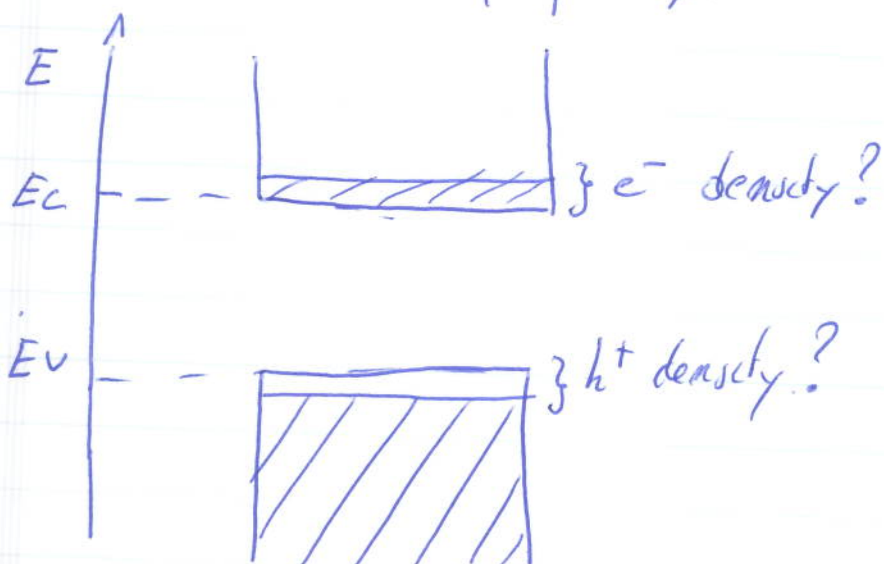
III Semiconductors : Fundamentals

(1) Carrier densities : basics

(a) Definition

we want to know the density of e^- (resp. h^+) inside the CB (resp. VB).



2 questions

(1) How many states are available at each energy?

we call $g(E)dE$ the density of states (DoS)

$\Rightarrow$ number of state that are available between E and $E+dE$

$g_c \Rightarrow$ DoS in CB.

$g_v \Rightarrow$ DoS in VB

(ii) What is the probability that a state is occupied by an electron (or hole) ?

we call $f(E)$ the distribution function for the $\bar{e}$.

$f(E)$ is also called the Fermi-Dirac statistics ~~which~~

(its derivation belongs to a statistical physics course).

$\Rightarrow$ we finally get for the $\bar{e}$

$$\boxed{n = \int_{E_c}^{+\infty} g_c(E) \cdot f(E) \, dE} \quad (2.8a)$$

and for the holes

$$\boxed{p = \int_{-\infty}^{E_V} g_V(E) \left[1 - f(E) \right] dE} \quad (2.8b)$$

In the following we propose to address the two questions above in order to calculate n and p.

b) Density of States

we want to know the DOS in energy $g(E)\,dE$;

however it is much easier to calculate the DOS in

k-space $g(k)\,dk$.

Since $\boxed{g(E)\,dE = g(k)\,dk.}$

[if we know $E(k)$ (dispersion relation for CB and VB).

as well as $g(k)$, one can get $g(E)$

$$\boxed{g(E) = g(k)\cdot\left(\frac{dE}{dk}\right)^{-1}}$$

in 3D

$$\boxed{g(k) = 2\cdot\frac{4\pi k^2}{(2\pi)^3}}$$

Also we know that $E = \dfrac{\hbar^2 k^2}{2m_n^*} + E_c$ for CB $k = \sqrt{\dfrac{2m_n^*(E-E_c)}{\hbar^2}}$

$E = \dfrac{\hbar^2 k^2}{2m_p^*} + E_v$ for VB $k = \sqrt{\dfrac{2m_p^*(E_v-E)}{\hbar^2}}$

So for CB $\dfrac{dE}{dk} = \dfrac{\hbar^2 k}{m_n^*}$

So $g_c(E) = g(k)\left(\dfrac{dE}{dk}\right)^{-1} = 2\cdot\dfrac{4\pi k^2}{(2\pi)^3}\cdot\dfrac{m_n^*}{\hbar^2 k} = \dfrac{2\cdot 4\pi m_n^*}{(2\pi)^3\hbar^2}\cdot\sqrt{\dfrac{2m_n^*(E-E_c)}{\hbar^2}}$

$$\boxed{g_c(E) = \frac{m_n^*\sqrt{2m_n^*(E-E_c)}}{\pi^2\hbar^3}} \qquad [2.6a]$$

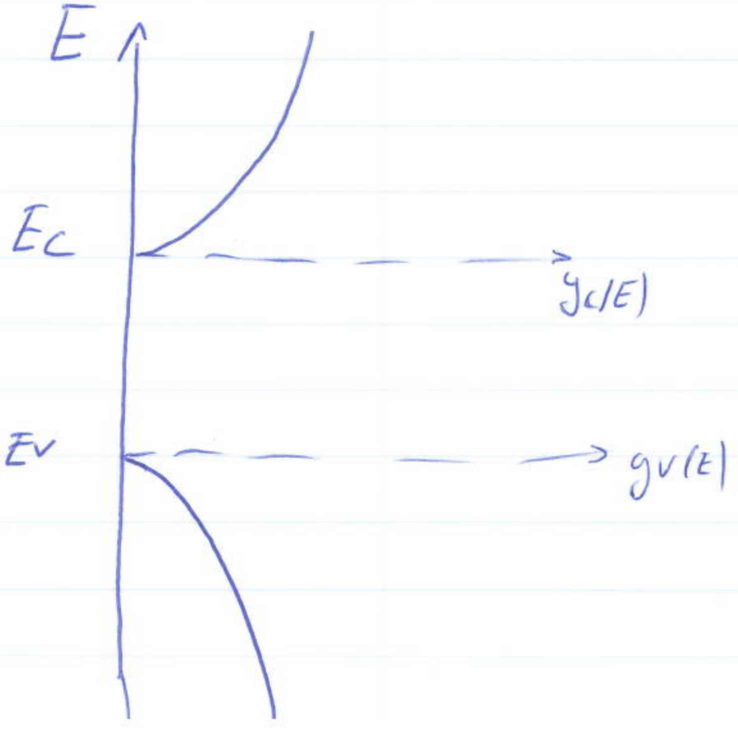
$E \geq E_c$.

and for the VB

we also get

$$\boxed{g_v(E) = \frac{m_p^*\sqrt{2m_p^*(E_v-E)}}{\pi^2\hbar^3}} \qquad [2.6b]$$

$E \leq E_v$.



c) Distribution function.

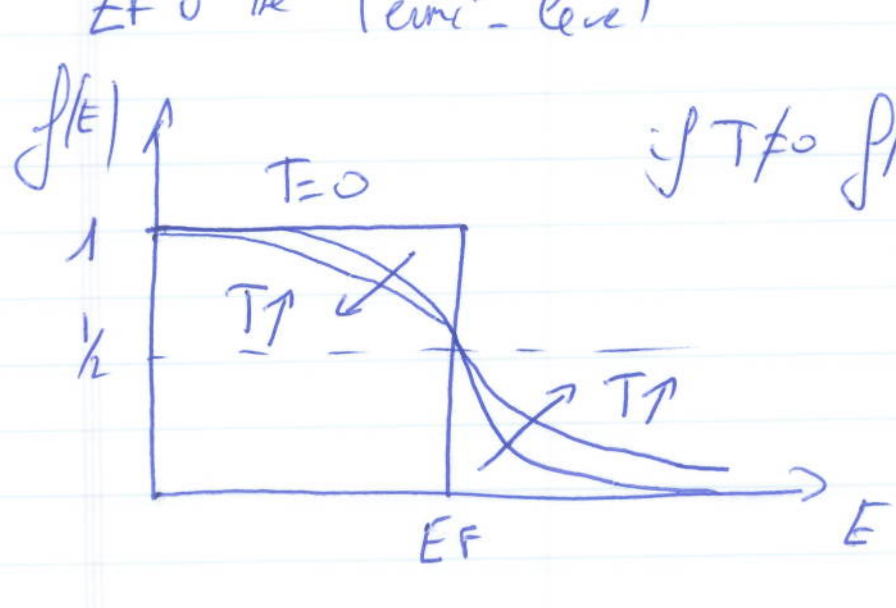
From the energy band theory and definitions of SC, we know that the probability for a state to be occupied by an e^- should depend on the temperature.
$\Rightarrow$ e^- obey the Fermi-Dirac statistics
(which accounts for the Pauli exclusion principle).

$$[2.7] \quad \boxed{f(E) = \frac{1}{1+\exp(\beta(E-E_F))}} \qquad \boxed{\beta = \frac{1}{k_B T}}$$

k_B = Boltzmann constant

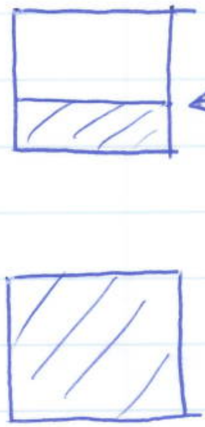
E_F is the Fermi-level

if $T \neq 0$ $f(E_F) = \frac{1}{2}$.



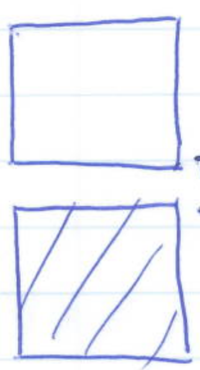
* what is happening for $f(E)$ in SC ?

. Conductors ($T=0$)



E_F is the maximum energy that can take the $\bar{e}$ at $T=0$.

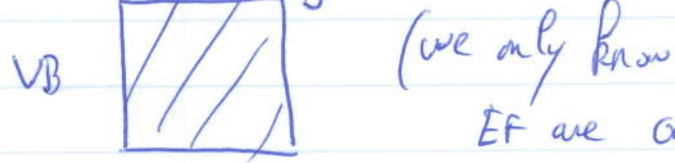
. semiconductor ($T=0$)



E_F lies between VB and CB.
(we only know that states below E_F are occupied)

. semiconductor $T>0$

[Fig 2.15]



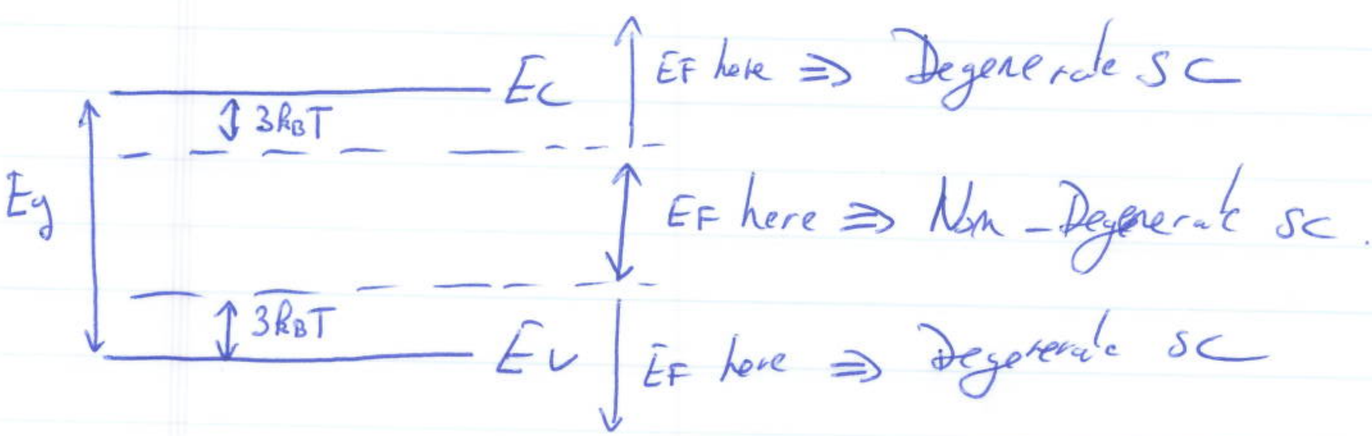
if $T\nearrow$, probability to find an $\bar{e}$ in CB increases. ($f(E)\neq 0$)

if $T\nearrow$, probability $(1-f)$ to find an hole in VB increases. ($1-f(E)\neq 0$).

(d) Non-degenerate Semiconductor

Depending on the position of E_F in the bandgap, the semiconductor can be classified as degenerate or non-degenerate.

[Fig 2.19]



For non-degenerate SC

$$\boxed{E_V + 3k_BT \leq E_F \leq E_C - 3k_BT}$$

So. $(E - E_F) > 3k_BT$ for e^-

. $(E_F - E) > 3k_BT$ for h^+

one can then approximate the Fermi-Dirac distribution by a Maxwell-Boltzmann distribution (classical distribution)

$$\boxed{\begin{aligned} f(E) &\simeq \exp\left(-\beta(E-E_F)\right) \quad \text{for } e^- \\ 1 - f(E) &\simeq \exp\left(-\beta(E_F-E)\right) \quad \text{for holes} \end{aligned}}$$

Summary

Degenerate SC $\Rightarrow$ Fermi Dirac distribution

Non-degenerate SC $\left(E_V + 3k_BT \leq E_F \leq E_C - 3k_BT\right) \Rightarrow$ Maxwell Boltzmann distribution can be used

e) Carrier densities calculations

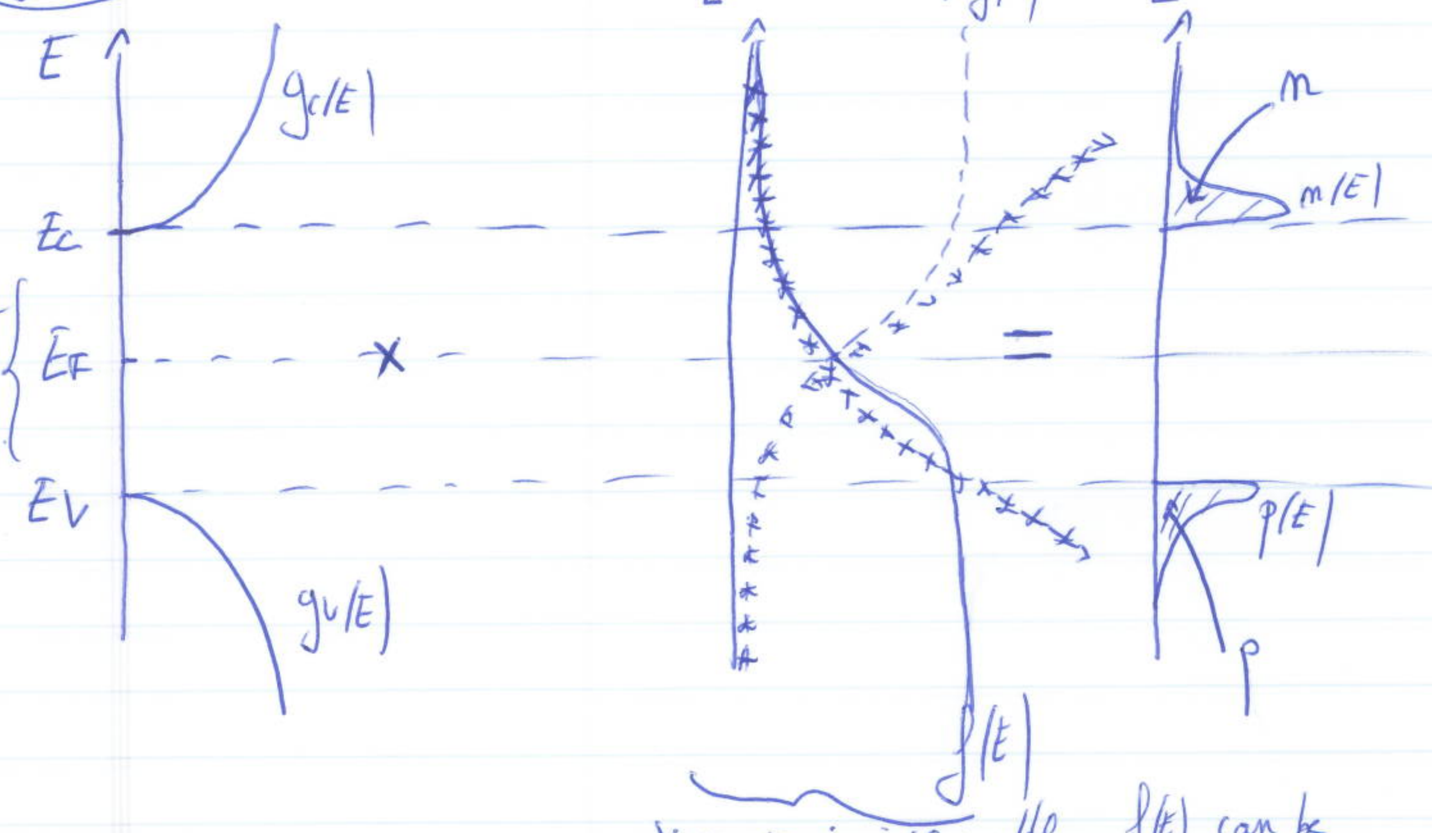
Let us call $n(E) = g_c(E)\, f(E)$; $p(E) = g_v(E)\,[1 - f(E)]$

[$n(E)$ (resp $p(E)$) is the density of $\bar{e}$ (resp. h^+) per unit of volume and per unit of energy

$$n = \int_{E_C}^{+\infty} n(E)\,dE \qquad p = \int_{-\infty}^{E_V} p(E)\,dE \ .$$

Fig 2.16b (graphical integration) $\underline{T > 0}$

Let us suppose E_F in the middle



Since E_F is in the middle, $f(E)$ can be approximated by Maxwell-Boltzmann distribution (*)

* <u>In the general case</u> (Fermi-Dirac distribution).

<u>for e^-</u>

$$n = \frac{m^* \sqrt{2 m_n^*}}{\pi^2 \hbar^3} \int_{E_c}^{+\infty} \frac{\sqrt{E - E_c}}{1 + \exp(\beta(E - E_F))} dE$$

we set $\eta = \beta(E - E_c)$ $\eta_c = \beta(E_F - E_c)$

$$n = \frac{m_n^* \sqrt{2 m_n^*} (k_B T)^{3/2}}{\pi^2 \hbar^3} \underbrace{\int_0^{+\infty} \frac{\eta^{1/2} d\eta}{1 + e^{\eta - \eta_c}}}_{F_{1/2}(\eta_c) \text{ Fermi-Dirac Integral.}}$$

<u>we get for e^-</u>

(2.14a) $$\boxed{n = N_c \frac{2}{\sqrt{\pi}} F_{1/2}(-\beta(E_c - E_F))}$$

N_c is called the effective DOS in the CB

(2.13a) $$\boxed{N_c = 2 \left[\frac{m_n^* k_B T}{2\pi \hbar^2}\right]^{3/2}}$$

an for holes (by analogy).

(2.14b) $$\boxed{p = N_V \frac{2}{\sqrt{\pi}} F_{1/2}\left(\beta(E_V - E_F)\right)}$$

N_V is the effective DOS in the VB.

(2.13b) $$\boxed{N_V = 2\left[\frac{m_p^* k_B T}{2\pi\hbar^2}\right]^{3/2}}$$

* For non-degenerate SC

2.16a $$n = N_C \exp\left(-\beta(E_C - E_F)\right)$$

for e^- $F_{1/2} \simeq \frac{\sqrt{\pi}}{2} \exp\left(\beta(E_F - E_C)\right)$

2.16b $$p = N_V \exp\left(\beta(E_V - E_F)\right)$$

for h^+ $F_{1/2} \simeq \frac{\sqrt{\pi}}{2} \exp\left(\beta(E_V - E_F)\right)$