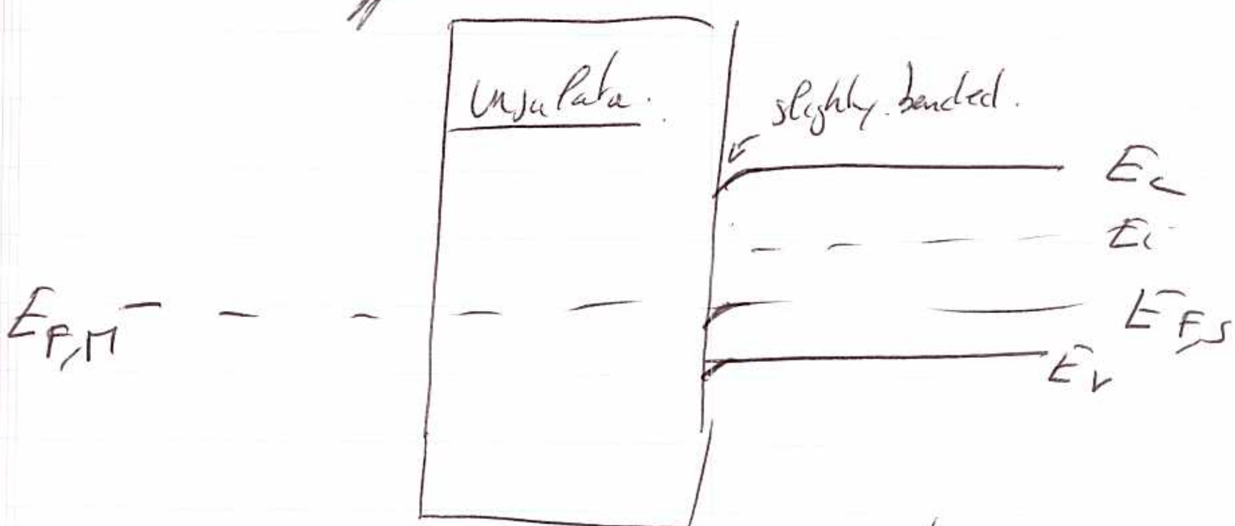
In the ideal case, charge would flow across the insulator so that E_{FM} and $E_{F,S}$ would line-up and the difference, V_{MS}, between the metal and SC workfunctions would vanish.

$$V_{MS} = V_m - \left(\chi + \frac{E_g}{2} + V_B\right) = 0 \quad [\text{p-type semiconductor}]$$

In practice the ideal situation is never achieved (extremely long-time).

The application of a small bias V_{FB}, is required to line-up the Fermi-level. Its value is given by the non-vanishing V_{MS}.

So the flat-band ~~e.~~ diagram is obtained when the applied gate voltage equal to the work function difference.



$\boxed{\text{Zero bias}}$ ~~cl~~ condition.

~~(b) Accumulation, Depletion, Inversion~~

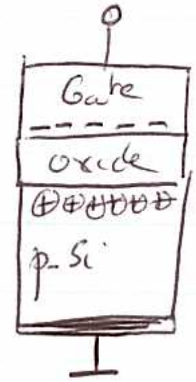
(b) Biased junction

To understand the different bias modes of an MOS capacitor, we now consider 3 different bias voltages. ~~these bias~~ $V_G < 0$, $0 < V_G < V_T$, $V_T < V_G$

(V_T: threshold voltage)

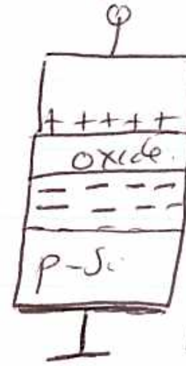
These bias regime are called = accumulation, depletion and inversion ~~mode~~.

(i) accumulation



negative charge on the gate attracts holes from the substrate to the interface.

(ii) Depletion



negative charge builds up in the semiconductor [depletion layer]

(iii) inversion

if potential ↗
another type of negative charges emerges at the oxide-semi interface ⟹ due to minority carriers which form a so-called inversion layer.
This charge increases exponentially with the surface potential.

(2) Accumulation

Negative bias applied to the metal gate (silicon substrate is grounded), the structure behaves like parallel-plate capacitor.

V_s = surface potential