I] P_N junctions

(1) Introduction

P-N junction is formed when a P-type and an N-type semiconductor are in contact.

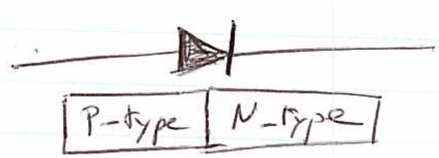
- if the N-type and P-type are made out of the same semic. material ⟹ homojunction
- if the semicond. are different ⟹ heterojunction. (we will study this later)

Application of P-N junctions :

- diode = ~~...~~ single P-N junction _ (2 terminal devices) present a highly non-linear current_voltage characteristics and it is often used as a rectifying element_ Some diode can emit light
- BJT = two P_N junction _ (3 terminal devices) capable of amplifying electrical signals_

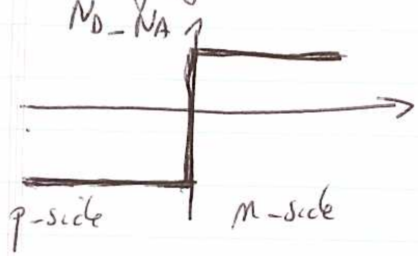
Properties :

P_N junction allows the current flow in one bias direction, but not in the other..

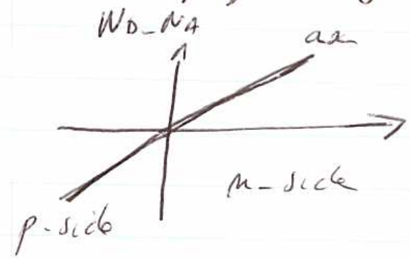


1 P-N junction can be separated into two major categories:

- step junction



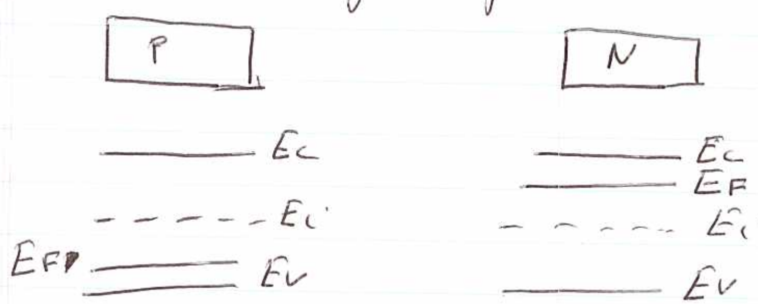
- Linearly-graded junction



2 unbiased junction

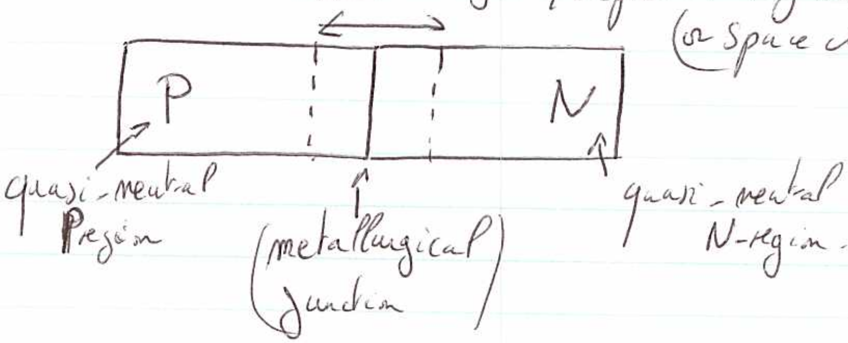
P-N junction is considered at equilibrium.
(no applied bias)

- If we consider 2 separate pieces:

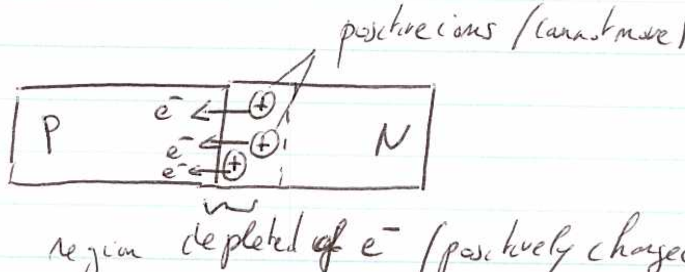


$$E_i - E_F = \frac{1}{\beta} \ln\left(\frac{N_a}{n_i}\right) \qquad E_F - E_i = \frac{1}{\beta} \ln\left(\frac{N_d}{n_i}\right)$$

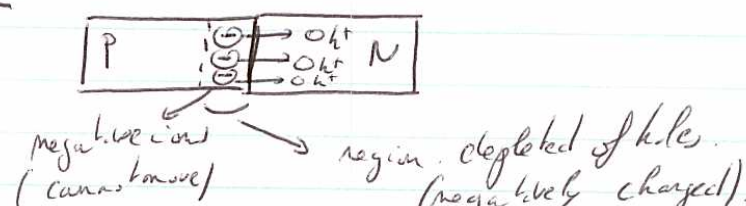
Let us connect the 2 pieces and form a PN step junction.



- **e^- diffusion**

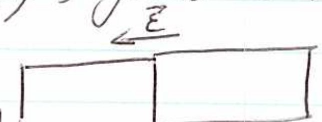


- **hole diffusion**

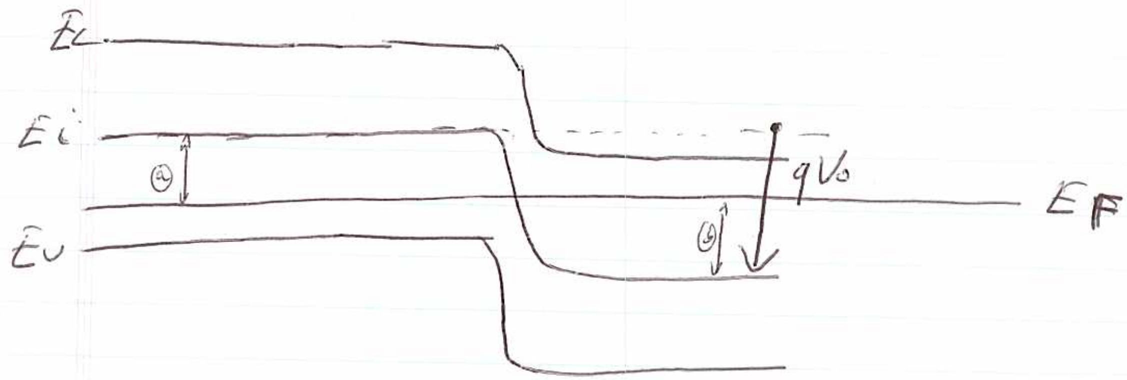


- **Built-in potential**

as a result to the diffusion processes, a built-in potential, V_0, is formed at the junction.



$\vec{E}$ recalls the e^- back to the N-type region.
$\vec{E}$ " holes back to the P-type region.



$$qV_0 = \overbrace{(E_i - E_F)_p}^{(a)} + \overbrace{(E_F - E_i)_n}^{(b)}$$

$$= \frac{1}{\beta} \ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow \boxed{V_0 = \frac{k_B T}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

- What are the potential variation and electric field in the depletion region?

→ Poisson equation

$$-\frac{d^2 V(x)}{dx^2} = \frac{-q}{\epsilon_s}\left(n + N_a^- - p - N_d^+\right)$$

~~with~~ it is non-linear since

$$\left[\begin{array}{l} n(x) = n_{no}\, e^{qV(x)\beta} \\ p(x) = p_{po}\, e^{-qV(x)\beta} \end{array}\right\}$$

Boltzmann relationship. with n_{no}, p_{po} are the e^- (hole) densities in the quasi-neutral region N (and P) we cannot solve the Poisson equation analytically, so we use here approximations.

'depletion approximation' $\Rightarrow$ $\vec{E}$ vanishes outside the depletion region (neutrality) and we ignore the charge due to free carriers in the depletion region. $\boxed{-l_p \le x \le l_n}$

so $N_D^+ \gg n$ for $0 \le x \le l_n$.
$N_A^- \gg p$ for $-l_p \le x < 0$

$$-\frac{d^2\psi(x)}{dx^2} = +q\frac{N_D}{\epsilon_s} \quad \text{for } 0 \le x \le l_n.$$

$$-\frac{d^2\psi(x)}{dx^2} = -q\frac{N_A}{\epsilon_s} \quad \text{for } -l_p \le x \le 0$$

with B. condition $\left.\frac{dV}{dx}\right|_{-l_p} = \left.\frac{dV}{dx}\right|_{l_n} = 0$

By definition $\begin{cases} \psi(-l_p) = \psi_{p0} \\ \psi(l_n) = \psi_{n0} \end{cases}$

$$\Rightarrow \boxed{\psi(x) = \begin{cases} -\frac{qN_D}{2\epsilon_s}(x-l_n)^2 + V_{n0} & (0 \le x \le l_n) \\ \frac{qN_A}{2\epsilon_s}(x+l_p)^2 + V_{p0} & (-l_p \le x \le 0) \end{cases}}$$

it comes for the electric field $\mathcal{E} = -\frac{dV}{dx}$.

$$\Rightarrow \mathcal{E}(x) = \begin{cases} \frac{qN_D}{\epsilon_s}(x - l_n) & (0 \le x \le l_n) \\ -\frac{qN_A}{\epsilon_s}(x + l_p) & (-l_p \le x \le 0) \end{cases}$$

- ~~the~~ continuity of $\mathcal{E}(x)$ at $x=0$ $\Rightarrow$ $\boxed{N_D l_n = N_A l_p}$ (*)

charge neutrality equation

- Continuity of the potential at $x=0$

$$\Rightarrow 0 = V_{p0} - V_{n0} + \frac{qN_A}{2\epsilon_s} l_p^2 + q\frac{N_D}{2\epsilon_s} l_n^2$$

with (*)

$$\Rightarrow 0 = V_{p0} - V_{n0} + q\frac{N_A}{2\epsilon_s}\frac{N_D^2 l_n^2}{N_A^2} + q\frac{N_D}{2\epsilon_s} l_n^2$$

we know that

$$V_0 = V_{n0} - V_{p0} = q\frac{N_A N_D^2 l_n^2}{2\epsilon_s N_A^2} + q\frac{N_D}{2\epsilon_s} l_n^2 \left(= \frac{k_B T}{q} \log \frac{N_a N_d}{n_i^2}\right.$$

$$\Rightarrow \boxed{\begin{array}{l} l_n = \sqrt{\frac{2\epsilon_s}{q}\frac{V_0 N_a}{N_d(N_a+N_d)}} \\ l_p = \sqrt{\frac{2\epsilon_s}{q}\frac{V_0 N_d}{N_a(N_a+N_d)}} \end{array}}$$

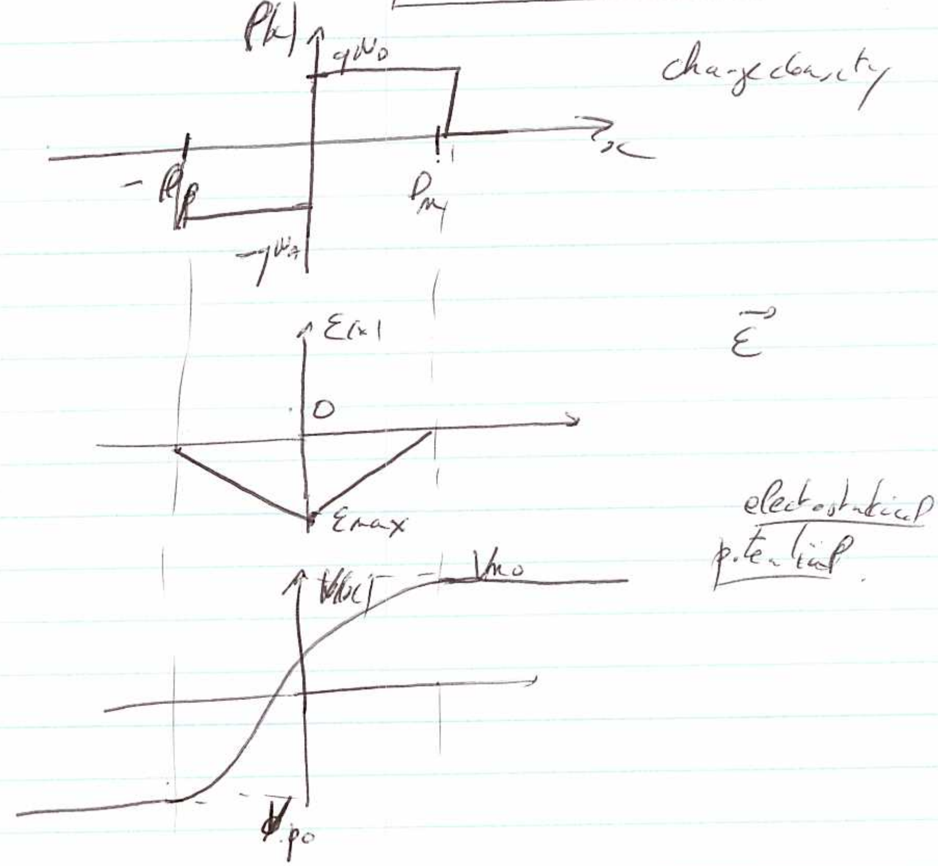
width of the depletion region

$$\boxed{W = l_n + l_p = \sqrt{\frac{2\epsilon_s (N_a + N_d) V_0}{q \, N_a N_d}}}$$

* Maximum electric field at $x=0$

$$\Rightarrow \boxed{\mathcal{E}_{max} = -q\frac{N_A}{\epsilon_s} l_p = -\frac{qN_D}{\epsilon_s} l_n.}$$

<u>Summary</u>



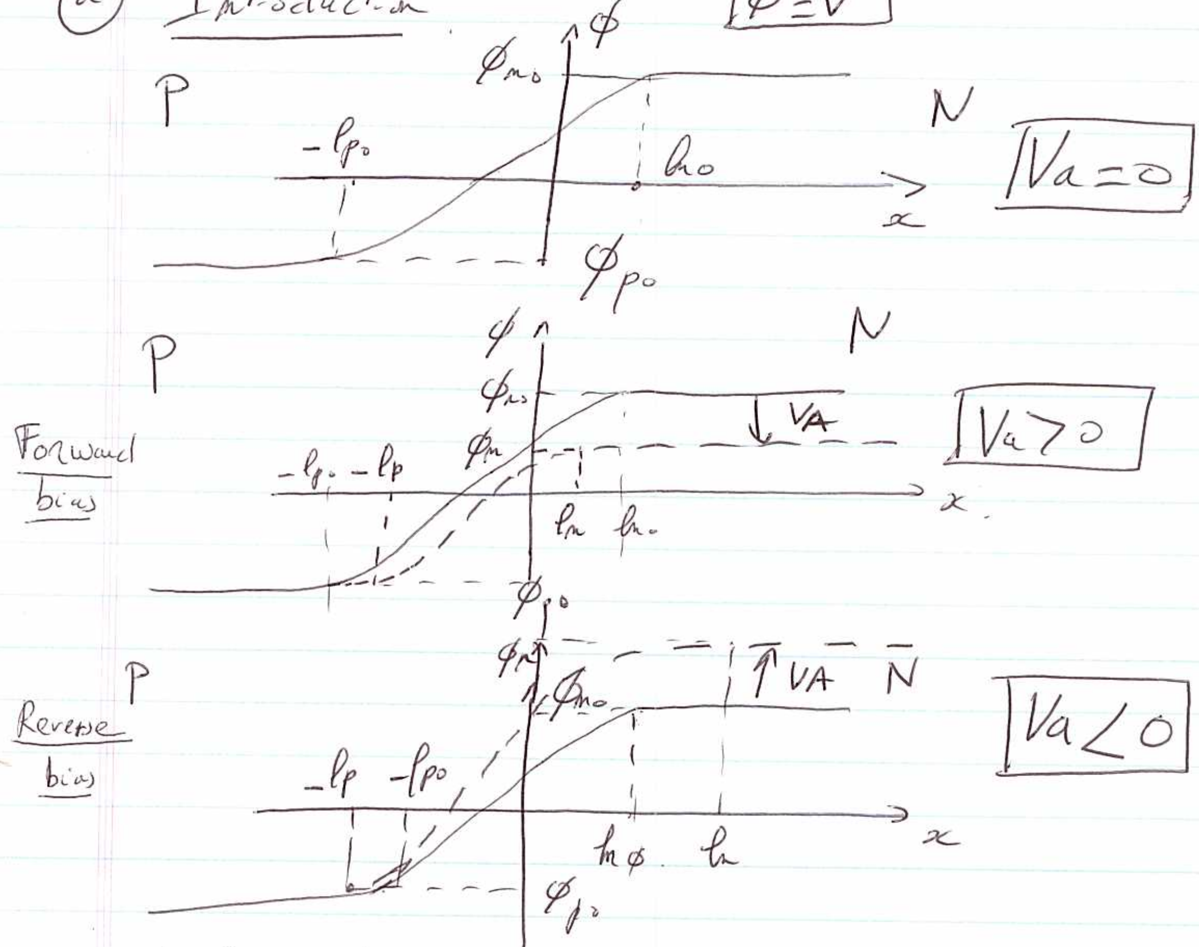
3 Biased Junction

an external bias, V_a, is now applied to the junction

we want to obtain the current_voltage characteristics.

a Introduction

$\boxed{\phi \equiv V}$



Assumption Potential drops across the quasi-neutral region are negligible (small current) $\Rightarrow$ $\boxed{\begin{aligned}\phi &= \phi_N - \phi_P \\ \phi &= \phi_0 - V_A\end{aligned}}$

potential drops across the transition region.

we get for l_n and l_p (using previous expressions where we replace $\phi_0 \to \phi_0 - V_A$).

$$\Rightarrow \quad l_p = \sqrt{\frac{2\epsilon}{q} \frac{(\phi_0 - V_A) N_d}{N_a (N_a + N_d)}} \qquad l_n = \sqrt{\frac{2\epsilon}{q} \frac{(\phi_0 - V_A) N_A}{N_d (N_d + N_A)}}.$$

$$W = l_n + l_p = \sqrt{\frac{2\epsilon}{q} \frac{(\phi_0 - V_A)(N_A + N_d)}{N_A N_d}}.$$

Note. $W \nearrow$ if $V_A < 0$
. $W \searrow$ if $V_A > 0$

~~* if $V_A > 0$~~

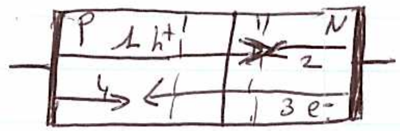
* Forward bias

Diffusion and electric field (drift) forces are no longer equal and of opposite sign.

$\Rightarrow$ Non-equilibrium situation where
Diffusion process > drift phenomena.

$\Rightarrow e^-$ can flow from $N \to P$
holes can flow from $P \to N$.

These carriers diffuse into N or P-type region with an average distance "diffusion length" before they start recombining with majority carriers.



[1: holes injected from P→N. 2: $\bar{e}$ recombining.
3: $\bar{e}$ injected from N→P ; 4: hole recombining..

[current 1 == current 2.
current 3 == current 4]

* Reverse Bias

the electric field in the depletion region ↗ and the associated drift current > diffusion current

the magnitude of the resulting net current is very small. (few $\bar{e}$ in the P region, few hole in the N region).

⇒ [P-N junction behave like a diode rectifying current flow.

* the derivation of current-voltage characteristics will make use of currents 1 and 3 (minority carriers injection).

the total current is equal to the sum of these two currents.

b) Ideal diode

$$\boxed{\text{No G/R in the depletion region}}$$

Let us make the following simplifying assumptions:

→ ① Low-level injection: the concentration of free carriers injected in a quasi-neutral region is low compared to the charge of the majority carriers while solving the Poisson equation.

→ ② The Boltzmann relationships are valid in the quasi-neutral regions as well as in the transition region.

→ ③ Current flow in the quasi-neutral region is due to a diffusion mechanism. (no electric field in these region).

→ ④ The quasi-neutral regions of the diode are infinitely long

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→ ① Low-level injection:

excess concentration of minority carriers are

$p'_n(x)$ in the N-type quasi-neutral region $p'_n(x) = p_n(x) - p_{no}$

$n'_p(x)$ in the P-type region $n'_p(x) = n_p(x) - n_{po}$

$$\boxed{\begin{array}{ll} p'_n(x) \ll n_{no} & \text{(charge of majority carriers)} \\ n'_p(x) \ll p_{po} & (\quad \text{"} \quad) \end{array}}$$

Also ~~[illegible]~~ the concentration of the majority carriers is not modified by the injection of minority carriers.

$n_n(x) = n_{n0} = N_d$ in the N-type quasi-neutral region.
$p_p(x) = p_{p0} = N_a$ in the P-type quasi-neutral region.

→ ② Boltzmann relationship

$$n(x) = n_i \exp\left[\beta(E_{FN} - E_{i0})\right] \exp\left[q\beta V(x)\right]$$

- $n_n(x) = n_{n0} = N_d$ for $l_n \leq x \leq \infty$ and $\boxed{V(x) = V_n}$
- so $n_{n0} = \exp\left[\beta(E_{FN} - E_{i0})\right] \exp\left[q\beta V_n\right]$

$$\Rightarrow \quad n(x) = n_{n0} \exp\left[\beta q \left(V(x) - V_n\right)\right]$$

- if this equation valid also in the transition region.
 at $x = -l_p$ where $V(x) = -V_p$.

$$n_p(-l_p) = n_{n0} \exp\left[\beta q (V_p - V_n)\right]$$

and since $V_p - V_n = V_A - V_0$ it comes.

$$\boxed{n_p(-l_p) = n_{n0} \exp\left[-qV_0\beta\right] \exp\left[qV_A\beta\right]}$$

we know that $$V_0 = \frac{1}{q\beta} \ln\left(\frac{N_a N_d}{n_i^2}\right) = \frac{1}{q\beta} \ln\left(\frac{p_{p0}\, n_{n0}}{n_i^2}\right)$$

we get ~~finally~~

$$n_p(-\ell_p) = \frac{n_i^2}{p_{p0}} \exp[\beta q V_A] \quad , \; \cancel{\text{since}}$$

Since

$$n_{p0}\, p_{p0} = n_i^2$$

$$\Rightarrow \boxed{n_p(-\ell_p) = n_{p0} \exp[q\beta V_A]}$$

Similarly, $$\boxed{p_n(\ell_n) = p_{n0} \exp(q\beta V_A)}$$

So excess of e^- at the P-side edge of the transition region

$$n'_p(-\ell_p) = n_{p0}\left[\exp(q V_A \beta) - 1\right]$$

and excess of holes

$$p'_n(\ell_n) = p_{n0}\left[\exp(q V_A \beta) - 1\right]$$

(3) $\vec{\mathcal{E}} = 0$ in the quasi neutral region.

$$\Rightarrow \quad J_p = -q D_p \frac{dp}{dx} \quad \text{N-type quasi-neutral region.}$$

$$J_n = q D_n \frac{dn}{dx} \quad \text{P-type quasi neutral region.}$$

[we need first to derive p and n]

Using the continuity equation (no external source)

$$\frac{\partial p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{p_n - p_{no}}{\tau_p}$$

if we replace J_p, we get.

$$\frac{\partial p_n}{\partial t} = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{no}}{\tau_p} = 0$$

if steady state conditions

So $$D_p \frac{\partial^2 p_n}{\partial x^2} = \frac{p_n - p_{no}}{\tau_p}$$

$$\Rightarrow \frac{\partial^2 p_n}{\partial x^2} - \frac{1}{D_p \tau_p}(p_n - p_{no}) = 0$$

$$p_n(x) = p_{no} + A\exp(-x/L_p) + B\exp(x/L_p) \qquad \boxed{L_p = \sqrt{D_p \tau_p}}$$

diffusion length.

<u>Similarly</u>

$$n_p(x) = n_{po} + A\exp(-x/L_n) + B\exp(x/L_n) \qquad \boxed{L_n = \sqrt{D_n \tau_n}}$$

→ 4 quasi-neutral regions are infinitely long ⇒ "long-base diode"

$p_n(\infty) = p_{no}$ (Boundary condition due to thermodynamic equilibrium far from the junction)

$\Rightarrow B = 0$

$$p_n(l_n) = p_{no}\exp[qV_A\beta] = p_{no} + A\exp\left(\frac{-l_n}{L_p}\right)$$

$$\Rightarrow \quad A = p_{no}\left[\exp(qV_A\beta) - 1\right]\exp\left(\frac{l_n}{L_p}\right)$$

we get finally for the holes

$$\boxed{p_n(x) = p_{no} + p_{no}\left[\exp(qV_A\beta) - 1\right]\exp\left(\frac{-(x-l_n)}{L_p}\right)}$$

in the N-region $x > l_n$

So $$\boxed{J_p = -qD_p\frac{dp_n}{dx} = q\frac{D_p\, p_{no}}{L_p}\exp\left(\frac{-(x-l_n)}{L_p}\right)\left[\exp(qV_A\beta) - 1\right]}$$

Similarly in the P-region $x < -l_p$

$$\boxed{J_n = qD_n\frac{dn_p}{dx} = q\frac{D_n\, n_{po}}{L_n}\exp\left(\frac{x+l_p}{L_n}\right)\left[\exp(qV_A\beta) - 1\right]}$$

Remarks:

These currents depend on x, for example $J_p(x) \searrow$ if $x \nearrow$. This is due because of the recombination process. Here the hole current is transformed into an e^- current (via recombination). So, the total current is independent of x.

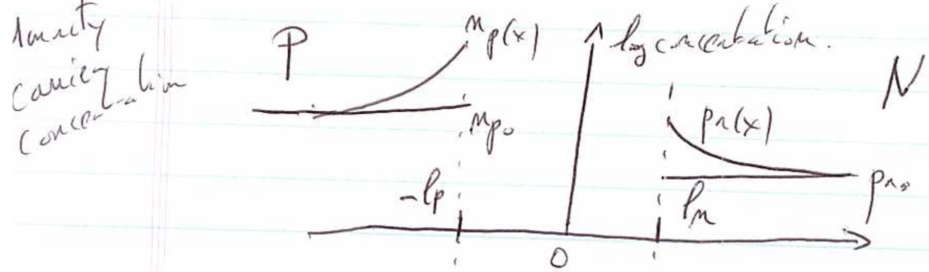
the cte potential can be obtained using our assumption that there is no G/R in the depletion region.
we can evaluate these current at a position in which they have not yet started to decay: at $x = l_n, -l_p$

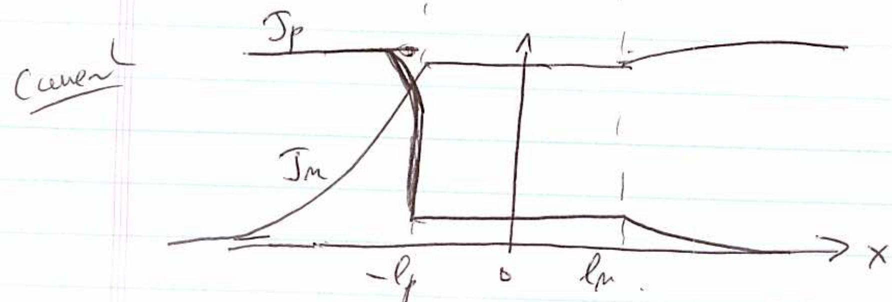
$\Rightarrow$ the total current is then given by:

$$J_{total} = J_p(l_n) + J_n(-l_p) = \left[q\frac{D_n n_{po}}{L_n} + q\frac{D_p p_{no}}{L_p} \right] \exp\left[q\beta V_A \right] - 1$$

$$\boxed{J_{total} = J_s \left(\exp\left[q\beta V_A \right] - 1 \right)} \Rightarrow \text{Shockley's equation}$$

$\Rightarrow$ with J_s the 'saturation current density'





Remark if $V_A < 0$, the total current is I_S independent of V_A, and the electric field in the structure.

(c) Deviation from ideality

G/R in the depletion region; these processes are proportional to $np - n_i^2$.

if the external bias is applied, excess carriers are injected ($V_A > 0$) into or extracted ($V_A < 0$) from the transition region.

so $pn \neq n_i^2$

We consider two quasi Fermi-level, E_{Fn} for e^- and E_{Fp} for holes

According to class notes p62.

we get

$$\boxed{pn = n_i^2 \exp\left[\frac{E_{FN} - E_{FP}}{k_B T}\right] = n_i^2 \exp\left[\frac{qV_A}{k_B T}\right]}$$

since $E_{FN} - E_{FP} = qV_A$

we know that the SRH G/R rate is

$$U = \frac{pn - n_i^2}{\tau_0 \left(p + n + 2n_i \cosh\left[\frac{E_t - E_i}{k_B T}\right]\right)}$$

* we assume that only traps at energy $E_T = E_i$ contribute effectively to the G/R processes.

$\Rightarrow$ it comes (using the two last equations)

$$\boxed{U \simeq \frac{n_i^2 \left[\exp(q\beta V_A) - 1\right]}{\tau_0 (p + n + 2n_i)}}$$

using the continuity equation in steady state

$$\boxed{\frac{dJ_n}{dx} = -\frac{dJ_p}{dx} = q\,U(x)}$$

so far we have calculated $J_n(-\ell_p)$ that we ~~supposed~~ assumed equal to $J_n(\ell_n)$ since no G/R.

we want now to evaluate the influence of G/R:

$$\int_{-\ell_p}^{\ell_n} \frac{dJ_n}{dx}\,dx = \int_{-\ell_p}^{\ell_n} q\,U(x)\,dx$$

$$\Rightarrow \boxed{J_n(\ell_n) = J_n(-\ell_p) + q \int_{-\ell_p}^{\ell_n} U(x)\,dx.}$$

the net current is then

$$\boxed{J = J_p(\ell_n) + J_n(\ell_n) = J_p(\ell_n) + J_n(-\ell_p) + J_{GR}.}$$

$$\boxed{= J_s\left[\exp(q\beta V_A) - 1\right] + J_{GR}}$$

we want to evaluate J_{GR} (correction to the Schockley term) using a simple approximation.

$J_{G/R}$ will be dominated at a position where $p+n$ is a minimum; Also pn is constant.

we get $d(p+n)=0 \Rightarrow dp=-dn$.

and $pn = c^{te} \Rightarrow d(pn)=0 \quad n\,dp + p\,dn = 0$

it comes $p=n$

so $\boxed{n = p = n_i \exp\left(\frac{qV_A\beta}{2}\right)}$

$$\Rightarrow U = \frac{n_i\left[\exp\left(\frac{qV_A\beta}{2}\right)-1\right]}{2\tau_0} = U_{max}$$

assuming that G/R takes this value U_{max} over the entire transition region [not correct - we overestimate the effects here!]

$$\Rightarrow J_{GR} \simeq q\,U_{max}\,(l_n+l_p) \qquad \boxed{W = l_n + l_p}$$

$$J_{tot} \simeq J_s\left[\exp(qV_A\beta)-1\right] + q\frac{W n_i}{2\tau_0}\left[\exp\left(\frac{qV_A\beta}{2}\right)-1\right]$$

if $V_A > 0$ but small, the $J_{R/G}$ is larger than the diffusion current
if $V_A < 0$ $J_{R/G}$ adds to the reverse current.

if $V_A > 0$ — small bias

the I-V follows $\exp\left(\frac{qV_A}{2}\beta\right)$ laws $\Rightarrow$ characteristics of a recombination dominated current

high bias

I-V follows $\exp(qV_A\beta)$ $\Rightarrow$ diffusion current takes over and completely overshadows the recombination current

if $V_A < 0$

$$pn = n_i^2 \exp\left(\frac{qV_A}{k_BT}\right) < n_i^2$$

so generation current can be observed in the reverse bias.

I-V characteristics

(d) Junction Breakdown

if $V_A < 0$ strongly reversed biased, $\vec{E}$ near the metallurgical junction can reach high value. Carriers accelerated in that field can accumulate enough kinetic energy that they can, through a collision process, generate e^--hole pairs through impact ionization "impact ionization"

when an e^- is accelerated to high $\vec{E}$, its kinetic energy can be equal or larger than the bandgap E_g.
That energy can be released through a collision event while creating e^--hole pair. So instead of having $1e^-$ at high energy, we have 2 free e^- + 1 hole.
we call it generation "by impact ionization".
$\Rightarrow$ avalanche multiplication phenomenon.

In the P-N junction a sudden increase of current is observed $\Rightarrow$ breakdown.

[BV = junction breakdown voltage.

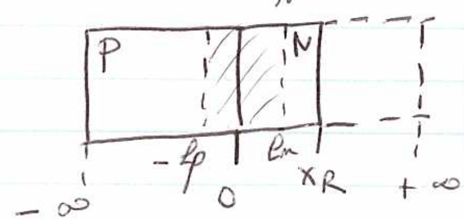
Remark:
If N-type and P-type region are heavily doped, the width of the depletion region is very small, and e^- can directly tunnel from P-type valence into N-type conduction band.
these diodes are called "Zener diodes" where can be accurately controlled while adjusting concentration.
Zener diode are then used as voltage reference.

(e) Short-base diode

previously we assume that the length of the P and N quasi-neutral region $\gg$ diffusion length of the minority carriers

the term 'short base' comes from bipolar transistors that consist in two P-N junctions PNP or NPN. and the central region is called 'base'.

Consider a P-type region $\gg L_n$
a N-type region 'short base' compared to L_p.



the previous assumption $p_n(x \to +\infty) = p_{n0}$ must be replaced by $p_n(x_R) = p_{n0}$.

In addition the minority holes will not be able to recombine in the base $\Rightarrow J_p = \text{cte}$. (constant)

$$J_p = -q D_p \frac{dp}{dx} \Rightarrow p(x) = -\frac{J_p x}{q D_p} + B \quad \left(\begin{array}{c}\text{linear}\\ \text{function}\end{array}\right)$$

at $x = l_n$ $\quad p(x = l_n) = p_{n0} \exp(q V_A \beta)$.

$$\Rightarrow B = p_{n0} \exp(q V_A \beta) + \frac{J_p l_n}{q D_p}$$

* Also

$$p(x_R) = p_{n0} = -\frac{J_p x_R}{qD_p} + p_{n0}\exp\left(\frac{qV_A}{\beta}\right) + \frac{J_p l_n}{qD_p}$$

$$\Rightarrow \boxed{J_p = \frac{qD_p p_{n0}}{x_R - l_n}\left[\exp\left(\frac{qV_A}{\beta}\right) - 1\right]}$$

(4) PN junction Capacitance

So far we considered only steady state characteristics. In practical application, it is important to know how quickly the device can adjust to a new bias condition ("transient effects").

Capacitance calculations will help to estimate this "time response" of the device.

Capacitance is measure of charge stored per unit change of voltage $\Rightarrow$ if capacitance is large, this means more charge must be moved in or out, so that for a fixed current $\Rightarrow$ more time is needed to complete the process.

In P-N junction two major capacitance =

(i) capacitance associated with the charge which must be moved in or out of the depletion region.
$\Rightarrow$ depletion capacitance

(ii) "diffusion capacitance" under forward bias due to minority carriers.

(1) depletion capacitance

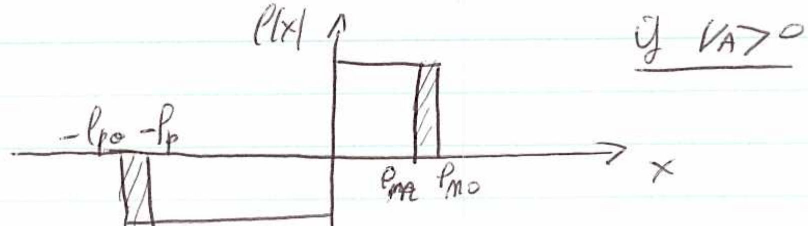
we know that
$$\begin{cases} l_p = \sqrt{\dfrac{2\epsilon}{q}\dfrac{(V_0 - V_A) N_d}{N_A (N_A + N_d)}} \\[2ex] l_n = \sqrt{\dfrac{2\epsilon}{q}\dfrac{(V_0 - V_A) N_A}{N_d (N_A + N_D)}} \end{cases}$$

the charge of the fixed, ionized doping impurities in each depletion region is (absolute value)

$$\boxed{Q = A q N_d l_n = A q N_A l_p}$$

in Coulombs.

A = cross-section of the junction.



capacitance is $C_T = \left|\dfrac{dQ}{dV_A}\right| = A q N_d \left|\dfrac{dl_n}{dV_A}\right|$

$$\Rightarrow \boxed{C_T = A \frac{\epsilon}{l_n + l_p}}$$

== capacitance of parallel plate capacitor with a dielectric of permittivity ϵ.

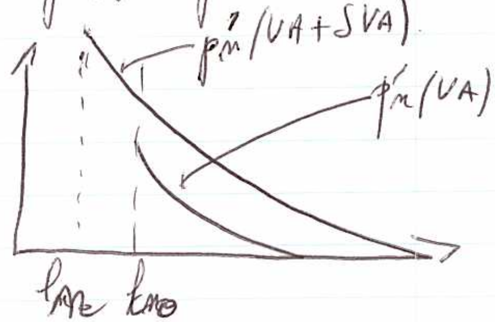
(ii) Diffusion capacitance

hole concentration in the n-type region

$$p_n(x) = p_{n0} + p_{n0}\left[\exp(q\beta V_A) - 1\right]\exp\left(\frac{-(x-l_n)}{L_p}\right)$$

excess concentration

$$p'_n(x) = p_n(x) - p_{n0}$$



$\int_{l_n}^{+\infty} \ldots$ ||

$$Q = q\int_{l_n}^{\infty} p'_n(x) = q\,p_{n0}\left[\exp(qV_A\beta) - 1\right] \times L_p$$

$$\boxed{C_{Dp} = \frac{dQ}{dV_A} = \beta q^2 L_p\, p_{n0} \exp(qV_A\beta)}$$

Also $\boxed{C_{Dp} = \beta q \tau_p J_p(l_n)}$

Similarly $C_{DN} = \beta q \tau_n J_n(-l_p)$

$$\Rightarrow \boxed{C_D = C_{Dp} + C_{DN} = \beta q\left[\tau_p J_p(l_n) + \tau_n J_n(-l_p)\right]}$$

II Metal - semiconductor contacts

2 differents types of metal-semiconductor contacts:

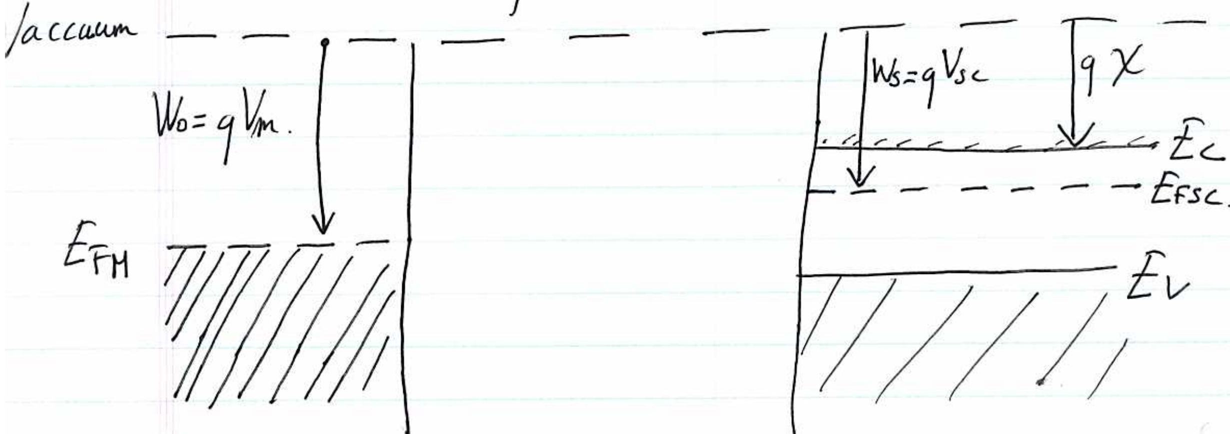
(i) - a Schottky contact = non-linear, rectifying current voltage characteristics.

(ii) an Ohmic contact = linear, non-rectifying

I Schottky diode

a) Unbiased junction

Consider a N-type semiconductor and a metal.



Metal

Semiconductor - N-type

E_{FM} = Fermi-level of the metal.

W_0 = Work function
energy required to extract an e^- at E_{FM}

E_{Fsc} = Fermi level

W_s = Work-function.

$q\chi = e^-$ affinity

$\Rightarrow$ energy required to extract an e^- from the CB to vaccum.
(energy of e^- in CB $\sim E_c$).

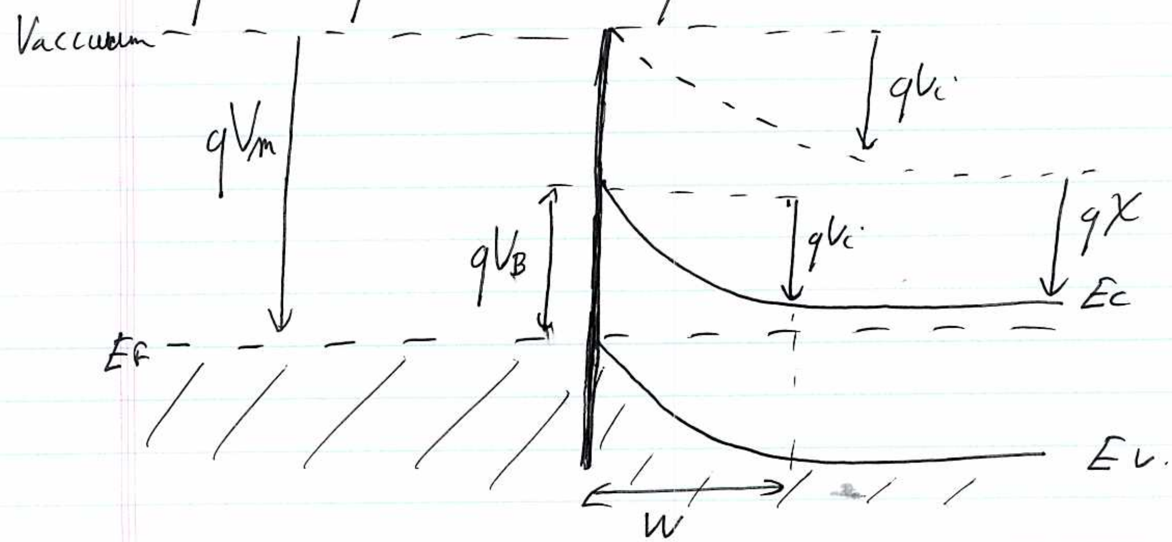
We consider $E_{FM} < E_{FSC}$.

* Since $E_C > E_{FM}$, e^- flow from the semiconductor to the metal until a new equilibrium will be reached
e^- leave behind positively charged donor atoms and a depletion region is formed in the semiconductor.
(W its width).

Because of the alignment of the Fermi-levels $\Rightarrow$

we get a band curvature $qV_i = q(V_m - V_{SC})$
which correspond to a potential barrier preventing further e^- to migrate into the metal.
E^- in the metal see a potential barrier V_b.

$$qV_b = q(V_m - \chi) = qV_i + (E_C - E_F).$$



qV_i, qV_B much larger than $\frac{k_B T}{q}$, only few $\bar{e}$ can overcome them at room temperature.
the resulting current of $\bar{e}$ migrating from the semiconductor to the metal is $I_{M\to S}$ (negative charge).

At thermodynamic equilibrium $I_{M\to S} = -I_{S\to M}$.

(b) biased junction

* if a forward bias $V_A > 0$ is applied to the junction

$$V_i \to V_i - V_A$$

more $\bar{e}$ can flow from the semiconductor to the metal
however $I_{S\to M}$ remains constant since V_b is unchanged

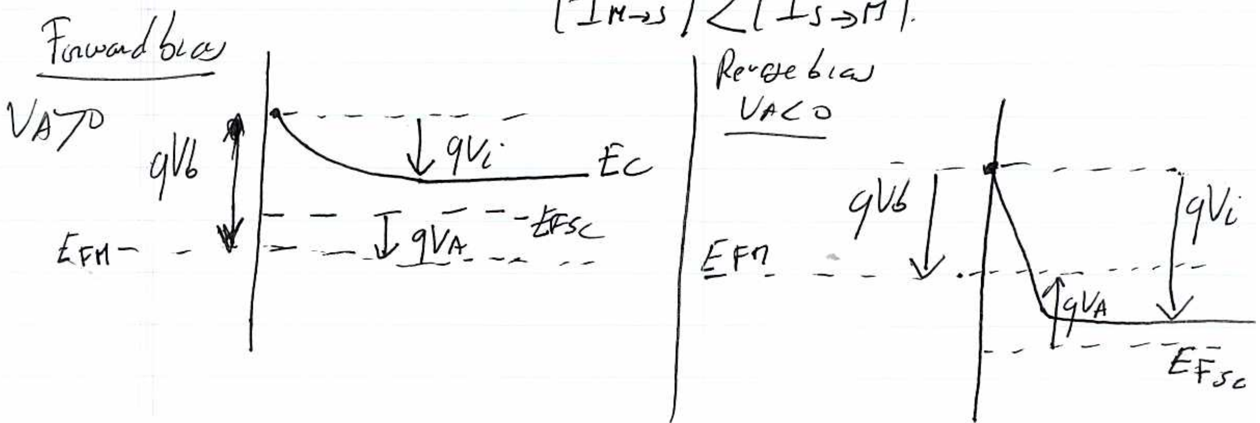
$$|I_{M\to S}| > |I_{S\to M}|$$

* if reverse bias $V_A < 0$ is applied to the junction.

$$V_i \to V_i - V_A$$

the electron flow from the semiconductor to the metal is reduced. • $I_{S\to M}$ remains unchanged

$$|I_{M\to S}| < |I_{S\to M}|.$$



The width of the depletion zone can be calculated using the Poisson equation + depletion approximation.

$$-\frac{d^2 \cancel{\phi}(x)}{dx^2} = \frac{\rho}{\epsilon} = +\frac{qN_d}{\epsilon}$$

at $x = W$ $\quad \cancel{V}(W) = 0$ and $\frac{dV(W)}{dx} = 0$.

$$\Rightarrow \quad +\frac{dV(x)}{dx} = \frac{qN_d}{\epsilon}(W - x)$$

$$\boxed{V(x) = -\frac{qN_d}{2\epsilon}(W - x)^2}$$

at $x = 0$ $\quad V(0) = V_i - V_A$.

$$\Rightarrow \quad -(V_i - V_A) = -\frac{qN_d}{2\epsilon}W^2$$

$$\Rightarrow \quad \boxed{W = \sqrt{\frac{2\epsilon}{qN_d}(V_i - V_A)}}$$

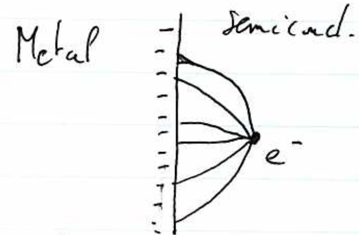
$$\Rightarrow \quad \boxed{\mathcal{E}(0) = -\frac{qN_d}{\epsilon}W = -\sqrt{\frac{2qN_d}{\epsilon}(V_i - V_A)}}$$

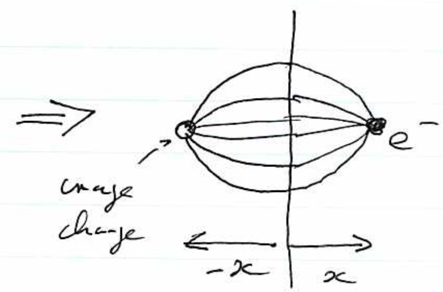
$$\boxed{\mathcal{E}(x) = -\frac{qN_d}{\epsilon}(W - x)}$$

(c) Schottky effect

A lowering of V_b is observed due to a mirror charge produced in the metal by e^- in the semicond

From Electrostatics, we know that when a charge is near a "perfect conductor", a mirror charge of same magnitude but opposite sign is created inside the conductor.





Field lines and surface charges due to an e^- in close proximity to a perfect conductor

electrostatic force between 2 particles $\Rightarrow$

$$F(x) = \frac{-q^2}{4\pi\epsilon(2x)^2} \Rightarrow \cancel{\text{potential energy } \phi(x) = -\int_x^\infty \frac{F}{q}\,dx}$$

$$\left.\begin{aligned} F &= q\mathcal{E} \\ \mathcal{E} &= -\nabla V \\ U &= -qV \end{aligned}\right\} \quad \mathcal{E} = \frac{\nabla U}{q} \Rightarrow F = \nabla U \Rightarrow \boxed{U = \frac{-q^2}{16\pi\epsilon x}} \quad \cancel{\frac{-q^2}{16\pi\epsilon x}}.$$

~~For an e in the conduction band the potential energy is~~ $\cancel{\frac{-q^2 N_d (W-x)^2}{2\epsilon} + E_c}$

For an e^- in the CB, potential energy is $-q^2 \frac{N_d (W-x)^2}{2\epsilon}$ (31)

To simplify the expression, we suppose that the electric field in the depletion region is cte == $\mathcal{E}_{max}$ [$\mathcal{E}$ = $\mathcal{E}$

potential energy for the e^- $\Rightarrow$ ~~...~~ $-q\,\mathcal{E}_{max}\,x$ (linear).

So total potential energy (from the mirror charge and the depletion region).

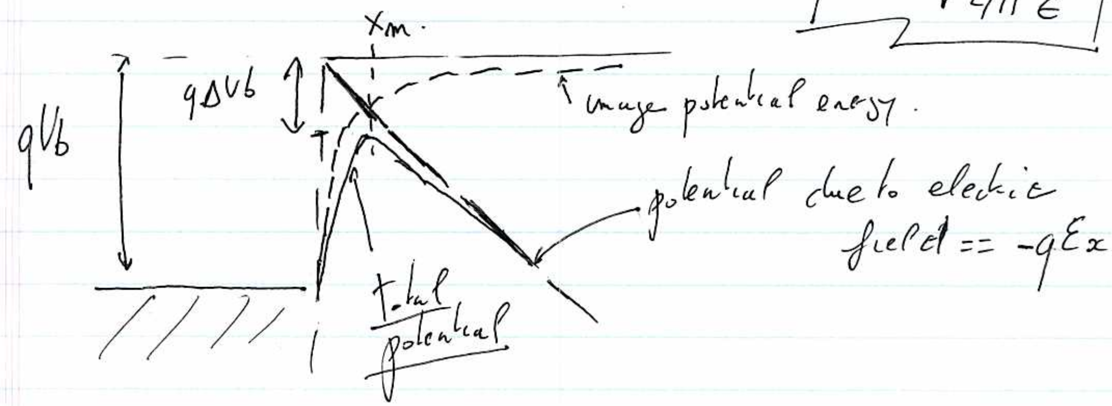
$$U(x) = -q\,\mathcal{E}_{max}\,x - \frac{q^2}{16\pi\epsilon x}.$$

The maximum potential at $x = x_m$ $\Rightarrow$ $\left.\frac{dU}{dx}\right| = 0$

$$\Rightarrow x_m = \sqrt{\frac{q}{16\pi\epsilon\,\mathcal{E}_{max}}}$$

potential due to image charge ~~...~~ $-q\,\Delta V_B = -q\sqrt{\frac{q\,\mathcal{E}_{max}}{4\pi\epsilon_{sc}}} < 0.$

$\Rightarrow$ ΔV_B is the barrier height reduction $\boxed{\Delta V_B = \sqrt{\frac{q\,\mathcal{E}_{max}}{4\pi\epsilon}}}$



2 I_V characteristics of Schottky diode

the current across a M-SC junction is mainly due to majority carriers.
3 different mechanisms exist:
(i) diffusion of carriers from the SC → M
"assumes that the driving force is distributed over the length of the depletion region."
(ii) thermionic emission of carriers across the Schottky barrier
"only energetic carriers, which energy ≥ CB energy at the interface, contribute to the current"
(iii) Quantum mechanical tunneling through the barrier.
"wave nature of the $\bar{e}$".

- The analysis reveals that the diffusion and thermionic currents can be written in the following form:

$$\left[J_n = q \underbrace{v}_{\text{velocity}} \underbrace{N_c \exp(-q\beta V_B)}_{\text{density of available carriers, located next to the interface}} \left[\exp(q\beta V_a) - 1\right] \right]$$

$V = \mu E$ for the diffusion current

$V = V_R$ Richardson velocity for thermionic current

⇒ if no applied voltage $J_n = 0$.

- for the tunneling current $\left[J_n = q v_R n \Theta \right.$

Θ is the tunneling probability term..

a) Diffusion current

we assume that the depletion layer W is large compared to the mean free path $\Rightarrow$ concept of drift-diffusion are valid.

$$J_n = q\left(\mu_n n \mathcal{E} + D_n \frac{dn}{dx}\right)$$

also equal to

$$J_n \exp(-V \times q \times \beta) = q D_n \left(-q\beta n \frac{dV}{dx} + \frac{dn}{dx}\right) \exp(-q\beta V)$$

$$= q D_n \frac{d}{dx}\left[n \exp(-V q \beta)\right]$$

in steady state integration over W $\Rightarrow$ J_n does not depend on x

$$\Rightarrow \quad J_n = \frac{q D_n \left[n \exp(-q\beta V)\right]_0^W}{\int_0^W \exp(-q\beta V)\, dx}$$

Using the following B.C

x	$n(x)$	$V(x)$
0	$N_c \exp(-q\beta V)$	$-(V_i - V_A)$
W	$N_d = N_c \exp(-q\beta V_B) \exp(+q\beta V_i)$	0

$$\Rightarrow \quad J_n = \frac{q D_n N_c \exp(-q\beta V_B)\left[\exp(q\beta V_A) - 1\right]}{\int_0^W \exp(-q\beta V^*)\, dx}$$

where $\boxed{V^* = V + V_i - V_A}$

the denominator can be solved using the potential obtained from the full depletion approximation.

$$V = -\frac{qN_d}{2\epsilon}(x-W)^2$$

so

$$V^* = V - V(0) = -\frac{qN_d}{2\epsilon}\left[x^2 \cancel{+W^2} - 2xW \cancel{-W^2}\right] = \frac{qN_d}{\epsilon}x\left(W - \frac{x}{2}\right)$$

$V^* \simeq \frac{qN_d}{\epsilon} xW$ since the linear term is dominant if $x \ll W$.

$$\boxed{V^* = (V_i - V_A)\frac{2x}{W}}$$

$$\Rightarrow \int_0^W \exp(-V^* q\beta)\, dx = -\frac{W}{2(V_i-V_A)q\beta}\left[\exp(-q\beta V^*)\right]_0^W$$

$$= -\frac{W}{2(V_i-V_A)q\beta}\left[\underbrace{\exp\left[-2q\beta(V_i-V_A)\right]}_{\to 0 \;\; (V_i-V_A) \gg k_BT} - 1\right]$$

$$\simeq \frac{W}{2(V_i-V_A)q\beta} = \frac{1}{q\beta}\sqrt{\frac{\epsilon}{2qN_d(V_i-V_A)}}$$

$$\Rightarrow \boxed{J_n = q^2\beta D_n N_c \underbrace{\sqrt{\frac{2qN_d(V_i-V_A)}{\epsilon}}}_{|E_{max}|} \exp(-V_B q\beta)\left[\exp(q\beta V_A) - 1\right]}$$

we can also write "physically"

$$J_n = \underbrace{q\mu_n |E_{max}| N_c \exp(-q\beta V_B)}\left[\exp(q\beta V_A) - 1\right]$$

Drift current at the interface, if $V_A = 0$ balance the diffusion current.

(b) Thermionic current

The thermionic emission theory assumes that e^- which have an energy larger than the top of the barrier will cross it if they move toward. ~~...~~.
The actual shape of the barrier is ignored.

$$J = \int_{E_c(x=\infty)+qV_i} q V_x \frac{dn}{dE} dE \qquad \boxed{\frac{dn}{dE} = n(E)}$$

For non-degenerate semiconductors, the density of e^- between E and $E+dE$ is given by:

$$\frac{dn}{dE} = g_c(E) F(E) = \frac{4\pi (2m^*)^{3/2}}{h^3} (E-E_c)^{1/2} \exp-\left[\frac{E-E_{F_n}}{k_B T}\right]$$

if parabolic CB with m^* constant

$$\Rightarrow \left(E - E_c = \frac{m^* v^2}{2} \qquad \begin{array}{l} dE = m^* v\, dv \\ \sqrt{E-E_c} = v\sqrt{\frac{m^*}{2}} \end{array} \right.$$

$$\boxed{\frac{dn}{dE} dE = 2\left(\frac{m^*}{h}\right)^3 \exp\left[-\left(\frac{E_c(x=\infty) - E_{F,N}}{k_B T}\right)\right] \exp\left(\frac{-m^* v^2}{2k_B T}\right) 4\pi v^2 dv}$$

if we replace v^2 by $v_x^2+v_y^2+v_z^2$ and $4\pi v^2 dv$ by $dv_x\, dv_y\, dv_z$

$$\Rightarrow J = 2\left(\frac{m^*}{h}\right)^3 \int_{-\infty}^{\infty} \exp\left(-\frac{m^* v_y^2}{2k_B T}\right) dv_y \int_{-\infty}^{+\infty} \exp\left(-\frac{m^* v_z^2}{2k_B T}\right) dv_z$$

$$\int_{V_{0x}} q v_x \exp\left(\frac{-m^* v_x^2}{2k_B T}\right) dv_x \quad \exp\left[-\frac{(E_c - E_{F,N})}{k}\right]$$

Setting V_{ox} = the minimal velocity in the quasi-neutral region that is needed to cross the barrier

Using $$\int_{-\infty}^{+\infty} \exp\left(-\frac{m^* v_y^2}{2k_BT}\right) dv_y = \int_{-\infty}^{+\infty} \exp\left(-\frac{m^* V_z^2}{2k_BT}\right) dV_z = \sqrt{\frac{2\pi k_BT}{m^*}}$$

$\Rightarrow$

$$J = 2q\left(\frac{m^*}{h}\right)^3 \frac{2\pi k_BT}{m^*} \exp\left[-\frac{(E_C - E_{F,N})}{k_BT}\right] \exp\left[-\frac{m^* V_{ox}^2}{2k_BT}\right] \frac{k_BT}{m^*}$$

V_{ox} is obtained by setting the kinetic energy == potential across the n-type region.

$$\frac{m^* V_{ox}^2}{2} = q V_i$$

Using $V_i = V_B - V_A - \frac{1}{q}[E_C - E_{F,N}]$

$$\Rightarrow J_{MS} = A^* T^2 \exp[-(V_B q\beta)][\exp(q\beta V_A)]$$

for the total current.

- 1 term is added to account for the current flowing from right to left; when $V_A = 0 \Rightarrow J_{MS} = -J_{SM}$.

$$\Rightarrow J_n = A^* T^2 \exp\left[-\frac{qV_b}{k_BT}\right]\left[\exp(q\beta V_A) - 1\right]$$

A^* is the Richardson constant

$$\boxed{A^* = \frac{4\pi q m^* k_B^2}{h^3}}$$

one can also define an average velocity ~~with~~ for which the e⁻ at the interface approach the b...

This velocity is referred as the Richardson velocity

$$v_R = \sqrt{\frac{k_B T}{2\pi m}}.$$

$$\Longrightarrow \boxed{J_n = q v_R N_c \exp(-q\beta V_B)[\exp(q\beta V_A) - 1]}$$

(c) Tunneling current

e^- in the metal can tunnel across the Schottky barrier and enter in the semiconductor. Similarly, e^- can tunnel from the SC $\rightarrow$ M.

we start from the Schrodinger equation

$$-\frac{\hbar^2}{2m^*}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = \frac{2m^*(V-E)}{\hbar^2}\psi$$

Assuming that V is independent of position in a section between x, $x+dx$ (~~slowly varying function~~)

we can solve it yielding:

$$\psi(x+dx) = \psi(x)\exp(-k\,dx) \qquad k = \sqrt{\frac{2m^*(V(x)-E)}{\hbar}}$$

For a slowly varying ~~function~~ potential, the wave function at $x = L$ can be related to ψ at $x = 0$

$$\psi(L) = \psi(0) \exp\left[-\int_0^L \frac{\sqrt{2m^*(V(x) - E)}}{\hbar}\, dx\right]$$

$\Rightarrow$ this equation is referred as the

WKB approximation (Wentzel, Kramers, Brillouin) 1926

we suppose a triangular barrier $\boxed{(V(x) - E) = qV_B\left(1 - \frac{x}{L}\right)}$

tunneling probability $\Theta = \frac{|\psi(L)|^2}{|\psi(0)|^2} = \exp\left[-2\int_0^L \frac{\sqrt{2m^*}}{\hbar}\sqrt{qV_B\left(1 - \frac{x}{L}\right)}\, dx\right]$

$\Rightarrow \boxed{\Theta = \exp\left(-\frac{4}{3}\frac{\sqrt{2qm^*}}{\hbar}\frac{V_B^{3/2}}{\mathcal{E}}\right)}$ where $\boxed{\mathcal{E} = \frac{V_B}{L}}$ we replace L by

electric field

tunneling current is then defined by:

$$J_n = q\underbrace{v_R}_{\text{Richardson velocity}}\underbrace{n\,\Theta}$$

carrier density = density of available e^- multiplied with the tunneling probability.

③ Additional Comment for the Schottky contact

(1) ● Lowering barrier effect (already described) due to mirror charge previously.

ⓐ Influence of interface states

So far, we considered "an ideal junction".

In practice, the periodic nature of the SC crystal is disturbed at the interface, which gives rise to a large number of permitted states in the bandgap of the SC near the interface.

⟹ interface states or interface traps.

with Energy ranging from E_V to E_C, and occupied by e^- if they are below ~~of~~ the Fermi-level.

when we put the M and SC together
the Fermi level is 'pinned' at some particular energy in the gap.

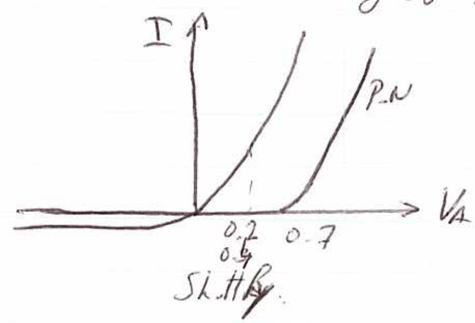
We do not know what pins the Fermi level (complicated physics at the interface)

⟹ Schottky barrier height as to be ~~determined~~ determined experimentally

(b) Comparison with the P-N junction.

* Reverse bias = the saturation current of a Schottky diode is 100 to 1000 times larger than that of PN junction (junction accounts for a larger leakage current)

* In forward bias = I-V characteristics show strong conduction at 0.2/0.4V for Schottky compared to 0.7V for PN

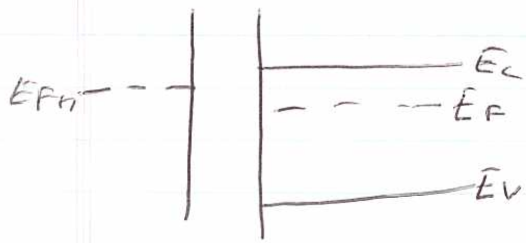


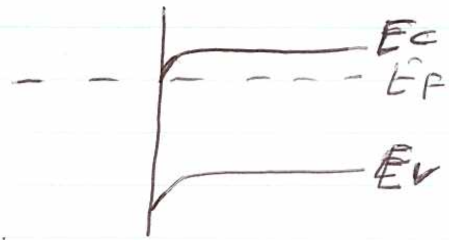
⇒ Schottky diodes are capable of very fast switching
⇒ due to majority carrier.
⇒ for P-N junction, device operation is slowed by recombination of excess minority carriers.

4 Ohmic contact

it is a non-rectifying contact.
the I-V should obey Ohm's law $U = RI$,
and R should be as low as possible.

~~we~~ we consider M-SC contact with $E_{Fm} > E_F$ [N-type semiconductor]

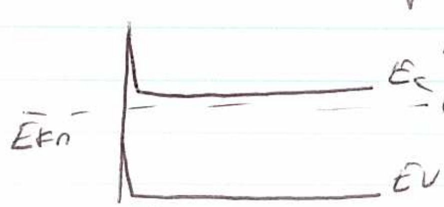




the magnitude of the band bending and its extension to the semiconductor are very small.
⇒ there is virtually no potential barrier and e^- can flow freely through this Ohmic contact.

* it is also possible to obtain an Ohmic contact from a Schottky diode. $E_{Fm} < E_F$ if the impurity concentration is high enough. ($N_d = 10^{20}\ cm^{-3}$)

since $W(V_A) = \sqrt{\dfrac{2\varepsilon (V_i - V_A)}{q N_d}}$ if $N_d \nearrow$ $W \searrow$



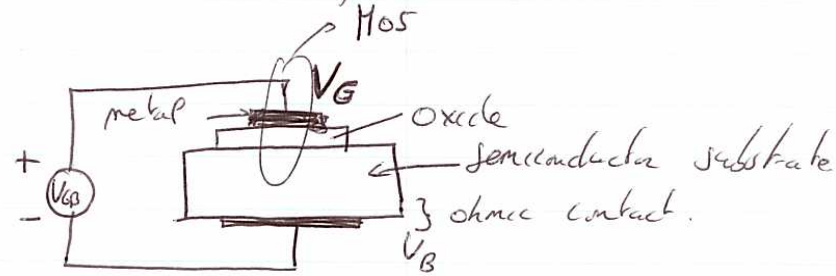
⇒ tunnel contact ~~with~~ low-resistance
⇒ Ohmic contact

III the MOS capacitors

MOS = Metal - Oxide - Semiconductor.
(or MIS = Metal - Insulator - Semiconductor)

Most used device in VLSI technology [Si/SiO_2]
oxide thickness typically from 5-50 nm.
oxide is an amorphous material

① Structure and principle of operation



- for a P-type substrate we talk about nMOS capacitor. (since inversion layer contains e^-).
- for a N-type substrate we talk about pMOS capacitor.

. we consider in the following nMOS capacitor (p-type semiconductor)

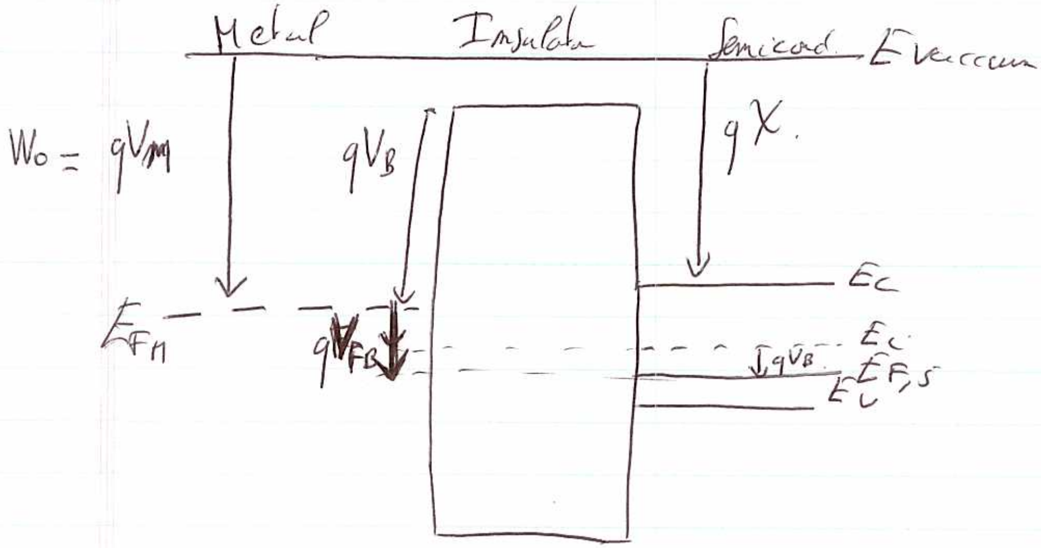
* 4 modes of operation,

flat-band, accumulation, depletion, inversion

(a) 'Unbiased' junction

~~(b) Unbiased junction Flat band diagram~~

The energy band diagram of the semiconductor is flat.



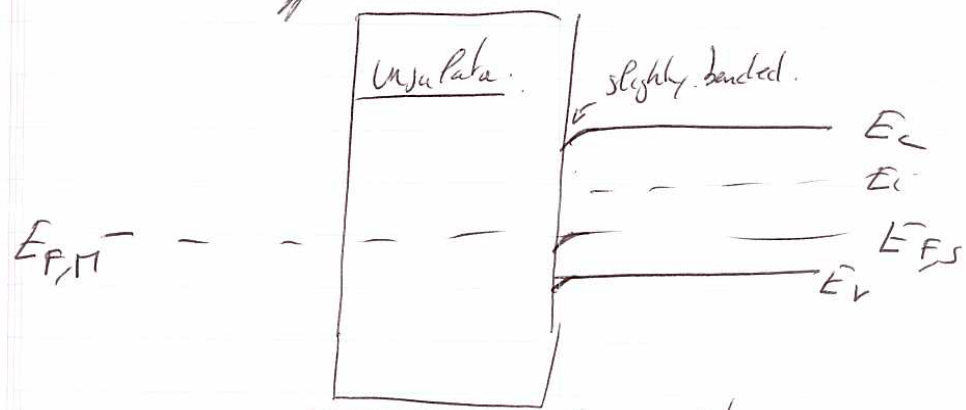
In the ideal case, charge would flow across the insulator so that E_{FM} and $E_{F,S}$ would line-up and the difference, V_{MS} between the metal and SC workfunctions would vanish.

$$V_{MS} = V_M - \left(\chi + \frac{E_g}{2} + V_B\right) = 0 \quad [p\text{-type semiconductor}]$$

In practice the ideal situation is never achieved (extremely long-time).

The application of a small bias V_{FB}, is required to line-up the Fermi-level. Its value is given by the non-vanishing V_{MS}.

So the flat-band ~~e.~~ diagram is obtained when the applied gate voltage equal to the work function difference.



$\boxed{\text{Zero bias}}$ ~~cond~~ condition.

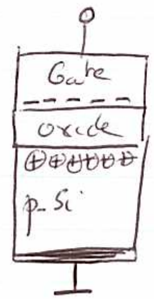
~~(b) Accumulation / Depletion / Inversion~~

(b) Biased junction

To understand the different bias modes of an MOS capacitor, we now consider 3 different bias voltages. ~~these bias~~ $V_G < 0$, $0 < V_G < V_T$ (threshold voltage), $V_T < V_G$

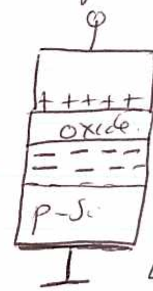
These bias regime are called = accumulation, depletion and inversion ~~mode~~.

(i) accumulation



negative charges on the gate attracts holes from the substrate to the interface.

(ii) Depletion



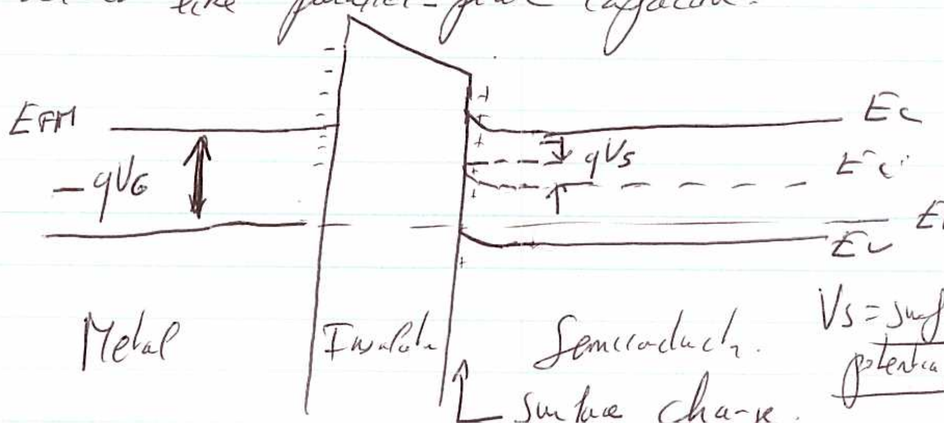
negative charge builds up in the semiconductor [depletion layer]

(iii) inversion

if potential ↗ another type of negative charges emerges at the oxide-semi interface ⇒ electrominority carriers which form a so-called inversion layer.
this charge increases exponentially with the surface potential.

(2) Accumulation

Negative bias applied to the metal gate (silicon substrate is grounded), the structure behaves like parallel-plate capacitor.



V_s = surface potential

the thickness of the obtained accumulation layer is $\sim 10\ nm$.

$\text{permittivity of } SiO_2$

Capacitance is $C = \dfrac{\varepsilon_{ox}}{t_{ox}}$ ~~[illegible]~~

thickness of the oxide $\boxed{\varepsilon_{ox} = \varepsilon_{r_{SiO_2}} \times \varepsilon_0}$

* **Derivation of the thickness of the accumulation layer**

Poisson equation $\Rightarrow$ $- \dfrac{d^2V(x)}{dx^2} = \dfrac{\rho}{\varepsilon_{si}} = + \dfrac{q}{\varepsilon}(p - n + N_d - N_a)$

equilibrium

$$\begin{cases} p(x) = p_{p0} \exp\left(-\dfrac{qV(x)}{k_BT}\right) = N_a \exp\left(-\dfrac{q\,V(x)}{k_BT}\right) \\ n(x) = n_{p0} \exp\left(\dfrac{q\,V(x)}{k_BT}\right) = \dfrac{n_i^2}{N_a} \exp\left(\dfrac{q\,V(x)}{k_BT}\right) \end{cases}$$

Far from the surface $V(x \to \infty) = 0$.

in the hole accumulation layer formed : we assume

$n \ll p$; $N_d \ll N_a$.

$$- \frac{d^2 V(x)}{dx^2} = + \frac{q}{\varepsilon} N_a \left[\exp\left(-\frac{qV(x)}{k_BT}\right) - 1 \right]$$

Also we assume that there ; the hole concentration is greater than the hole concentration due to doping.

$p \gg N_a$.

$$+\frac{d^2V(x)}{dx^2} \simeq -q\frac{N_A}{\epsilon}\exp\left(-\frac{qV(x)}{k_BT}\right)$$

we can integrate this equation if we multiply by $2\frac{dV(x)}{dx}$

$$\Rightarrow \quad 2\frac{dV(x)}{dx}\frac{d^2V(x)}{dx^2} = -2q\frac{N_A}{\epsilon}\exp\left(-\frac{qV(x)}{k_BT}\right)\frac{dV(x)}{dx}$$

$$\Rightarrow \quad \frac{d}{dx}\underbrace{\left(\frac{dV}{dx}\right)^2}_{\mathcal{E}^2} = 2\frac{N_a k_BT}{\epsilon}\frac{d}{dx}\left[\exp\left(-\frac{qV(x)}{k_BT}\right)\right]$$

we integrate $\int_x^{x_{acc}}$, x_{acc} thickness of the layer

with $\underbrace{\mathcal{E}(x=x_{acc}) = V(x=x_{acc}) = 0}_{\text{neutrality.}}$

$$\boxed{\mathcal{E}^2(x) = \frac{2}{L_D^2}\left(\frac{k_BT}{q}\right)^2\left[\exp\left(-q\beta\, V(x)\right) - 1\right]}$$

with $L_D = \sqrt{\frac{\epsilon\, k_BT}{q^2 N_a}}$ == Debye length.

Definition of Debye length = distance in semiconductor over which local electric field affects distribution of free charge carriers.

$L_D \searrow$ if concentration $\nearrow$

See textbook p 172 - 173 for the derivation of $V(x)$ from $\mathcal{E}(x)$. $\left[\text{BC}: \; V(x_{acc}) = 0, \; V(0) = V_S \right.$

$\Rightarrow$ after calculations

$$\left[\begin{array}{l} V(x) = -\dfrac{k_B T}{q} \ln \left[\dfrac{1}{\cos^2\left[\alpha - \dfrac{x}{\sqrt{2} L_D}\right]} \right] \\ \alpha = \cos^{-1}\left(\exp\left(\dfrac{q V_S}{2 k_B T} \right) \right) \end{array} \right.$$

$V(x_{acc}) = 0 \Rightarrow \boxed{x_{acc} = \sqrt{2}\, L_D \cos^{-1}\left(\exp\left(\dfrac{q V_S}{2 k_B T} \right) \right)}$

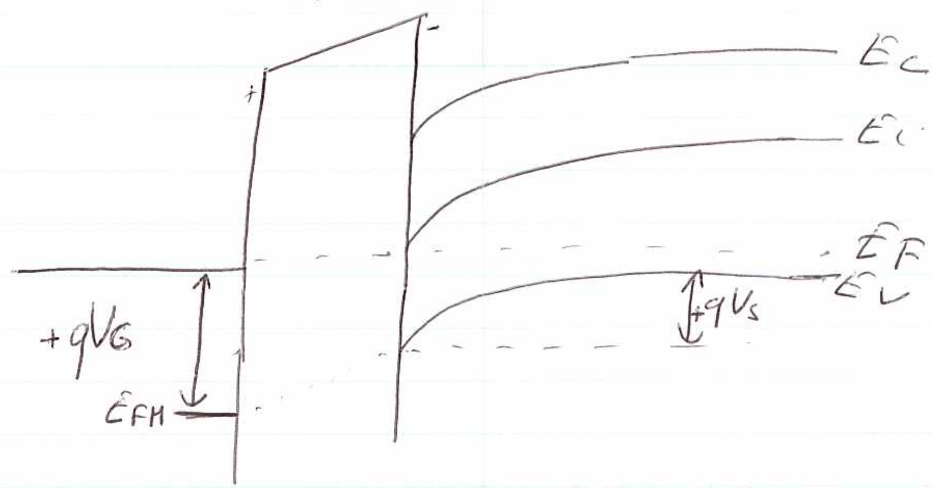
V_S is in practice very small even for large V_G.
$\Rightarrow$ x_{acc} is small. Since the hole concentration is an exponential function of the potential, the charge density increases rapidly close to the surface.
$\Rightarrow$ surface charge

So $\Rightarrow$ $C = \dfrac{\epsilon_{ox}}{t_{ox}}$ is a good approximation.

~~So[illegible]~~

3 Depletion

if a small positive bias is applied to the gate $V_G > 0$, holes near the silicon surface are repelled by the gate.
A negative charge appears due to the acceptor doping atoms.



- A positive charge appear at the Gate/oxide interface $\Rightarrow$ Surface charge
- At the Oxide/Silicon interface it appears a depletion region which extends into the Silicon from $x=0$, $x=x_d$.

* The potential in the depletion region is solution of the Poisson equation.

$$\frac{d^2V(x)}{dx^2} = \frac{-\rho}{\epsilon} = \frac{-q}{\epsilon}(p - N_A) = \frac{-q}{\epsilon} N_a\left[\exp\left(\frac{-qV(x)}{kT}\right) - 1\right]$$

Near the interface $p \ll Na$; (exponential term is small since the potential is positive)

$$\Rightarrow \quad \frac{d^2 V(x)}{dx^2} \simeq \frac{q Na}{\epsilon} \quad \Rightarrow \text{depletion approximation}$$

B.C $\Rightarrow$ $V(xd) = 0$; $\left.\frac{dV(x)}{dx}\right|_{xd} = 0$

$$\Rightarrow \boxed{V(x) = \frac{q Na}{2\epsilon}(x - xd)^2}$$

Surface potential $\boxed{Vs = V(x=0) = \frac{q Na}{2\epsilon} xd^2}$

$$\Rightarrow \boxed{xd = \sqrt{\frac{2\epsilon Vs}{q Na}}}$$

* "the depletion charge" between $x=0$, $x=xd$ is equal to:

$$Qd = -q Na\, xd = -\sqrt{2 q \epsilon Na Vs}$$

"depletion capacitance" $\boxed{C_D = \epsilon / xd.}$

* [the gate voltage, V_G, == potential drop across the oxide + potential variation in the semiconductor.

$$\boxed{V_G = Vs + Vox = Vs - \frac{Qd}{Cox}} \qquad \boxed{Cox = \frac{\epsilon_{ox}}{t_{ox}}}$$

- Capacitance of the structure is:

$$\left[C = \frac{dQ_G}{dV_G} = -\frac{dQ_d}{dV_G} = -\frac{dQ_d}{d\left(-\frac{Q_d}{C_{ox}} + V_s\right)} = \frac{-dQ_d/dV_s}{d\left(-\frac{Q_d}{C_{ox}} + V_s\right)/dV_s} \right.$$

$$C = \frac{C_D C_{ox}}{C_D + C_{ox}} = \frac{1}{\frac{1}{C_{ox}} + \frac{1}{C_D}} \Rightarrow \text{with } \left[C_D = \frac{-dQ_D}{dV_S} = \frac{\epsilon}{x_d} \right]$$

- C can also be expressed in function of the gate voltage:

$$V_G = -\frac{Q_d}{C_{ox}} + V_s = \frac{qN_a x_d}{C_{ox}} + \frac{qN_a}{2\epsilon} x_d^2$$

$$\Rightarrow x_d = -\frac{\epsilon}{C_{ox}} + \sqrt{\left(\frac{\epsilon}{C_{ox}}\right)^2 + \frac{2\epsilon}{qN_a} V_G}$$

since $C_D = \frac{\epsilon_{Si}}{x_d}$

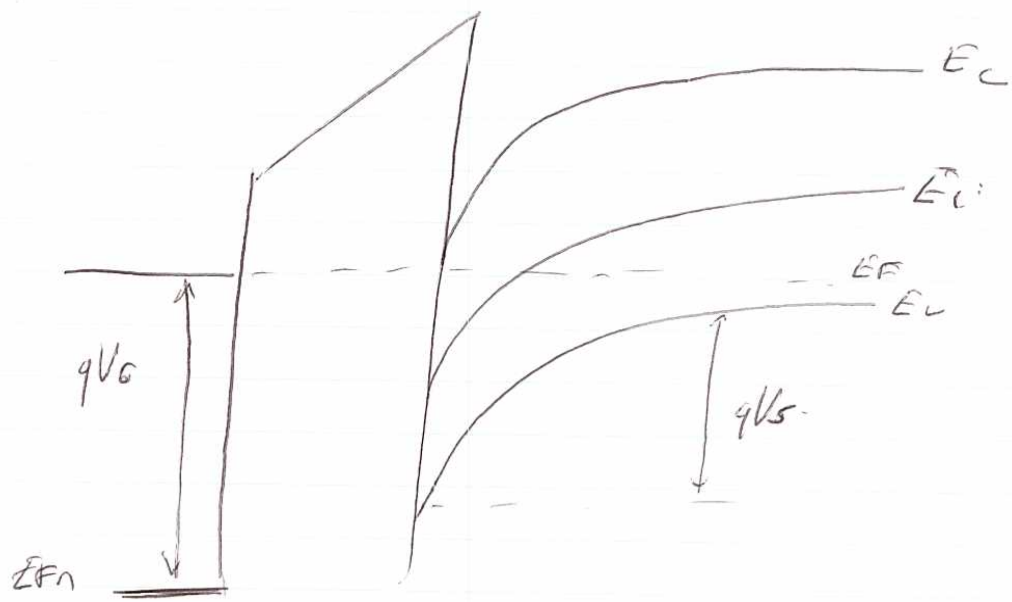
$$\Rightarrow \boxed{C = \frac{C_{ox}}{\sqrt{1 + \frac{2C_{ox}^2 V_G}{qN_a \epsilon}}}}$$

(4) Inversion

If a larger positive voltage is applied $V_G > 0$, the surface potential V_s will continue to increase according to the following relation:

$$\begin{cases} p(x=0) = N_a \exp\left(-\frac{qV_s}{k_BT}\right) & \text{majority carriers} \\ n(x=0) = \frac{n_i^2}{N_a} \exp\left(\frac{qV_s}{k_BT}\right) & \text{minority carriers} \end{cases}$$

$p \searrow$ and $n \nearrow$



the surface concentration of $\bar{e}$ and holes are equal iff

$n(0) = p(0) = n_i$ if $(E_i == E_F)$ at $x = 0$.

Since
$$\left\{\begin{array}{l} n = n_i \exp(\beta(E_F - E_i)) \\ p = n_i \exp(-\beta(E_F - E_i)) \end{array}\right]$$

$$\boxed{V_S = \frac{k_B T}{q} \ln\left(\frac{N_a}{n_i}\right)}$$

$\Rightarrow$ we then get. at $\underline{V_S = V_F}$

$$\left[N_a \exp\left(-q \frac{V_F}{k_B T}\right) = \frac{n_i^2}{N_a} \exp\left(q \frac{V_F}{k_B T}\right) \right] (*)$$

Also if ~~the further~~ V_G is increased further

~~.....~~ $n(0)$ becomes equal to $p_{po} = N_a$ which is the original concentration of hole in Silicon.

$\Rightarrow$

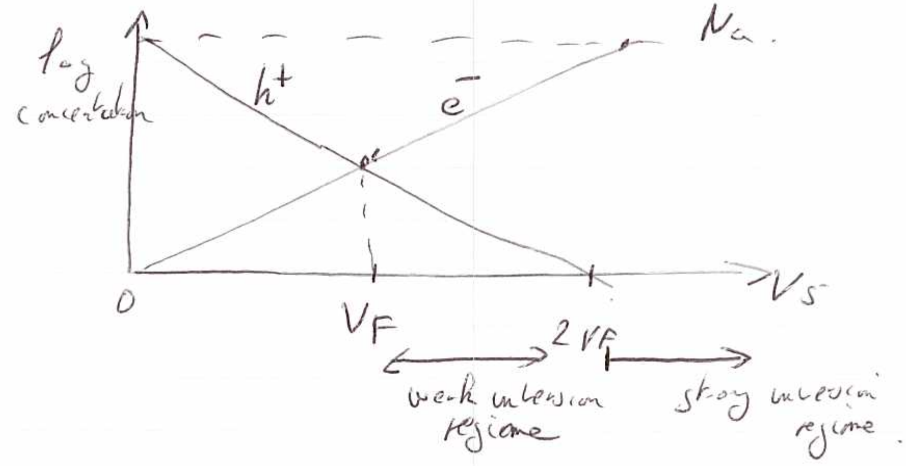
$$N_a = \frac{n_i^2}{N_a} \exp\left(q \frac{V_p}{k_B T}\right)$$

$(*$

$$N_a \exp\left(-q \frac{V_F}{k_B T}\right) = \frac{n_i^2}{N_a} \exp\left(q \left(\frac{V_p - V_F}{k_B T}\right)\right)$$

using (*) $\Rightarrow$ $V_p - V_F = V_F \Rightarrow V_p = 2 V_F$.

finally we get



the inversion layer is rich in electrons, and therefore, it is a good conductor.

the MOS capacitor consists then of two conducting electrodes, the metal gate and the inversion layer.
As in the case of accumulation, the capacitance of the MOS is once again equal to C_{ox}.

* When an inversion layer is formed e^- are locally majority carriers at the surface.
the thickness of the inversion layer remains very small if V_G increases. the electron charge in the layer can be considered as surface charge,
as in the case of accumulation, the e^- charge in the inversion layer depends exponentially on the surface potential $Q_{in} \sim \exp\left(q\frac{V_S}{k_B T}\right)$.

- We suppose that $V_s = 2V_F$ when an inversion layer is present [if $V_G \nearrow$ V_s increases only slightly above $2V_F$.

The depth of the depletion region is given by

$$\boxed{x_{dmax} = \sqrt{\frac{4\epsilon V_F}{qN_a}}}$$

(5) Threshold voltage

it is the voltage V_G that must be applied to form an inversion layer.

$$V_G = V_s + \frac{Q_G}{C_{ox}}$$ Q_G is the positive charge at the gate electrode.

~~a~~ $\cancel{V_G = V_s - \frac{Q_D}{C_{ox}}}$

~~take~~ ~~$-Q_G$~~

In practice, the flat-band voltage must be added for "non-ideal" threshold voltage:

$$V_G = V_{FB} + V_s + \frac{Q_G}{C_{ox}}$$ and $Q_G = -(Q_D + Q_{inv})$

We get

$$V_G = V_{FB} + V_S - \left(\frac{Q_d + Q_{inv}}{C_{ox}}\right)$$

with $\left[Q_d = -qN_a \cancel{x_d}\, x_{dT} = -\sqrt{2q\epsilon N_A V_s}\right]$

* in the depletion regime $Q_{inv} \simeq 0$ $\qquad 0 \leq V_S \leq 2V_F$

$$V_G = V_{FB} + V_S + \frac{\sqrt{2\epsilon q N_A V_S}}{C_{ox}}.$$

* in the inversion regime $Q_{inv} \neq 0$ $\qquad V_S > 2V_F$.

$$\boxed{V_G = V_{FB} + V_S + \frac{\sqrt{2\epsilon q N_A V_S}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}} \equiv V_T - \frac{Q_{inv}}{C_{ox}}.}$$

where $\boxed{V_T = V_{FB} + V_S + \frac{\sqrt{2\epsilon q N_A V_S}}{C_{ox}}}$ Threshold voltage.

Remark the flat band potential can be affected by the presence of charge in the oxide or oxide/semiconductor interface.

if Q_i is the charge at the surface and ρ_{ox} is the charge density distributed within the oxide.

Then the flat band voltage becomes.

$$V_{FB} = \underbrace{V_{MS}} - \underbrace{\frac{Q_i}{C_{ox}}}_{\text{charge at oxide/semi interface}} - \frac{1}{\epsilon_{ox}} \overbrace{\int_0^{t_{ox}} x \rho_{ox}(x)\, dy}^{\text{charge density in the oxide}}$$

already defined as

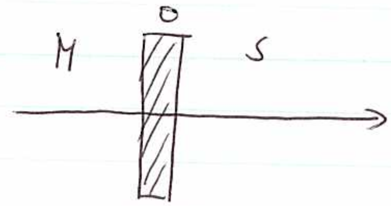
$$V_{MS} = V_M - \left(\chi + \frac{E_g}{2} + V_B\right)$$

* the actual calculation of the flatband voltage is further complicated by the fact that charge can move within the oxide. Also the charge at the oxide/semic. due to surface states depends on the position of the Fermi energy.

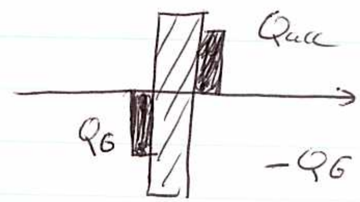
6 Summary

Charges in the MOS structure

Flat band



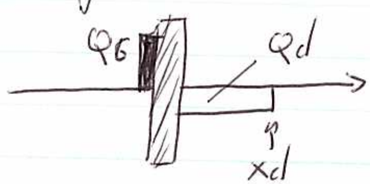
Accumulation



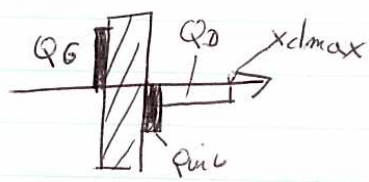
$-Q_G = Q_{acc}$

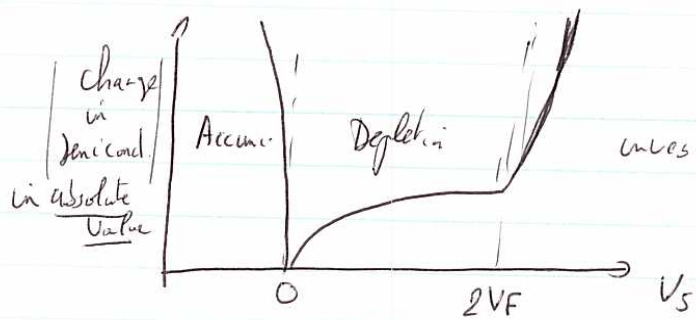
Depletion

$-Q_G = Q_d$

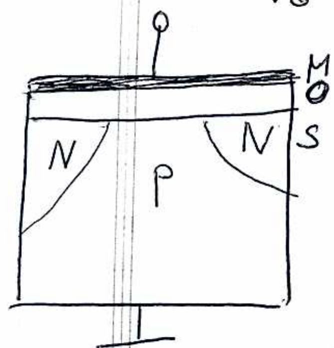


$-Q_G = Q_d + Q_{inv}$

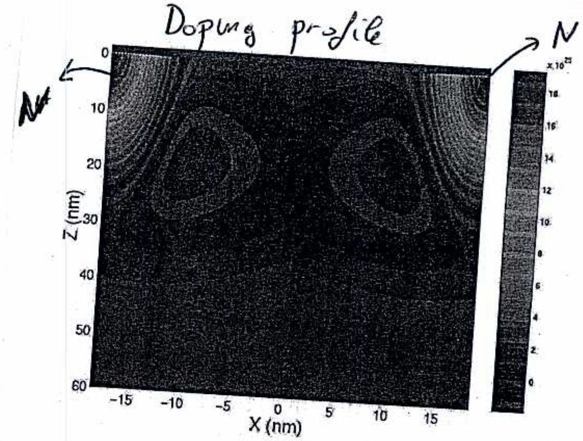




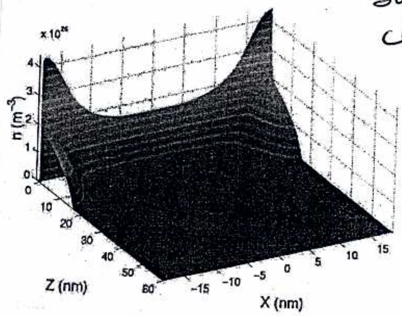
$V_G = 1V$



Doping profile

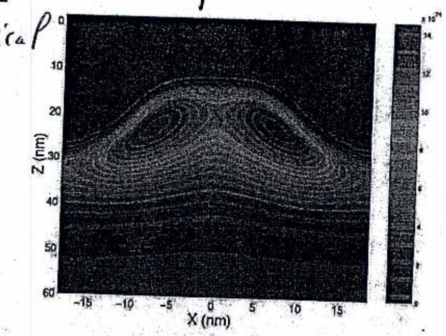


n

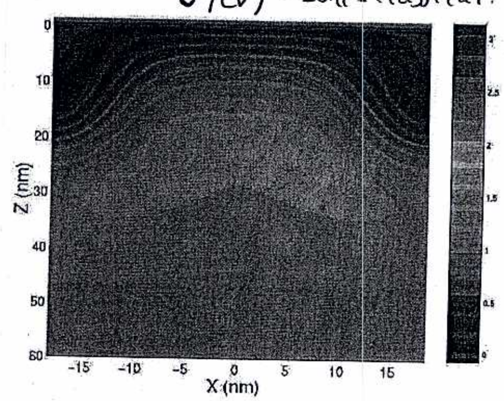


Semi-classical

p



U (eV) – semi-classical.



7 Quantum effects

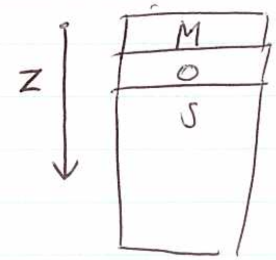
a confinement effects

one needs to solve self-consistently n and V ($p \ll n$ in inversion layer)

. V is solution of Poisson equation.

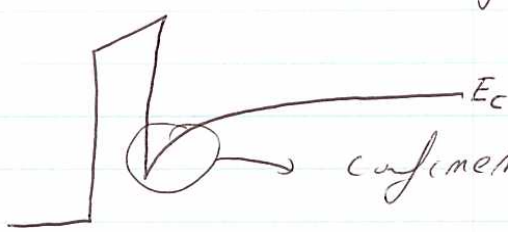
How to derive an expression for n?

$V_G > V_T$



$$-\frac{\hbar^2}{2m_z^*}\frac{d^2}{dz^2}\Psi_z(z) - qV\Psi(z) = E\Psi(z)$$

profile for $(-qV)$ ⇒ energy potential. $[E_c]$



confinement effects ⇒ energy levels.

[$E_{\Delta n}$ associated with $\Psi_{\Delta n}$]

$|\Psi_{\Delta n}(z)|^2$ is the density of probability to find an electron in a given subband (or mode) Δn.

So

$$n = \sum_{m=1}^{+\infty} n_m(E_F)\,|\Psi_m(z)|^2$$

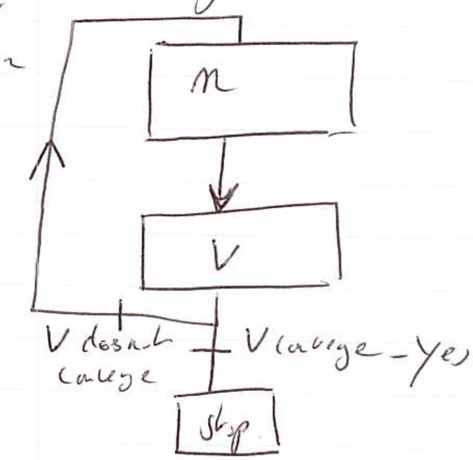
where $n_m(E_F)$ is the electron density ~~of~~ associated with a given subband ~~m~~. The e^- are then concentrated in a 2D layer (confined in 2D dimensions $\Rightarrow$ $\underbrace{\text{2DEG}}_{\text{2D electron gas}}$)

$$n_m(E_F) = \int_{E_m}^{+\infty} g_{2D}(E)\, f_{FD}(E-E_F)\, dE$$

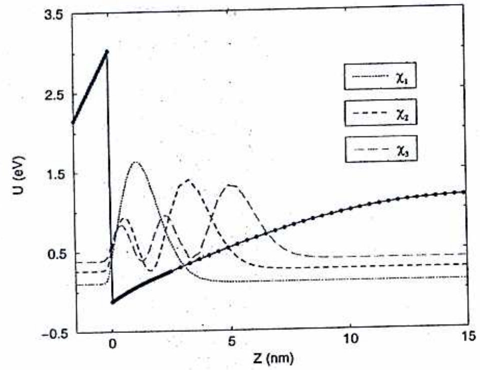
$$\Rightarrow \boxed{n_m(E_F) = \frac{m^*_{x,z}\, k_B T}{\pi \hbar^2} \ln\left[1+\exp\left(\frac{E_F - E_m}{k_B T}\right)\right]}$$

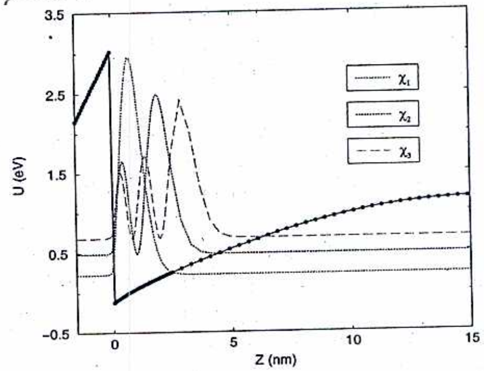
Since Ψ depends on V, n depend on V.

self-consistent simulation

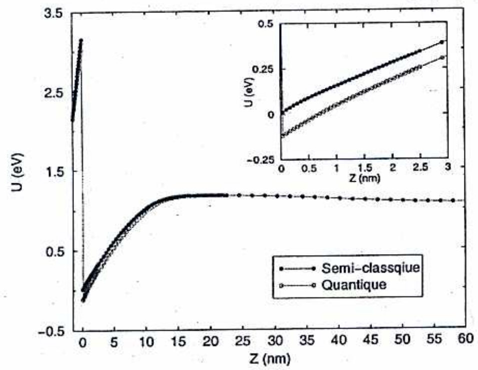


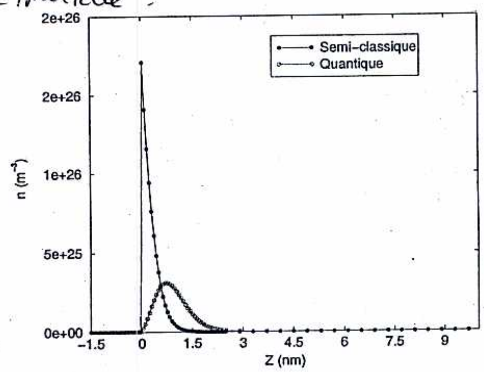
x in the middle.



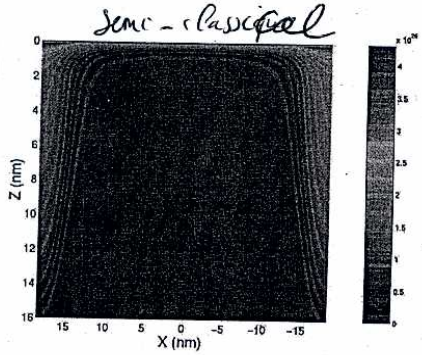


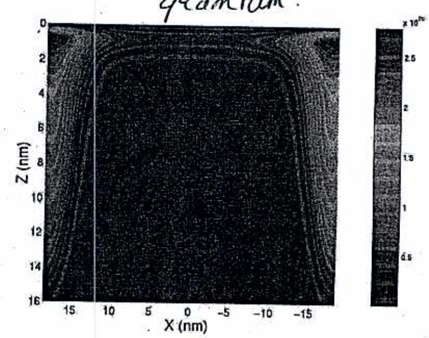
x in the middle.



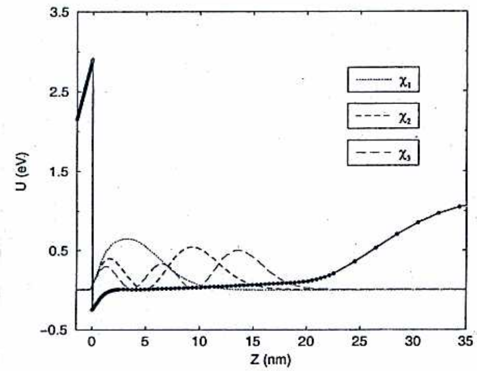


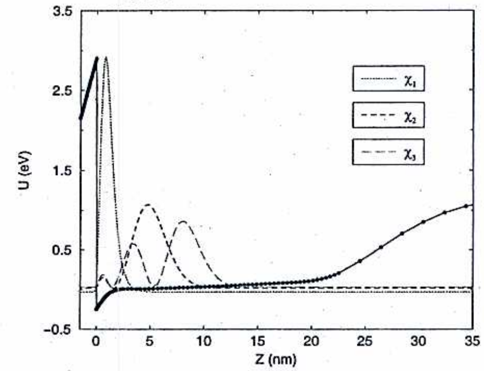
semi-classical n quantum.



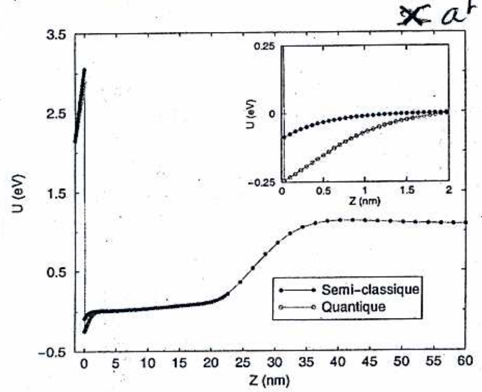


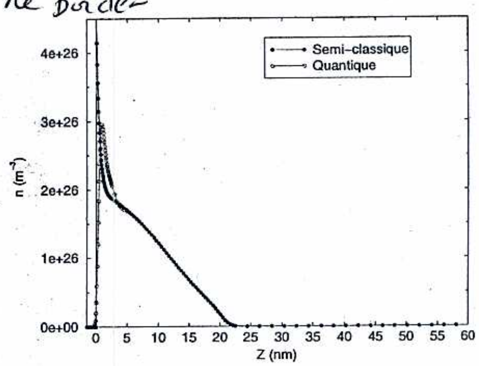
x at the border





x at the border





b) tunneling effects

gate leakage.

if scaling ↘, t_{ox} ↘, gate leakage becomes important. ⇒ physical limitation

Nanoelectronics research:

- to investigate the ultimate size of MOS devices.
- to ~~...~~ explore new class of device ~~...~~ whose operation principles relies on quantum effects

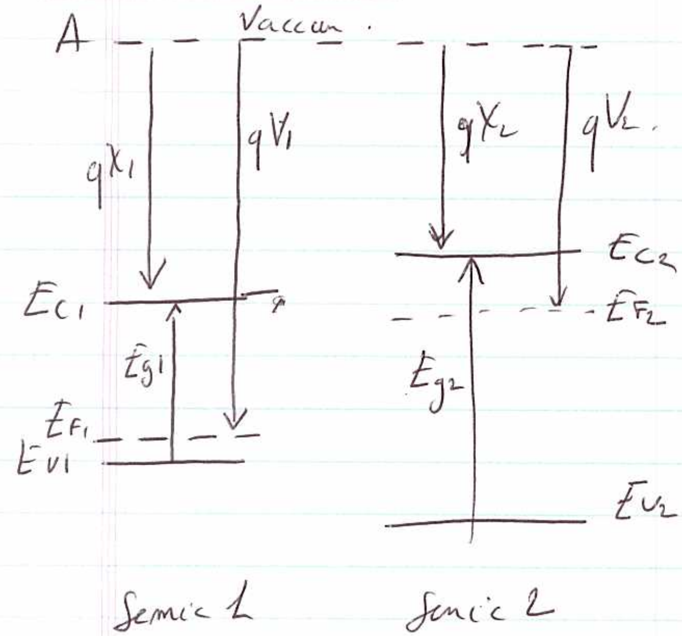
IV Heterojunction

a P_N junction ~~that~~ which is composed by two different semiconductors is called a heterojunction.

_ the P and N region have different bandgap.

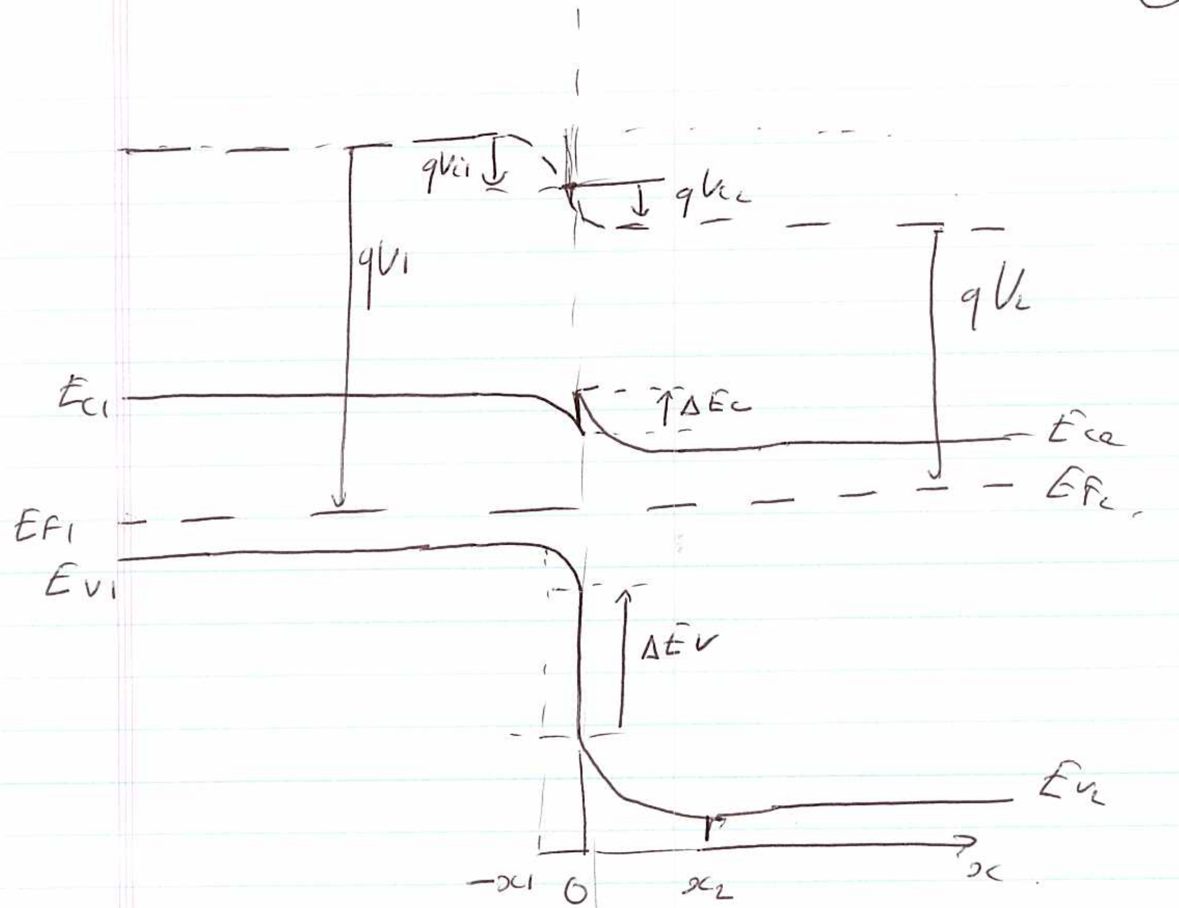
Remark
_ Using III_V semiconductors, such as $Ga_x Al_{1-x} As$ and $Ga_x In_{1-x} As_y P_{1-y}$, the desired bandgap energy can be reached adjusting the concentration x and y during the fabrication process_

1 Unbiased junction



Rules for plotting the band diagram

1. EF is equal and cte (under equilibrium)
   Far from the junction, the energy band ~~gap~~ diagram is known.
2. depletion region appear in the N_type and P_type sides.
3. a band - discontinuity appear at the junction



junction potential is $\boxed{V_i = V_{i1} + V_{i2} = \phi_1 - \phi_2}$

the discontinuity of the conduction band is $\Delta E_c = q(\chi_1 - \chi_2)$

of the valence band $\Delta E_V = (E_{g2} - E_{g1}) - q(\chi_1 - \chi_2)$

obviously $\Delta E_c + \Delta E_V = E_{g2} - E_{g1}$

The exact curvature of the energy bands within the transition region can be obtained solving the Poisson equation in both semiconductor materials using the depletion approx.

Semi 1	Semi 2
$\frac{d^2V}{dx^2} = \frac{qN_a}{\epsilon_1}$	$\frac{d^2V}{dx^2} = -\frac{qN_d}{\epsilon_2}$
$\mathcal{E}_1 = -\frac{dV}{dx} = -\frac{qN_a}{\epsilon_1}(x+x_1)$	$\mathcal{E}_2 = -\frac{dV}{dx} = -\frac{qN_d}{\epsilon_2}(x_2-x)$

$\mathcal{E}_1 = 0$ @ $x = -x_1$

$\mathcal{E}_2(x_2) = 0$

Gauss theorem at the junction $\Rightarrow$

$$\mathcal{E}_1 \epsilon_1 = \mathcal{E}_2 \epsilon_2 \Rightarrow N_a x_1 = N_d x_2.$$

[neutrality].

$V = A + \frac{qN_a}{2\epsilon_1}(x+x_1)^2$	$V = B - \frac{qN_d}{2\epsilon_2}(x_2-x)^2$
$V_{i1} = V(0) - V(-x_1) = \frac{qN_a}{2\epsilon_1}x_1^2$	$V_{i2} = V(x_2) - V(0) = \frac{qN_d}{2\epsilon_2}x_2^2$

$$\boxed{V_i = V_{i1} + V_{i2} = \frac{qN_a}{2\epsilon_1}x_1^2 + \frac{qN_d}{2\epsilon_2}x_2^2}$$

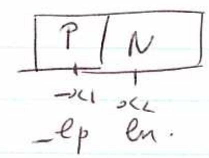
we want to know x_1 and x_2 in function of V_i.

So using neutrality equation.

$$\frac{qN_a}{2\epsilon_1}x_1^2 + \frac{qN_d}{2\epsilon_2}\frac{N_a^2}{N_d^2}x_1^2 = V_i$$

$$\Rightarrow \begin{cases} x_1 = \sqrt{\dfrac{2\epsilon_1\epsilon_2 N_d V_i}{qN_a(\epsilon_2 N_d + \epsilon_1 N_a)}} \\[2ex] x_2 = \sqrt{\dfrac{2\epsilon_1\epsilon_2 N_a V_i}{qN_d(\epsilon_2 N_d + \epsilon_1 N_a)}} \end{cases}$$

3 biased junction



V_a is applied to the diode.

→ hole diffusion current density injected at $x = +x_2$ in the P-region. $[x_2 = l_n]$ ~~...~~

$$\boxed{J_{p(x_1)} = q\frac{D_p}{L_p}p_{no}\left[\exp\left(\frac{qV_A}{k_BT}\right) - 1\right]} \quad \boxed{p_{no} = \frac{n_{i(2)}^2}{N_d}}$$

→ e^- diffusion current injected at $x = -x_1$ $[x_1 = l_p]$ in the N region ~~...~~.

$$\boxed{J_{n_1} = q\frac{D_n}{L_n}n_{po}\left[\exp\left(\frac{qV_A}{k_BT}\right) - 1\right]} \quad \boxed{n_{po} = \frac{n_{i(1)}^2}{N_a}}$$

taking the ratio $\frac{J_n}{J_p}$ will help to understand the current behavior in the P_N heterojunction.

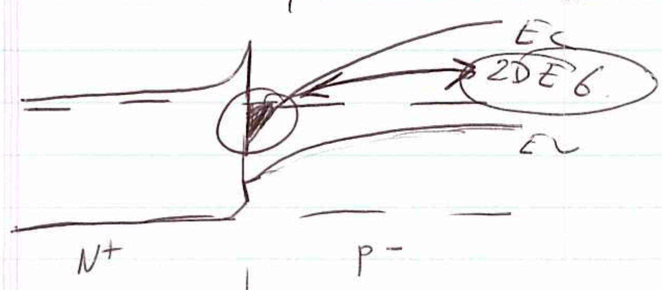
$$\frac{J_n}{J_p} = \frac{D_n L_p N_d n_{i1}^2}{D_p L_n N_a n_{i2}^2} = \frac{D_n L_p N_d N_{v1} N_{c1} \exp(-E_{g1}/k_B T)}{D_p L_n N_a N_{v2} N_{c2} \exp(-E_{g2}/k_B T)}$$

$$= \frac{D_n L_p N_d (m^*_{n1} m^*_{p1})^{3/2}}{D_p L_n N_a (m^*_{n2} m^*_{p2})^{3/2}} \exp\left[\frac{E_{g2} - E_{g1}}{k_B T}\right]$$

⇒ the ratio of electron to hole current in the P_N heterojunction is exponentially proportional to the difference of energy bandgap between the two semiconductors.

3) Applications: 2DEG

One can observe the formation of a 2DEG (2 dimensional Electron gas) in a N^+ AlGaAs/ P^- GaAs ~~junction~~ heterojunction



Very small thickness

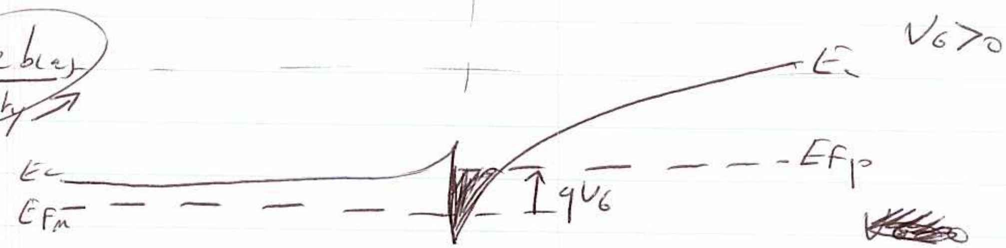
the mobility of e^- in lightly-doped GaAs is very high,
from 8,000 to 1,500,000 cm^2/Vs ~~[illegible]~~ –
T = 300K   T = 4.2K

[the surface mobility of e^- in the MOS channel, is ~ 650 cm^2/Vs

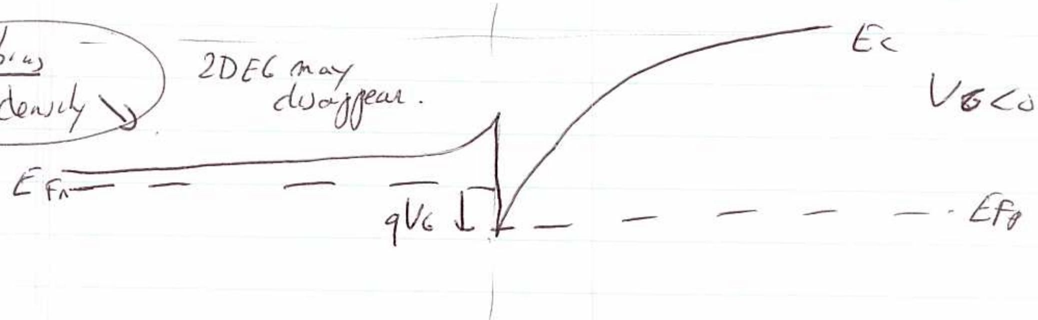
Rq: If impurity ↗ in GaAs mobility ↘ significantly.

* the e^- concentration in the 2DEG can be modulated by applying a bias to the heterojunction:

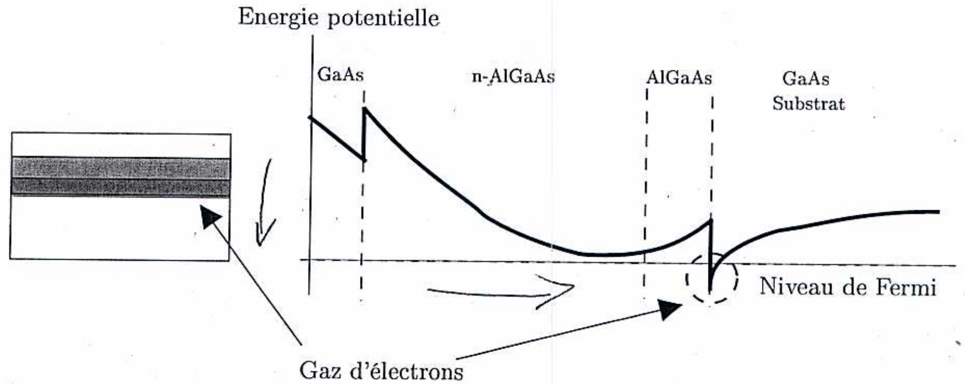
Reverse bias
electron density ↗

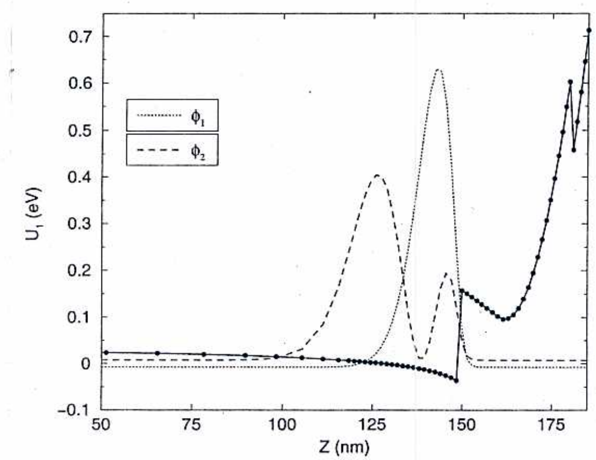


Forward bias
electron density ↘

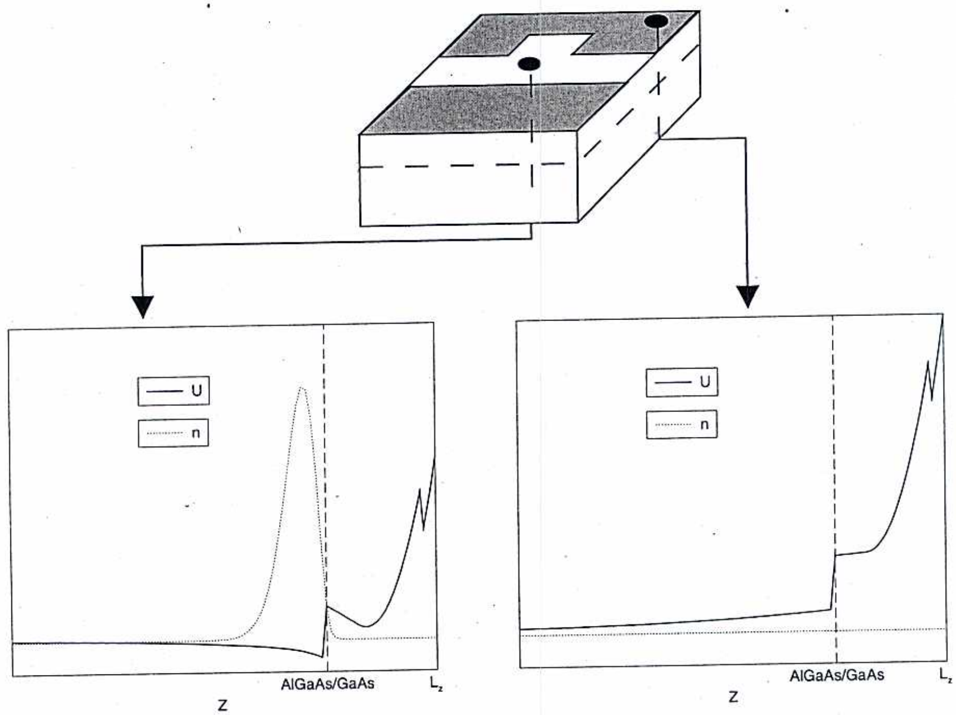


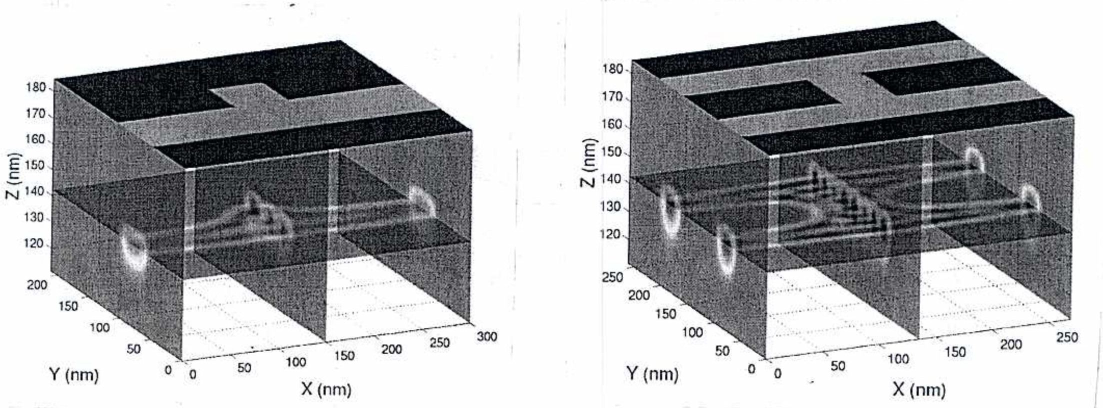
Example III-V Electron waveguide devices.





confinement effects


U
n
AlGaAs/GaAs
L_z
z
U
n
AlGaAs/GaAs
L_z
z


Z (nm)
Y (nm)
X (nm)
Z (nm)
Y (nm)
X (nm)