609 SQ9 HW1 100 ①

I] Photoelectric effect 30

① $W_0 = 4.3eV$ $K_{e^-} = 1eV.$

16

- $\underline{E_{ph}} = qW_0 + q = \underline{\underline{8.5 \cdot 10^{-19} J}} == \boxed{5.3eV}$ 4
- $\underline{\nu} = \frac{E_{ph}}{h} == \underline{1284\ THz}$ 4
- $\underline{\lambda} = \frac{hc}{E_{ph}} == \underline{0.234\ \mu m}$ 4
- momentum $\left[p = \frac{h}{\lambda} = 2.83 \cdot 10^{-27} \underline{kg \cdot m/s} \right.$ or $\underline{5\ 5/\mu m}$ 4

②

14

$W_0 = 2eV$

we know that $\boxed{K_{e^-} = h(\nu - \nu_0)}$ if $\underline{\nu > \nu_0}$ or $\underline{\lambda < \lambda_0}$

Threshold $\underline{\nu_0} = \frac{W_0}{h} == \underline{483.5\ THz}$

or $\lambda_0 = \frac{c}{\nu_0} = \frac{hc}{W_0} == \underline{620\ nm}$ 4

* [if incoming radiation is $\lambda = 610\,nm$

$\lambda < \lambda_0 \quad \nu > \nu_0$

$0.054\ 10^{-19} J$

so $Ke^- = h|\nu - \nu_0| = h\nu - W_0 == 34\,meV$ 4

$Ke^- = \frac{p^2}{2m} == \frac{1}{2} m v^2$

$\Rightarrow V = \sqrt{\frac{2Ke^-}{m}} == 1.09 \cdot 10^5\,m/s$ 4]

* [if incoming radiation is $\lambda = 630\,nm$ 2

$\lambda > \lambda_0$ below threshold, photoelectric effect
$\nu < \nu_0$ does not happen]

II Hydrogen atom 30

$$\Psi(r) = C \exp\left(\frac{-r}{a_0}\right)$$

① $\int_0^{+\infty} |\Psi|^2 4\pi r^2 dr = 1$ normalization

4

$$\Rightarrow 4\pi C^2 \int_0^{\infty} r^2 \exp\left(\frac{-2r}{a_0}\right) dr = 1$$

integration by parts

$$\int_0^{+\infty} r^2 \exp\left(-\frac{2r}{a_0}\right) dr = a_0 \int_0^{+\infty} r \exp\left(\frac{-2r}{a_0}\right) dr = \frac{a_0^2}{2} \int_0^{+\infty} \exp\left(\frac{-2r}{a_0}\right) dr$$

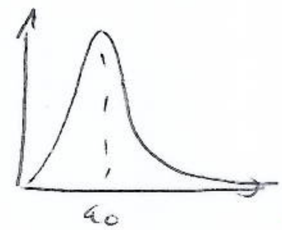
$$\boxed{= a_0^3/4}$$

$$\Rightarrow c = \left(\frac{1}{\pi a_0^3}\right)^{1/2}$$ 4

(2) $dP = |\Psi|^2 4\pi r^2 dr$

6/

so $\frac{dP}{dr} = \left(\frac{4}{a_0^3}\right) \exp\left(-\frac{2r}{a_0}\right) r^2$ 4



2

(3) $E(r) = \frac{dP}{dr}$ maximum for $E(r)$ obtained if.

6/

$\frac{dE(r_p)}{dr} = 0 \Rightarrow \underline{r_p = a_0}$ after calculations. 4

[The Bohr radius corresponds, in fact, to an ~~orbit~~ orbit where the probability to find the e^- is maximal. 2

(4) $P(0.9r_p < r < 1.1r_p) = \left(\frac{4}{a_0^3}\right) \int_{0.9r_p}^{1.1r_p} r^2 \exp\left(-\frac{2r}{a_0}\right) dr$

4/

After calculations we get $\underline{P = 0.108}$ $\underline{(10.8\%)}$ 4

(4)

(5) 4/ $\left[\langle r\rangle = \frac{4}{a_0^3}\int_0^{+\infty} r^3 \exp\left(-\frac{2r}{a_0}\right) dr = \frac{3a_0}{2}\right]$ after calculation

(6) 6/ $U(r) = -\frac{\alpha}{r^2}$ with $\alpha = \frac{q^2}{4\pi\epsilon_0}$

$$\langle U\rangle = -\alpha^2\langle\left(\frac{1}{r}\right)\rangle = -\alpha^2\left(\frac{4}{a_0^3}\right)\int_0^{+\infty} r\exp\left(-\frac{2r}{a_0}\right)dr = -\frac{\alpha^2}{a_0}$$

$$\boxed{\langle U\rangle = -27.1\,eV}$$ /2

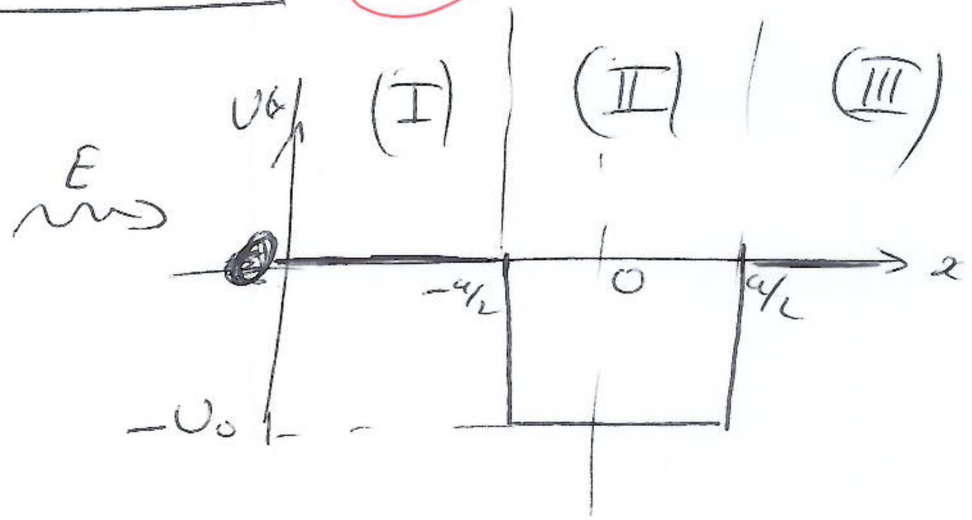
also $T = -\frac{h^2}{2mr^2}\frac{\partial}{\partial r}\left(\frac{r^2\partial}{\partial r}\right)$ in spherical coordinates

$$\Rightarrow \langle T\rangle = \frac{h^2}{2m}\frac{1}{a_0^2} = \boxed{13.6\,eV}$$ /2

Since $E = T+V$, it comes $\boxed{E = -13.6\,eV}$ /1

[equivalent to the fundamental energy obtained by the Bohr model /1

III Finite potential well 25



① 6

$$\begin{cases} \psi_I = e^{ikx} + r e^{-ikx} \quad /2 \\ \psi_{II} = E e^{iKx} + F e^{-iKx} \quad /2 \\ \psi_{III} = t e^{(ikx)} \quad /2 \end{cases}$$

2 ② $R + T = 1 \Rightarrow R = 1 - T$ /2

6 ③ See figure for $T(E)$ /6

④ $T = 1$ for $g(E) = n\pi$ $(n \in \mathbb{N}^*)$,

6 we get $\boxed{E_n = \dfrac{n^2 \pi^2 \hbar^2}{2ma^2} - U_0}$ /3

$\Rightarrow$ $E_1 = 0.7\,eV$ /1, $E_2 = 50.8\,eV$ /1, $\boxed{E_3 = 134.3\,eV}$ /1.

(see graph also)

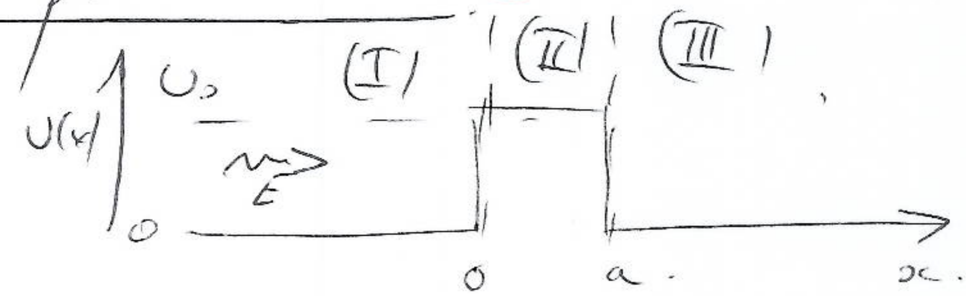
5) ✓ $E \to +\infty$ $f(E) \to 0$ $T \to 1$. /2

5/

at high energy, the e^- "does not feel" the effect of the potential well. Same result than that obtained using classical mechanics. /3

IV Finite potential well

15



1) ~~6~~/

$$a \gg \frac{1}{\alpha} \Rightarrow \exp(-\alpha a) \ll \exp(\alpha a)$$ /2

$$\Rightarrow T \simeq \frac{16 e^{-2\alpha a}}{\left(4 + \left(\frac{\alpha}{k} - \frac{k}{\alpha}\right)^2\right)}$$

$$\Rightarrow \boxed{T \simeq 16 \frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-2\alpha a}}$$ /4.

②

9/ From previous relation.

$$\mathrm{Log}(T) = \underbrace{\mathrm{Log}\left(16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)\right)} - 2\alpha a.$$

$\Rightarrow$ error $\qquad \epsilon$

$$\Rightarrow \boxed{\mathrm{Log}(T) = -2\alpha a + \epsilon} \quad /1$$

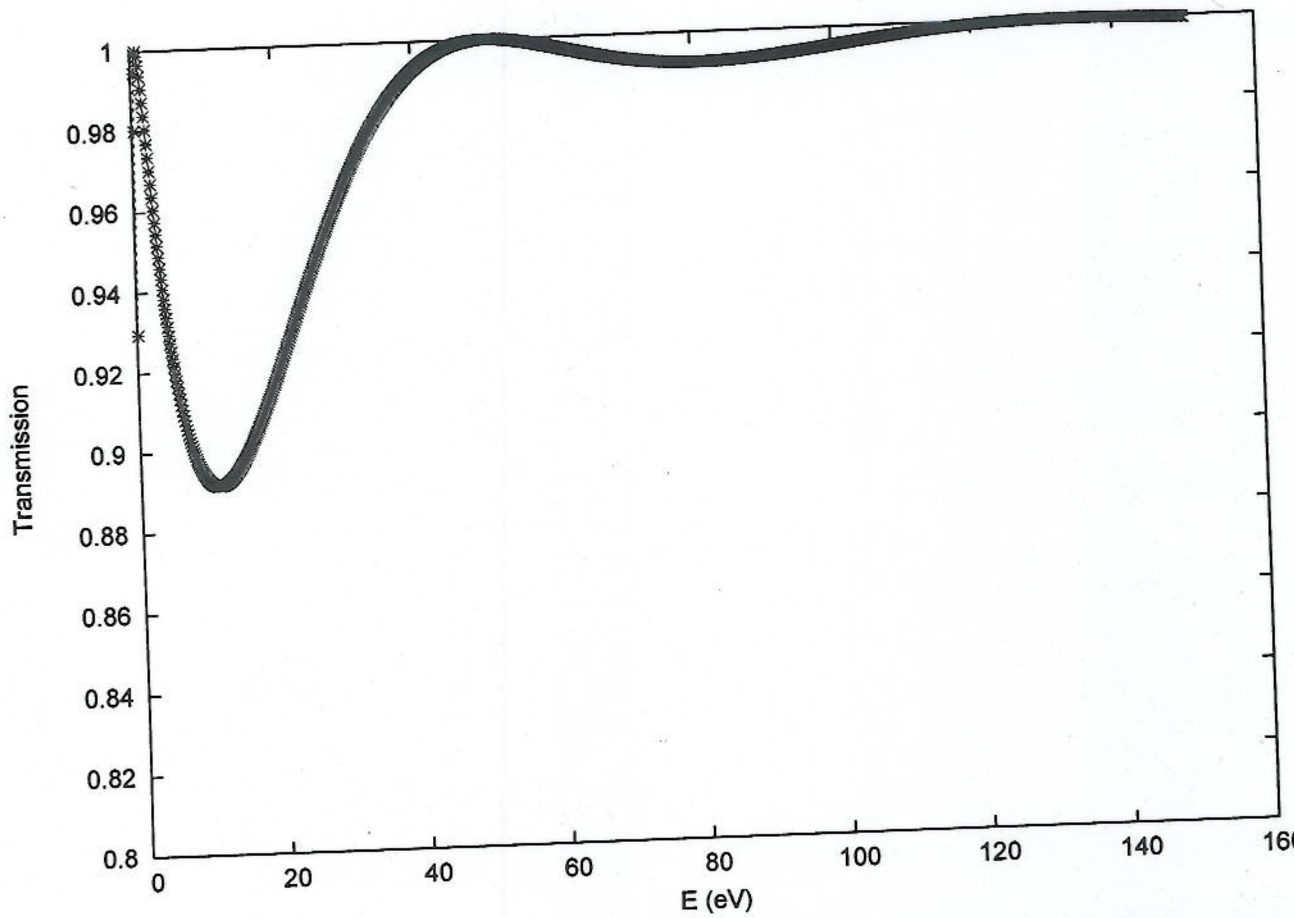
- $\epsilon(x) > 0$ is a necessary condition to obtain max error; indeed $T < e^{(2\alpha a+\epsilon)}$ only if $e^{\epsilon} > 1 \Rightarrow \epsilon > 0$ /2
- we set $x = \frac{E}{U_0}$, $\epsilon(x)$ is maximal if $\frac{d\epsilon}{dx} = 0$.
- so $\boxed{\epsilon(\tfrac{1}{2}) = \mathrm{Log}(4) \simeq 1.4}$ /2 $\Rightarrow x = \frac{1}{2}$.

when $\alpha a = 50$ we get $\mathrm{Log}(T) = -100 + 1.4$

the formula $\mathrm{Log}(T) = -2\alpha a$ is then exact at $\boxed{1.4\%}$ /2

- At the classical limit $\hbar \to 0$ $\log T \to -\infty$ then $\boxed{T \to 0}$

/2


Transmission
E (eV)
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
0
20
40
60
80
100
120
140
160