ECE609 Spring09

HOMEWORK 1

Review of Quantum Mechanics

1 The Photoelectric effect

(be careful with the units)

1. A metal has a workfunction of 4.3 eV. What is the minimum photon energy in Joule to extract an electron with $1eV$ kinetic energy from this metal through the photo-electric effect? What are the photon frequency in Terahertz and the photon wavelength in micrometer (we note that $\lambda = c/\nu$ where c is the velocity of the light $\simeq 3 * 10^8 m.s^-1$)? What is the corresponding photon momentum ?

2. A photocathode in Potassium has a workfunction of 2 eV. For an incoming radiation of $\lambda = 610nm$, what is the maximum velocity of an extracted electron ? For an incoming radiation of $\lambda = 630nm$, what is the maximum velocity of an extracted electron ?

2 Hydrogen atom (30pts)

The fundamental level (1s state) of the hydrogen atom can be described by the wave function (**spherical coordinates** $r \geq 0$):

$$\Psi(r) = Cexp(-r/a_0),$$

where C is a constant (complex) and a_0 is defined as the Bohr radius.
(Hint: for all the integrations, we will consider a volume $4\pi r^2 dr$ - elementary volume between two spheres with radius r and $r + dr$ (shell))

1. Find the value of C (Normalize Ψ),

2. What is the density of probability to find an electron in a volume $4\pi r^2 dr$ (shell). Plot the density of probability dP/dr in function of r.

3. We call $r = r_p$ the value for which this probability exhibits a maximum. What is r_p in function of a_0? Comment

4. What is the probability to find an electron at the distance between $0.9r_p$ and $1.1r_p$?

5. Give the average value (expectation value) of the distance that separate the proton from the electron $\langle r \rangle$. We note $\langle r \rangle = \langle \Psi | r | \Psi \rangle = \int_0^\infty \Psi^*(x) r \Psi(x) 4\pi r^2 \, dr$.

6. Give the expectation values (in eV) of the potential energy $\langle U \rangle$ and the kinetic energy $\langle T \rangle$. What is the value of the total energy ? Comment.

3 Finite potential well (25pts)

We consider a 1D finite potential well, where the variations of $U(x)$ are defined by:

$$U(x) = \begin{cases} 0 \text{ if } x < -a/2 \text{ (region I), or } x > a/2 \text{ (region III)} \\ -U_0 \text{ if } -a/2 < x < a/2 \text{ (region II)} \end{cases}$$

where a and U_0 are positive constants. We consider an incoming electron wave from the left of the system with energy $E > 0$ and amplitude equal to one.

1. Give the expressions of the wave function $\Psi(x)$ in the three regions. We will set:
$$k = (2mE)^{1/2}/\hbar \text{ and } K = (2m(E + U_0))^{1/2}/\hbar.$$
2. The Transmission coefficient T can take the following form:
$$T = \frac{1}{1 + f(E)sin^2 g(E)}$$
where
$$f(E) = \frac{U_0^2}{4E(E + U_0)}$$
and
$$g(E) = (2m(E + U_0))^{1/2} a/\hbar$$
What is the reflection coefficient R (no calculation is necessary here)?
3. Numerical application, $a = 1.5A$, $U_0 = 16eV$. Plot $T(E)$ for $E < 150eV$
4. Calculate (analytically) the energy values for which T is equal to 1 (total transmission). What are the numerical expressions of these energy values (we consider only the values smaller than $150eV$) ?
5. What is happening if $E \to \infty$. Comment.

4 Finite potential wall (15pts)

We consider a 1D finite potential wall, where the variations of $U(x)$ are defined by:
$$U(x) = \begin{cases} 0 \text{ if } x \leq 0 \text{ (region I), or } x \geq a \text{ (region III)} \\ U_0 \text{ if } 0 < x < a \text{ (region II)} \end{cases}$$
where a and U_0 are positive.
We consider an incoming electron wave from the left of the system with energy $0 < E < U_0$ and amplitude equal to one.

1. After calculations, one can find that the transmission coefficient T, is given by:
$$T = \frac{16}{|(1 + \frac{\alpha}{ik})(1 + \frac{ik}{\alpha})\exp(-\alpha a) + (1 - \frac{\alpha}{ik})(1 - \frac{ik}{\alpha})\exp(\alpha a)|^2},$$
where $k = (2mE)^{1/2}/\hbar$ and $\alpha = (2m(U_0 - E))^{1/2}/\hbar$. If we consider a very large barrier where $a >> 1/\alpha$, toward this limit, give the expression of T in function of E/U_0 and αa.
2. We consider the approximate formula:
$$Log(T) = -2\alpha a$$
What is the accuracy of this expression as compared to the result obtained above. If $\alpha a = 50$, show that the error is $\simeq 1.4\%$. What is happening at the classical limit (Hint: The Planck constant goes to zero)?