I/ P-N Junctions Diodes

1 Introduction

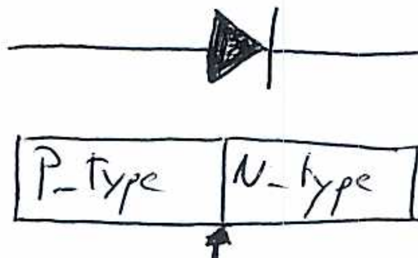
P-N junction is formed when a P-type and a N-type SC are in contact.

* if N-type and P-type are made out of the same material ⇒ homojunction.
* if N-type and P-type material are different ⇒ heterojunction.
* Application = Diode (2 terminal device)

It presents a highly non-linear I-V characteristics and it is often used as a rectifying elements.

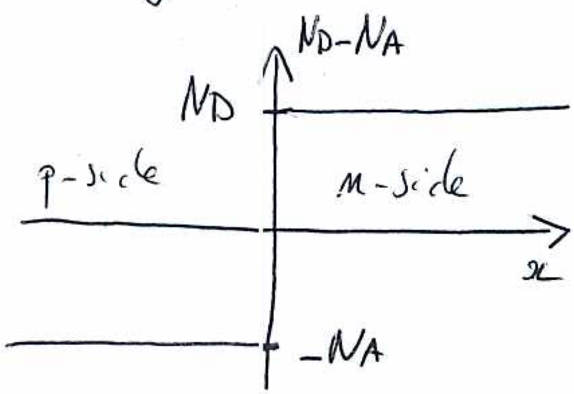
* <u>properties</u> :

P_N junction allows the current flow in one bias direction, but not in the other.

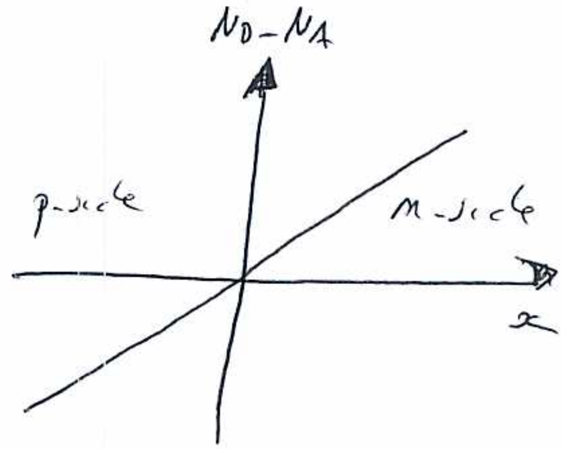


metallurgical junction

P_N junction can be separated into two major categories.



<u>Step junction</u>



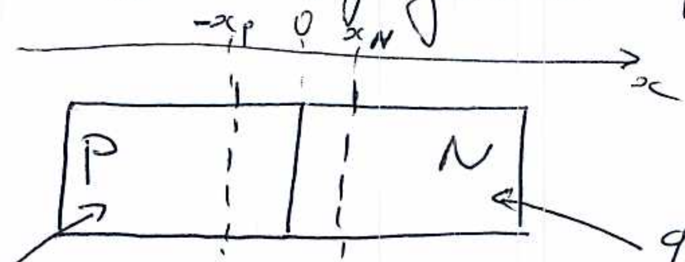
<u>Linearly graded junction</u>

⇒ <u>These are idealized doping profiles</u> see <u>Fig 5.2</u>

2 Built-in potential

a Basic

Let us connect 2 pieces (N-type and P-type) and form a PN step junction (Fig 5.5)



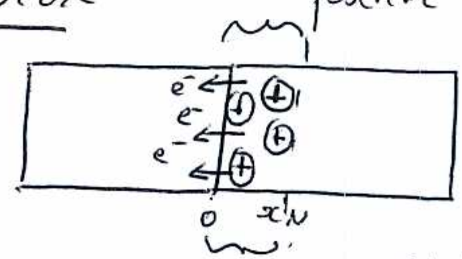
quasi-neutral P region

transition region or depletion region or space charge region.

quasi-neutral N-region.

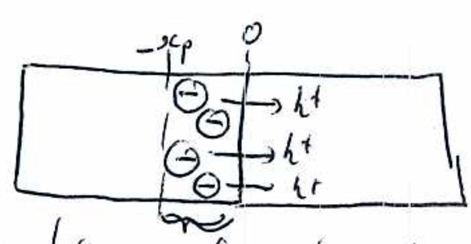
• e^- diffusion

positive ions (cannot move)



region depleted of e^- (positively charged)

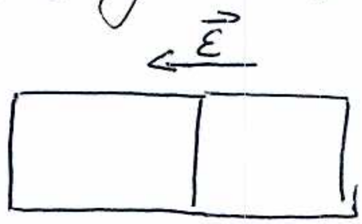
• hole diffusion



negative ions (cannot move) ⇒ region depleted of holes (negatively charged)

- **Built-in potential**

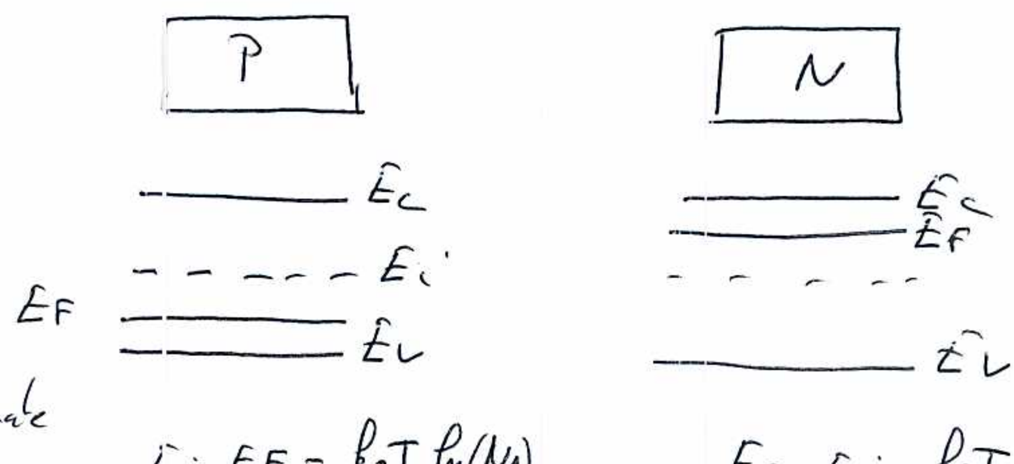
as a result of the diffusion process, a built-in potential, V_0, is formed at the junction.



Drift $\begin{cases} \vec{E} \text{ recalls the } e^- \text{ back to the N-type region} \\ \vec{E} \text{ " the } h^+ \text{ back to the P-type region.} \end{cases}$

$\Rightarrow$ equilibrium.

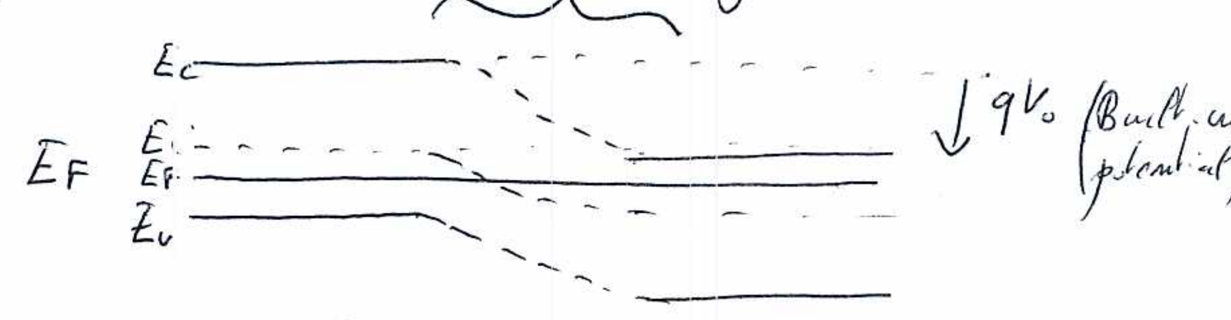
(b) * Energy band diagram Fig 5.3



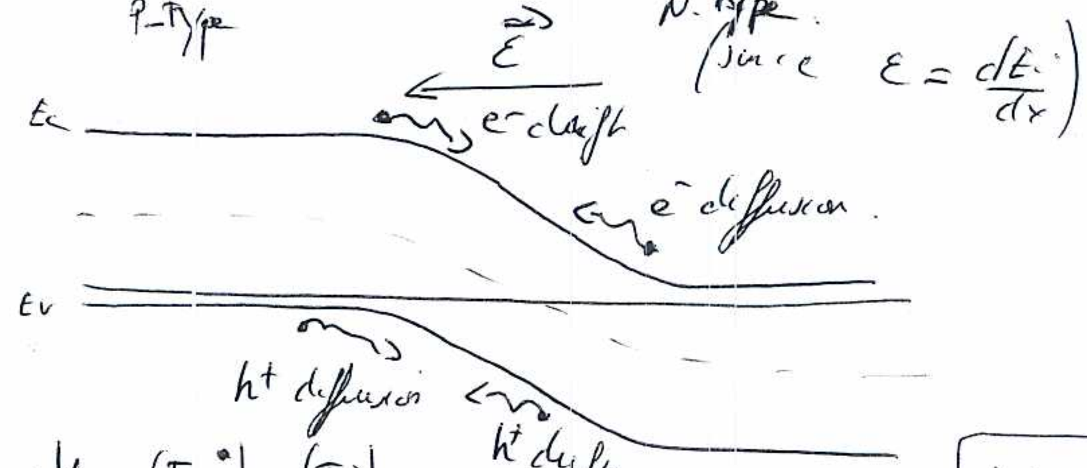
if non degenerate

$E_i - E_F = k_B T \ln\left(\frac{N_A}{n_i}\right)$ $E_F - E_i = k_B T \ln\left(\frac{N_D}{n_i}\right)$

E_F is unique and constant at equilibrium.
such that



From part 2 of the class we can say that



$$qV_0 = (E_i)_p - (E_i)_N$$

$$\boxed{V_0 \equiv V_{bi}}$$

(5.10)

$$qV_0 = (E_i - E_F)_p + (E_F - E_i)_N$$

$$= k_BT\left(\ln\frac{N_A}{n_i} + \ln\frac{N_D}{n_i}\right) \Rightarrow \boxed{V_0 = \frac{k_BT}{q}\ln\frac{N_a N_d}{n_i^2}}$$

3 Unbiased junction

what are the charge, potential and electric field variations in the transition region?

we propose to solve the Poisson equation.

~~a Depletion approximation~~

$\Rightarrow$ Poisson equation is

$$\boxed{-\frac{d^2V(x)}{dx^2} = \frac{\rho}{\epsilon_0 \epsilon_r}} \quad [5.2] \,/\, (5.14)$$

or

$$\boxed{\frac{d\mathcal{E}}{dx} = \frac{Q}{\epsilon_0 \epsilon_r}} \quad \text{since } \mathcal{E} = -\frac{dV}{dx}.$$

$\epsilon_r \equiv K_s$ dielectric constant

let us called $\epsilon_0 \epsilon_r = \epsilon_s$

$$\boxed{\rho : \text{total charge density} = -q(n - N_d + N_A - p)}$$

this equation is in practice non-linear since

$$\left[\begin{aligned} n(x) &= n_{n0}\, e^{\frac{qV(x)}{k_BT}} \\ p(x) &= p_{p0}\, e^{\frac{-qV(x)}{k_BT}} \end{aligned}\right.$$

since $E_{i(x)} = E_{i0} - qV(x)$

n, p depends on $V(x)$

n_{n0}, p_{p0} are $\bar{e}$ / h^+ density in the quasi-neutral region N (and P)

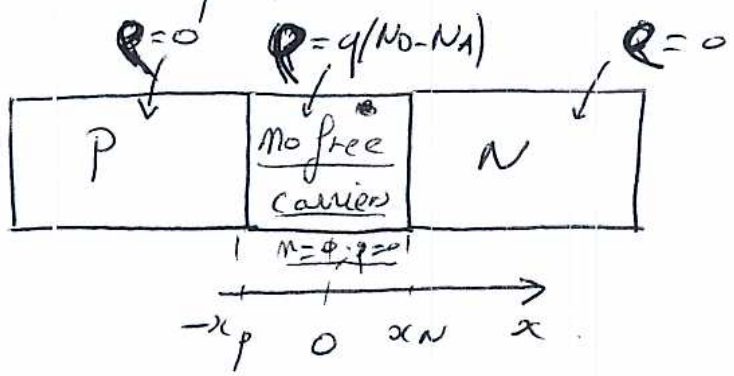
ⓐ Depletion approximation

We cannot solve the Poisson equation analytically so we use approximation.

"Depletion approximation" ⇒ $\vec{\mathcal{E}}$ vanishes outside the depletion region (neutrality) and we ignore the charge due to free carriers in the depletion region.

$$-x_p \leq x \leq x_n.$$

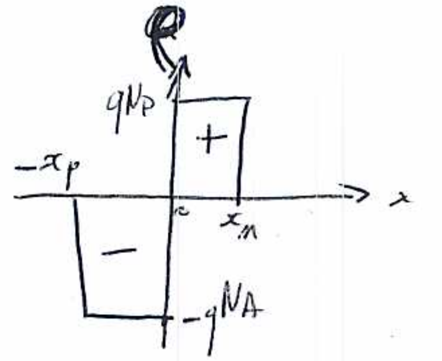
Depletion approximation.



5.6 a

Step junction

Fig 5.6b



See also Fig 5.7 for general doping profile

Depletion approximation
$$\begin{cases} N_D^+ \gg n & \text{for} \quad 0 \le x \le x_n \\ N_A^- \gg p & \text{for} \quad -x_p \le x \le 0 \end{cases}$$

$$\Rightarrow \quad (5.17) \quad \boxed{\frac{d\mathcal{E}}{dx} = \begin{cases} q\frac{N_D}{\epsilon_s} & \text{for} \quad 0 \le x \le x_n \\ -q\frac{N_A}{\epsilon_s} & \text{for} \quad -x_p \le x \le 0 \\ 0 & \text{for} \quad x \geq x_n \ ; \ x \le -x_p \end{cases}}$$

(b) Solution for Electric field $\mathcal{E}$

→ Solution for $-x_p \le x \le 0$.

2 options (for calculations)

(i) $\frac{d\mathcal{E}}{dx} = -\frac{qN_A}{\epsilon_s} \Rightarrow \mathcal{E}(x) = -\frac{qN_A}{\epsilon_s}x + \underset{\text{constant}}{a}$

$$\mathcal{E}(-x_p) = 0 = +q\frac{N_A x_p}{\epsilon_s} + a \Rightarrow a = -q\frac{N_A x_p}{\epsilon_s}$$

$$\Rightarrow \boxed{\mathcal{E}(x) = -\frac{qN_A}{\epsilon_s}(x+x_p)} \quad (5.19)$$

(ii) $$\int_0^{\mathcal{E}(x)} d\mathcal{E} = \int_{-x_p}^{x} -\frac{qN_A}{\epsilon_s}\,dx \Rightarrow \mathcal{E}(x) = -\frac{qN_A}{\epsilon_s}(x+x_p)$$

Solution for $0 \leq x \leq x_N$.

$$\frac{d\mathcal{E}(x)}{dx} = \frac{qN_D}{\epsilon_s} \qquad \mathcal{E}(x) = \frac{qN_D}{\epsilon_s}x + a$$

$$\mathcal{E}(x_N) = \frac{qN_D}{\epsilon_s}x_N + a = 0 \Rightarrow a = -\frac{qN_D x_N}{\epsilon_s}$$

$$\Rightarrow \boxed{\mathcal{E}(x) = \frac{qN_D}{\epsilon_s}(x-x_N)} \quad (5.21)$$

$\Rightarrow$ at $x=0$ it comes $\mathcal{E}(x) = -\frac{qN_A}{\epsilon_s}x_p = -\frac{qN_D}{\epsilon_s}x_N$

Continuity

$$\Rightarrow \boxed{N_A x_p = N_D x_N} \quad (5.22)$$

charge neutrality condition!

(c) Solution for Electrostatics potential V

$$\mathcal{E} = -\frac{dV}{dx} \Rightarrow \boxed{\frac{dV}{dx} = \begin{cases} \frac{qN_A}{\epsilon_s}(x_p + x) & -x_p \le x \le 0 \\ \frac{qN_D}{\epsilon_s}(x_N - x) & 0 \le x \le x_p \\ 0 & x \le -x_p;\ x \ge x_N \end{cases}} \quad (5.23)$$

Let us take the arbitrary reference $V = 0$ at $x = -x_p$

so $V = V_0$ ($\equiv V_{bi}$) at $x = x_N$ (built-in potential)

$\Rightarrow$ for $-x_p \le x \le 0$.

$$V(x) = \frac{qN_A}{\epsilon_s}\left(x_p x + \frac{x^2}{2}\right) + a$$

$$V(-x_p) = 0 = -\frac{qN_A}{\epsilon_s}\frac{x_p^2}{2} + a \Rightarrow a = \frac{qN_A}{\epsilon_s}\frac{x_p^2}{2}$$

$$\Rightarrow \boxed{V(x)} = \frac{qN_A}{\epsilon_s}\left(x_p x + \frac{x^2}{2} + \frac{x_p^2}{2}\right) = \boxed{\frac{qN_A}{2\epsilon_s}(x + x_p)^2} \quad (5.26)$$

$\rightarrow$ for $0 \le x \le x_N$

$$V(x) = \frac{qN_D}{\epsilon_s}\left(x_N x - \frac{x^2}{2}\right) + a$$

$$V(x_N) = V_{bi} = \frac{qN_D}{\epsilon_s}\left(\frac{x_N^2}{2}\right) + a$$

$$\Rightarrow \boxed{a = V_{bi} - \frac{qN_D}{\epsilon_s}\frac{x_N^2}{2}}$$

So $V(x) = \frac{qN_D}{\epsilon_s}\left(x_N x - \frac{x^2}{2} - \frac{x_N^2}{2}\right) + V_{bi}$

$$\Rightarrow \boxed{V(x) = -\frac{qN_D}{2\epsilon_s}(x - x_N)^2 + V_{bi}} \quad (5.28)$$

at $x = 0$ (continuity).

$$\boxed{\frac{qN_A}{2\epsilon_s}x_p^2 = V_{bi} - \frac{qN_D}{2\epsilon_s}x_N^2} \quad (5.29)$$

(d) ~~...~~ dimension of the transition region

we know ~~that~~ 2 equations (5.29) and (5.22) for x_N, x_p

After substitution, we get.

$$x_N = \sqrt{\frac{2\epsilon_s}{q}\frac{V_{bi} N_A}{N_D(N_A+N_D)}} \quad \underline{(5.30a)}$$

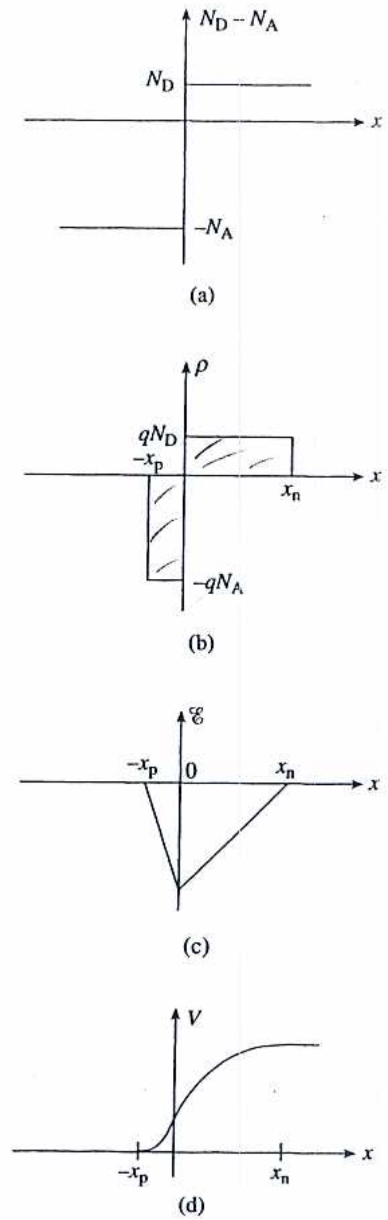
$$x_p = \sqrt{\frac{2\epsilon_s}{q}\frac{V_{bi} N_D}{N_A(N_A+N_D)}} \quad \underline{(5.30b)}$$

$$\boxed{W = x_N + x_p = \sqrt{\frac{2\epsilon_s}{q}\frac{(N_A+N_D)}{N_A N_D}V_{bi}}} \quad \underline{(5.31)}$$

$\Rightarrow$ total width of the depletion region

(e) Summary

- $\rightarrow$ we know the doping profile $N_D - N_A$ | ~~depletion approx~~
- $\rightarrow$ we obtain the built-in potential V_{bi}
- $\rightarrow$ using the depletion approximation we get ρ
- $\rightarrow$ we calculate $\mathcal{E}$ between $-x_N \leq 0 \leq x_p$
- $\rightarrow$ we calculate V between $-x_N \leq 0 \leq x_p$
- $\rightarrow$ we calculate x_N, x_p and $W = x_N + x_p$.



$\left(\begin{array}{l} N_D x_N = \\ N_A x_p \end{array} \right)$

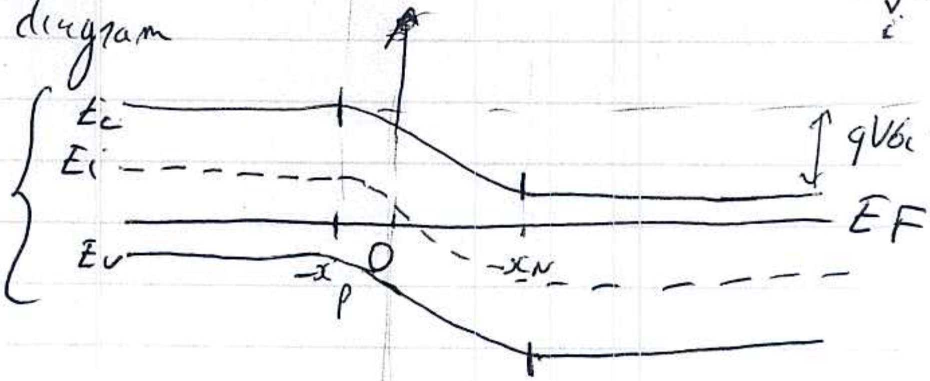
Figure 5.9 Step junction solution. Depletion approximation based quantitative solution for the electrostatic variables in a *pn* step junction under equilibrium conditions ($V_A = 0$). (a) Step junction profile. (b) Charge density, (c) electric field, and (d) electrostatic potential as a function of position.

Energy band diagram

$E_{C,V,i}(x) = E_{c,v,i}(ref) - qV(x)$

$\Rightarrow$

reference



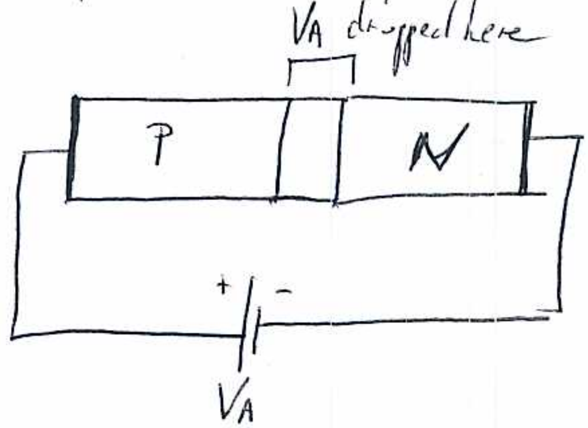
4 Biased junction

ⓐ Definition An external bias, V_A, is now applied to the junction.

Assumption The potential drop across the quasi-neutral regions are negligible ~~[illegible]~~.

Therefore the potential drops across the transition region

(See Fig 5.10)



Now ~~[illegible]~~: voltage drop (new potential barrier).

$$\boxed{V(x_n) - V(-x_p) = V_{bi} - V_A}$$

All the expressions we derived for $\mathcal{E}$, V, x_n, x_p, W are identical but V_{bi} is now replaced by $V_{bi} - V_A$

Equations (5.32) to (5.38)

Example: for W we get

$$\boxed{W = \left[\frac{2\epsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_A)\right]^{1/2}} \quad 5.38$$

if $V_A > 0 \Rightarrow$ Forward bias.

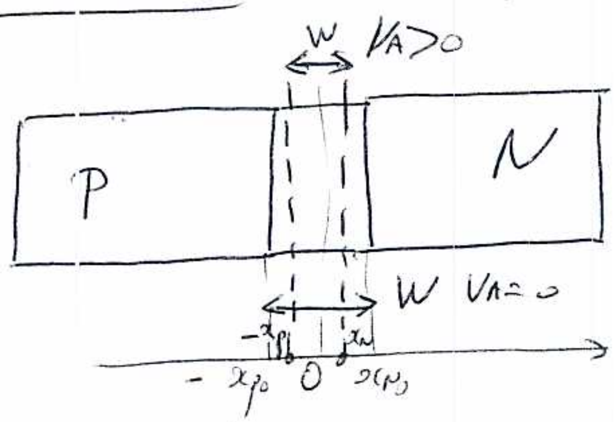
$W \searrow$ $x_n \searrow$ $x_p \searrow$

if $V_A < 0 \Rightarrow$ reverse bias

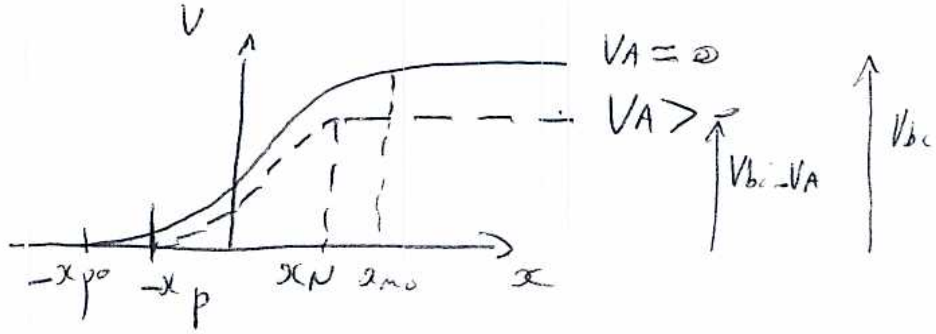
$W \nearrow$ $x_n \nearrow$ $x_p \nearrow$.

(b) Forward bias $V_A > 0$

Fig (5.11)

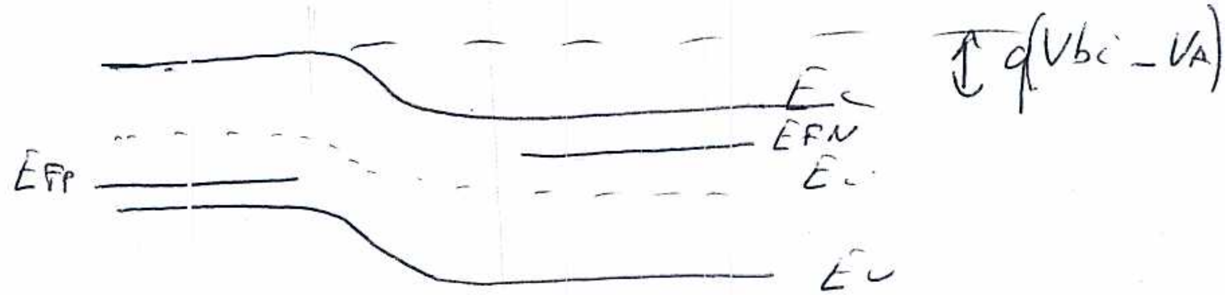


$\Rightarrow$



(Fig 5.12)

$\Rightarrow$



with $E_{FN} - E_{FP} = qV_A$ (E_F is not anymore unique and constant $\Rightarrow$ Non equilibrium)

Diffusion and Electric field (drift) forces are no longer equal and opposite sign.

Diffusion process > drift phenomena.

$\Rightarrow$ e^- can flow from $N \to P$

h^+ can flow from $P \to N$

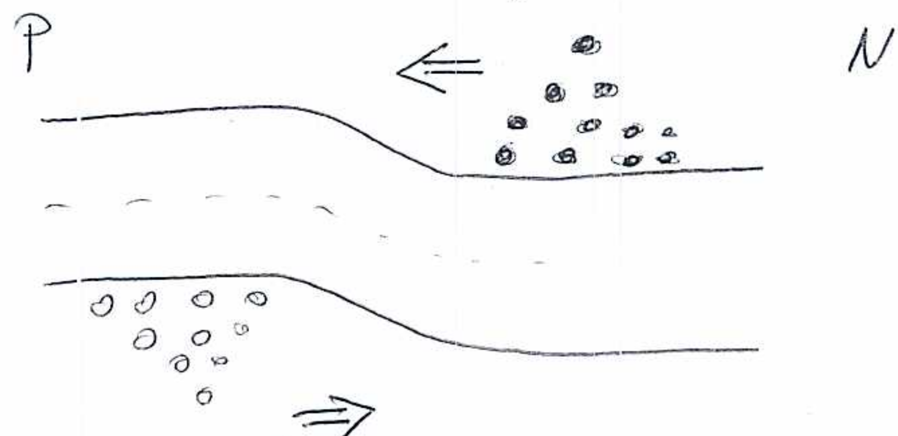
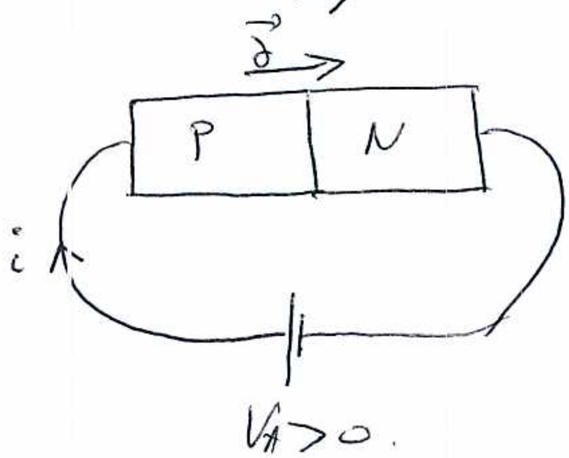
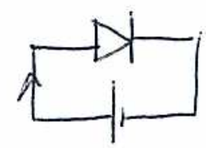
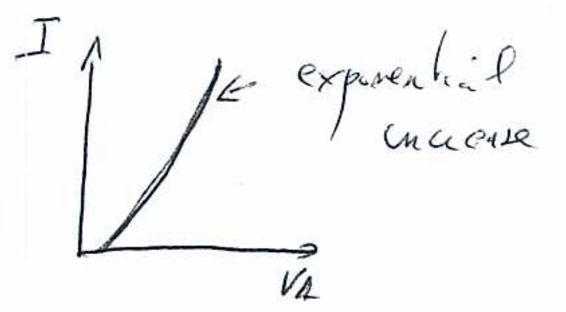


Fig 6.1







(c) Reverse bias $V_A < 0$

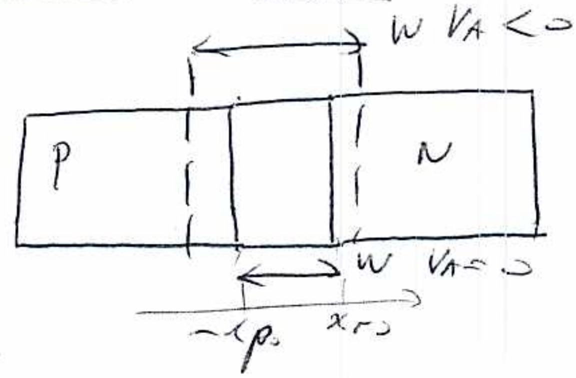
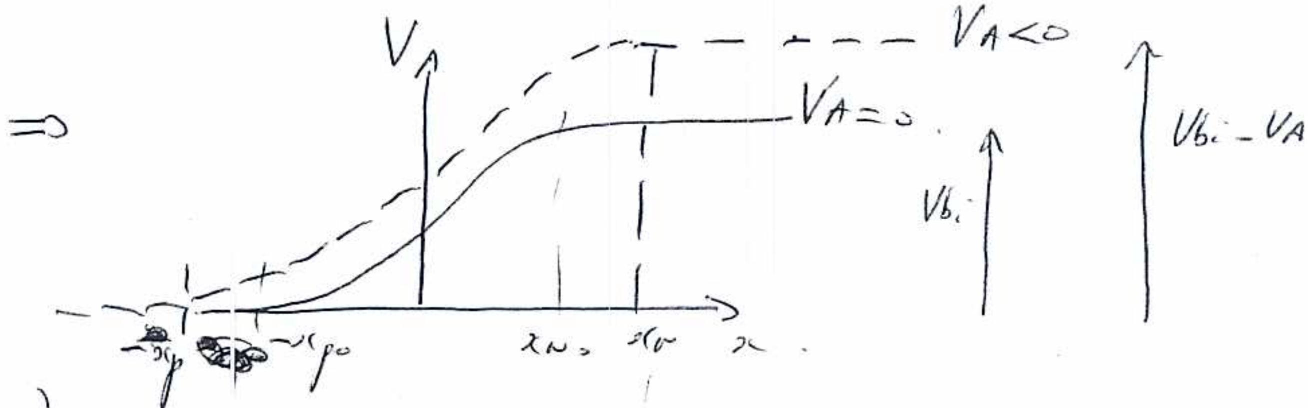
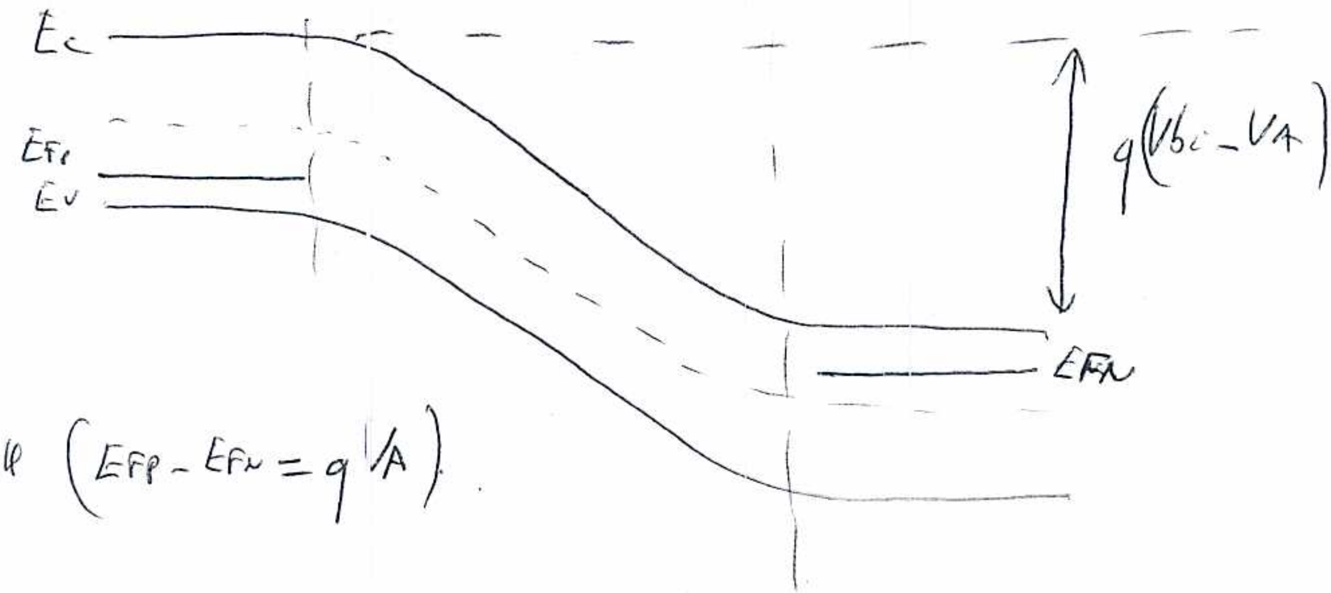


Fig 5.11



(Fig 5.12)

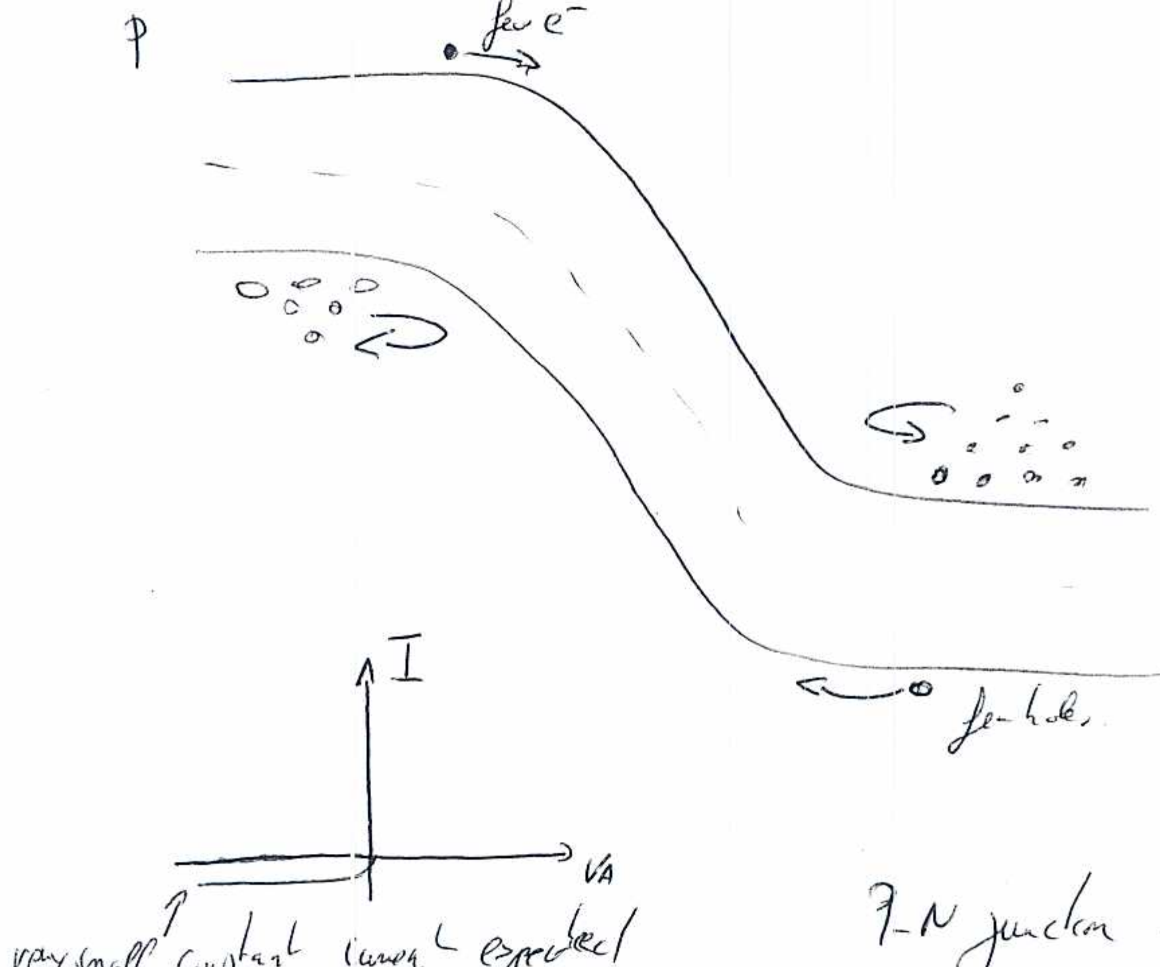


with $(E_{FP} - E_{FN} = qV_A)$.

the Electric field in the transition region increases and the associated drift current > diffusion current

⇒ the magnitude of the resulting net current is very small

(few e^- in the P-region, few holes in the N-region).

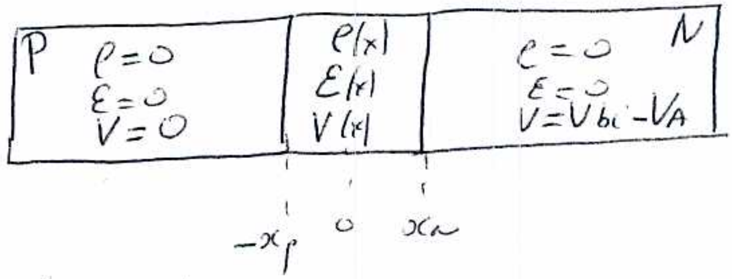


P-N junction behaves like a diode rectifying current flow.

(5) I-V Characteristics

(a) General considerations

So far, what do we know (using the depletion approx.)



V_A = applied bias

for computing the current, we can make use of

$$I = AJ \quad (A = \text{cross sectional area}) \qquad (6.2)$$

$$J = J_N(x) + J_p(x) \qquad (6.3)$$

with

$$\begin{cases} J_N = q\mu_n n \mathcal{E} + q D_n \dfrac{dn}{dx} \\ J_p = q\mu_p p \mathcal{E} - q D_p \dfrac{dp}{dx} \end{cases} \qquad (6.4)$$

the current J is constant for all position x.

if n and p were available, we are done. However we do not know them (or did not compute them numerically). Let us try first to derive an expression for the carrier current density J_N in the P-region and J_p in the N-region.

N-side

$$J_p = -q D_p \frac{dp}{dx} \qquad x \geq x_n \qquad (\mathcal{E}=0) \tag{6.6}$$

P-side

$$J_n = q D_n \frac{dn}{dx} \qquad x \leq -x_p \qquad (\mathcal{E}=0)$$

using the minority carrier diffusion equations

$$D_p \frac{d^2 p_n}{dx^2} - \frac{p_n - p_{n0}}{\tau_p} = 0 \qquad x \geq x_n \tag{6.5}$$

$$D_n \frac{d^2 n_p}{dx^2} - \frac{n_p - n_{p0}}{\tau_n} = 0 \qquad x \leq -x_p$$

p_n means "density of holes in the N-side"

p_{no} means "density of holes in the N-side at equilibrium"

Let us try to solve for n_p, J_N $\quad \underline{x \leq -x_p}$

$$\boxed{D_n \frac{d^2 n_p}{dx^2} - \frac{n_p - n_{po}}{\tau_n} = 0} \quad (-\infty \leq x \leq -x_p) \qquad (6.5a)$$

$\Rightarrow$ 2 Boundary conditions are necessary.

(i) $n_p(-\infty) = n_{po}$ $\Rightarrow$ the diode is infinitely long and we have thermodynamic equilibrium far from the junction.

(ii) $n_p(-x_p) = ?$

Techniques for obtaining this boundary conditions

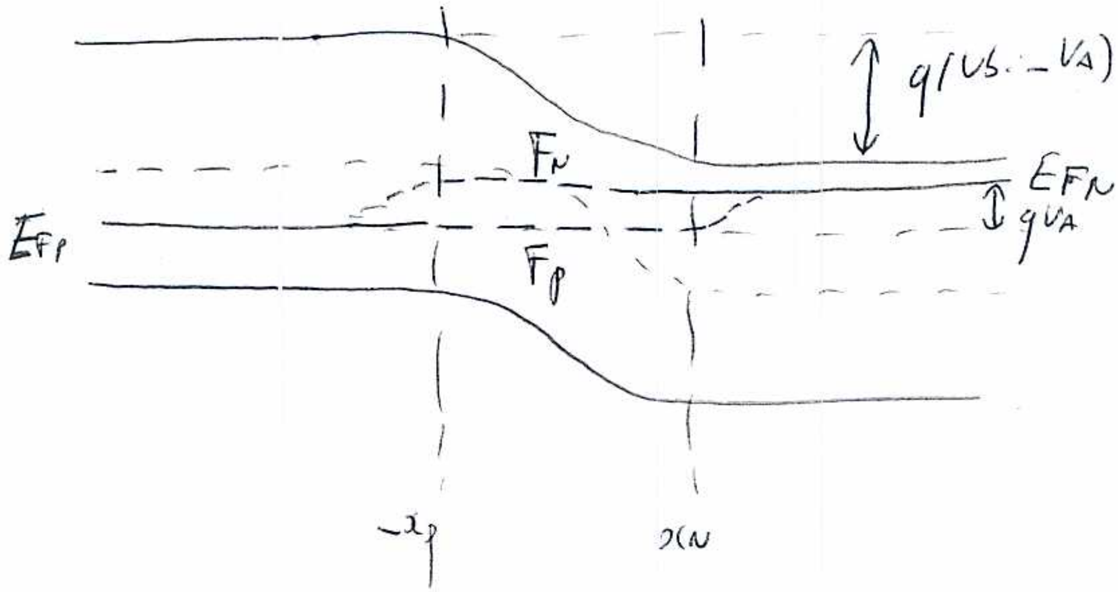
(3.72) we saw that (non equilibrium situation)
$$\begin{cases} n = n_i\, e^{(F_N - E_i)/k_B T} \\ p = n_i\, e^{(E_i - F_p)/k_B T} \end{cases}$$
$\underline{F_N \text{ and } F_p \text{ are } \text{imref}}$

From (3.72) we can obtain (6.11)

$$\boxed{np = n_i^2 e^{(F_N - F_P)/k_BT}}$$

Valid in the entire diode under non equilibrium condition.

If we look at the following figure (Fig 6.4b)



In the transition region we assume that

$$F_N - F_P = qV_A \qquad \text{(monotonic variation)}$$

$$\Rightarrow \quad \boxed{np = n_i^2 e^{qV_A/k_BT}} \quad -x_p \leq x \leq x_N \qquad 6.12$$

at $x = -x_p$

$$n(-x_p)\, p(-x_p) = n(-x_p)\, N_A = n_i^2 e^{qV_A/k_BT}$$

$$\Rightarrow \boxed{n_p(-x_p) = \frac{n_i^2}{N_A} e^{qV_A/k_BT}} \quad (6.14)$$

Also $\boxed{\frac{n_i^2}{N_A} = n_{p0}}$

we can then solve 6.5a

$$\cancel{D_n \frac{d^2 \delta n}{dx^2} - \frac{\delta n}{\tau_n} = 0}$$

$$D_n \frac{d^2 n_p}{dx^2} - \frac{n_p - n_{p0}}{\tau_n} = 0$$

$$\Rightarrow \boxed{\frac{d^2 n_p}{dx^2} - \frac{1}{L_n^2}(n_p - n_{p0}) = 0} \quad \boxed{L_n = \sqrt{D_n \tau_n}}$$

Diffusion Length.

$$\Rightarrow n_p(x) = n_{p0} + A \exp\left(\frac{-x}{L_n}\right) + B \exp\left(\frac{x}{L_n}\right)$$

using the Boundary conditions we derived

$$n_p(-\infty) = n_{po} \Rightarrow A = 0$$

$$n_p(-x_p) = n_{po}\, e^{qV_A/k_BT} = n_{po} + B\exp\left(-\frac{x_p}{L_N}\right)$$

$$\Rightarrow \boxed{B} = n_{po}\left[e^{qV_A/k_BT} - 1\right]\exp\left(\frac{x_p}{L_N}\right)$$

$$\Rightarrow \boxed{n_p(x) = n_{po} + n_{po}\left[e^{qV_A/k_BT} - 1\right]\exp\left(\frac{x + x_p}{L_N}\right)}$$

6.27a $$\boxed{J_N = qD_n\frac{dn}{dx} = q\frac{n_{po}D_N}{L_N}\left[e^{qV_A/k_BT} - 1\right]\exp\left(\frac{x + x_p}{L_N}\right)}$$

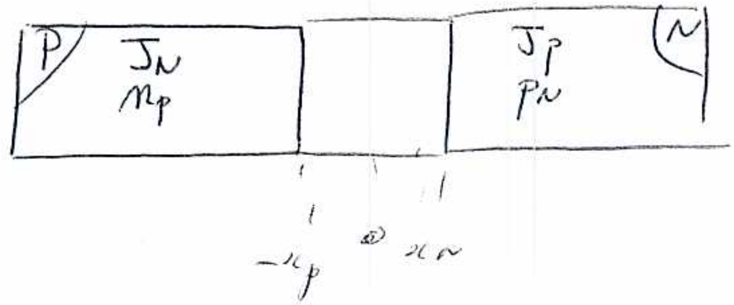
$x \leq -x_p$

Similarly we can obtain

$$p_n(x) = p_{no} + p_{no}\left[\exp\left(qV_A/k_BT\right) - 1\right]\exp\left(-\frac{(x - x_n)}{L_p}\right)$$

6.27b $$\boxed{J_p = \frac{qD_p\,p_{no}}{L_p}\left[e^{qV_A/k_BT} - 1\right]\exp\left(-\frac{(x - x_n)}{L_p}\right)}$$ $x \geq x_n$

Finally, we have obtained



Remark:

J_p and J_n depends on x.

For example $J_p(x) \searrow$ if $x \nearrow$. This is due because of the recombination process, here the hole current is transformed into an e^- current. So the total current is independent of x.

to calculate the total current we need to know either J_p in P-type region or J_n in N-type region. These currents involve majority carriers and are very difficult to derive. The total current can be obtained using the ideal diode approximation.

(b) Ideal diode

$\Longleftrightarrow$ No G/R in the depletion region
(Generation/Recombination)

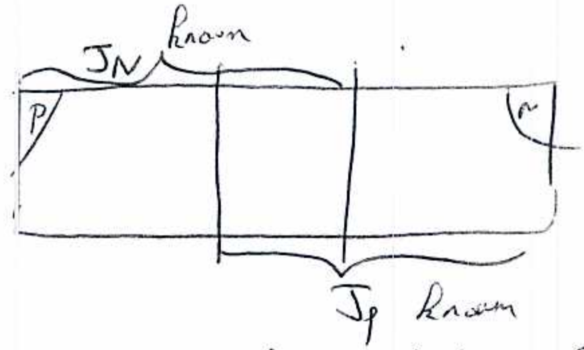
using continuity equations inside the depletion region

$$0 = \frac{1}{q}\frac{dJ_n}{dx} - U_n \qquad 6.7a$$

$$0 = \frac{1}{q}\frac{dJ_p}{dx} - U_p \qquad 6.7b$$

if $U_n = U_p = 0 \Rightarrow J_n$ and J_p are constant if $-x_p \le x \le x_n$.

So
$$\boxed{\begin{aligned} J_n(-x_p \le x \le x_n) &= J_n(-x_p) \\ J_p(-x_p \le x \le x_p) &= J_p(x_n) \end{aligned}} \qquad (6.8)$$



$\Rightarrow$ we have now over-lap between J_n and J_p !

The total current can be obtained by

$$\boxed{J = J_N(-x_p) + J_p(x_n)} \qquad 6.9$$

Shockley's equation $\Rightarrow$

$$\boxed{J = J_0\left(\exp\left(q\frac{V_A}{k_BT}\right) - 1\right)} \qquad 6.29$$

$$\boxed{I = AJ} \qquad 6.30$$

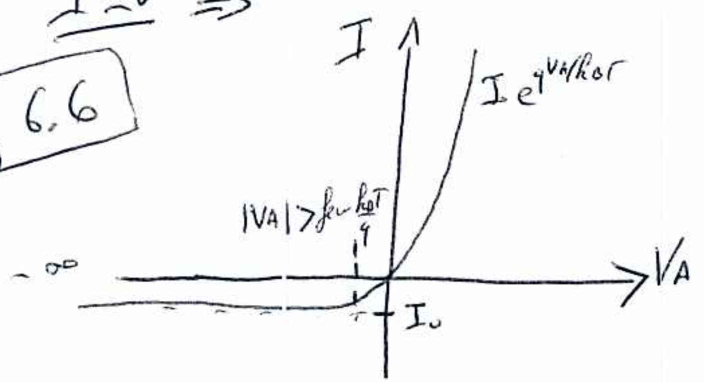
with

$$\boxed{J_0 = q\left[\frac{D_N}{L_N} n_{p0} + \frac{D_p}{L_p} p_{n0}\right]}$$

saturation current density

I–V $\Rightarrow$

Fig 6.6



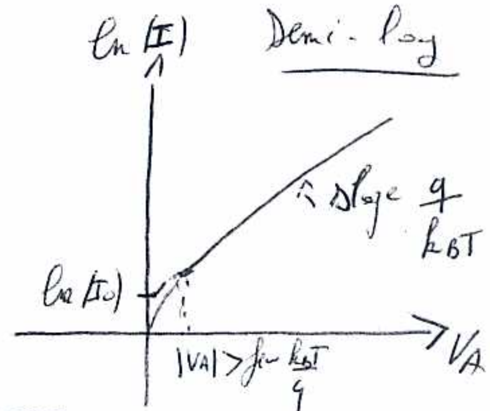
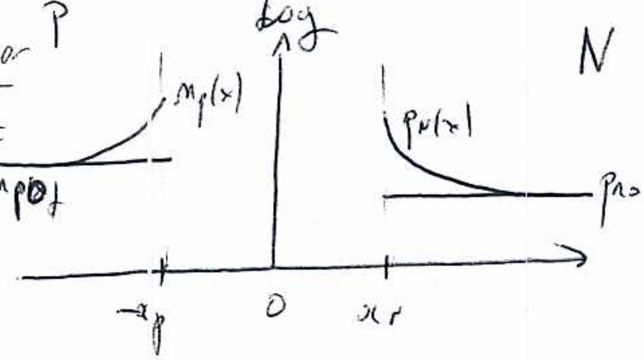
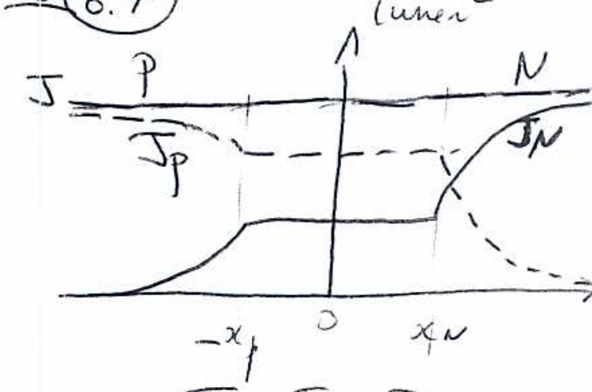


Fig 6.7

minority carrier concentration

Fig 6.8 a





$J = J_N + J_p$