SOLUTIONS

NAME:

ECE344 Fall08

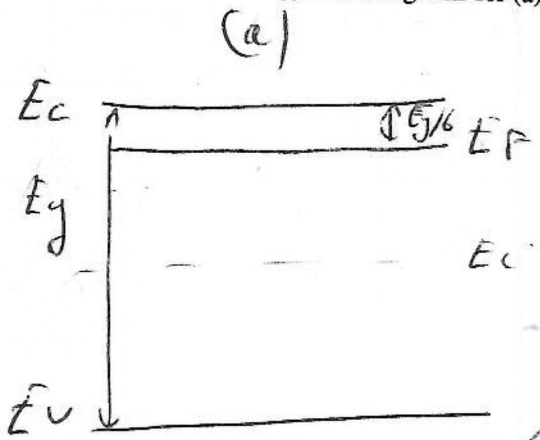
MID-TERM EXAM

1 Conceptual [50pts]

Let us consider two non-degenerate Silicon semiconductors at room temperature (a) a N-type Silicon with $E_F = E_c - E_g/6$, and (b) a P-type Silicon with $E_F = E_v + E_g/3$.

Part I – 22pts

4pts Plot the energy band diagrams for (a) and (b).



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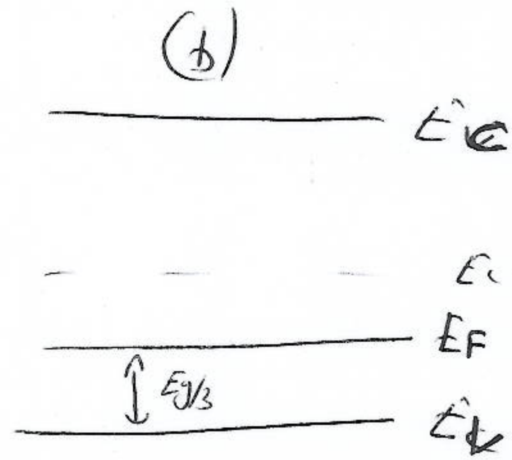

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3pts Fill up this table (you will give the results in function of E_g)

	(a) N-type	(b) P-type
$E_F - E_i$	$E_g/3$	$-E_g/6$
$E_F - E_c$	$-E_g/6$	$-2E_g/3$
$E_F - E_v$	$5E_g/6$	$E_g/3$

/3

3pts For the specific case of semiconductors (a) and (b), Is the basic assumption "non-generate semiconductor" verified ? What does this assumption mean in practice ?

- for (a) $E_C - E_F = E_g/6 > 3k_BT$
  for (b) $E_F - E_V = E_g/3 > 3k_BT$ } assumption is verified

/2

- For a non degenerate SC, one can approximate the Fermi-Dirac statistics by Maxwell-Boltzmann statistics

/1

4pts For both (a) and (b), give the expression of the electron density n and hole density p in function of E_g, n_i and $\beta = 1/(k_B T)$.

(a)

$n_A = n_i e^{\beta E_g/3}$ /1

$p_A = n_i e^{-\beta E_g/3}$ /1

(b)

$n_b = n_i e^{-\beta E_g/6}$ /1

$p_b = n_i e^{\beta E_g/6}$ /1

3pts From the previous results, is the doping concentrations higher in (a) or (b) ? Why was this result expected from the very first question ?

(a) is N-type $N_D \simeq n_a$ } since $n_A > p_b$ doping concentration is
(b) is P-type $N_A \simeq p_b$ } higher in (a). $\underline{N_D > N_A}$ /2

• Since E_F is closer to E_c ~~the~~ in (a) as compared to E_F and E_v in (b)
⇒ this result was expected. /1

2pts Let us suppose for this question only that (a) and (b) have comparable doping densities. Is the resistivity smaller for (a) or (b)? Explain why?

(a) $\rho_a = \dfrac{1}{q \mu_n N_D}$ (b) $\rho_b = \dfrac{1}{q \mu_p N_A}$

Since $N_D = N_A$ but in general $\mu_n > \mu_p$, one can expect $\rho_a < \rho_b$ ⇒ resistivity higher in (b). /2

3pts Let us suppose that the temperature $T \equiv 0$, what can you say about the value of n and p for both (a) and (b) ? What is then happening for very high temperature.

• If $T = 0$, SC is an insulator $n = p = 0$ (no thermal excitation)
[Freeze-out region] /1.5

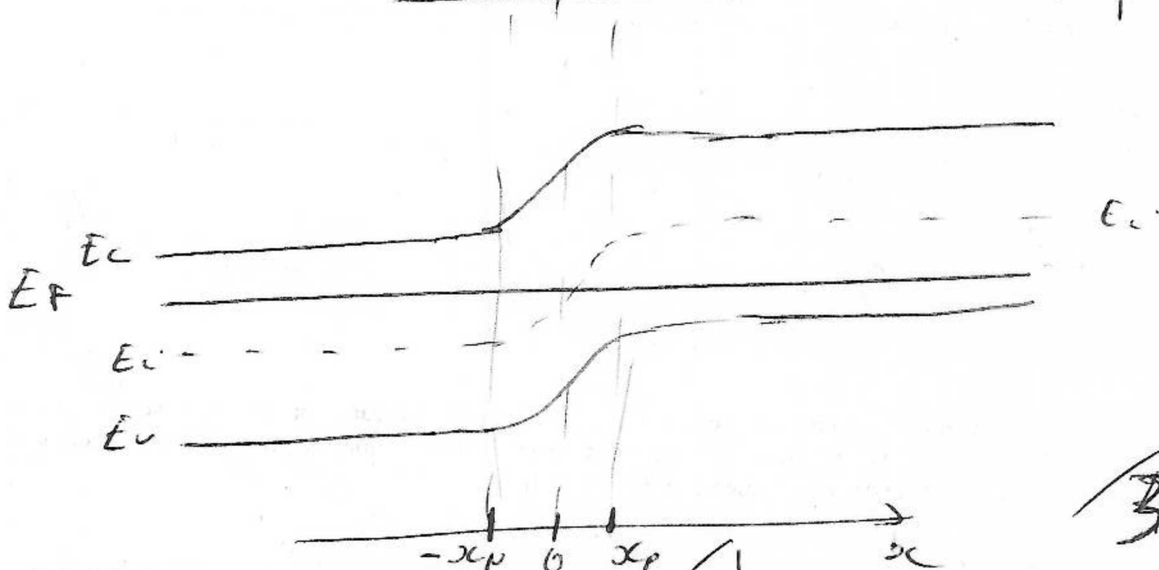
• If $\underline{T \nearrow}$ very high, $n = p = n_i$ for both (a) and (b).
$n_i \nearrow$
this is an intrinsic regime. /1.5

Part II – 18pts

5pts Let us suppose that we put these two materials (a) and (b) in contact. How can we call this junction? Draw the new equilibrium energy band diagram. You will specify the transition region going from $-x_n$ to x_p.

$\Rightarrow$ N-P junction

/1



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2pts The built-in potential is given looking at the absolute value of the relative difference of the intrinsic Fermi-level E_i of (a) and (b) (i.e. $V_o = |(E_i)_N - (E_i)_P|$). Give its expression in function of E_g.

$$V_0 = |(E_i)_N - (E_i)_P| = |\underbrace{(E_i)_N - E_F}| - [(E_i)_P - E_F]|$$

$$|-E_g/3 + E_g/6| = E_g/2$$

/2

2pts Find and justify the direction of the electric field in the transition region.

The slope is positive. $\mathcal{E} = \frac{1}{q}\frac{dE_c}{dx}$

$\vec{\mathcal{E}}$ from left to right $\longrightarrow$

/2

$\vec{\mathcal{E}} \longrightarrow$

4pts Fill up the following table with left or right direction arrows.

	Electrons CB	holes VB
Drift motion	$\longleftarrow$	$\longrightarrow$
Diffusion motion	$\longrightarrow$	$\longleftarrow$
Drift current	$\longrightarrow$	$\longrightarrow$
Diffusion current	$\longleftarrow$	$\longleftarrow$

/4

3pts What is the full ~~the~~ depletion approximation ? Why do we need the full depletion approximation ?

- $\vec{\mathcal{E}} = 0$ outside the transition region
- there is no free charge carriers inside the transition region /2

we need this approximation for solving analytically the Poisson equation /1

2pts Using the charge neutrality condition $N_D x_n = N_A x_p$ inside the transition region of the materials (a) and (b), what can you say about the width of the space charge region zone in the N region compared to the width of the corresponding zone in the P region ?

$N_D x_n = N_A x_p$

Since $N_D > N_A$ (as we demonstrated).

$\frac{N_D}{N_A} > 1 \quad \frac{N_D}{N_A} = \frac{x_p}{x_n} > 1$ so $x_p > x_n$

width in the P region is ~~be~~ larger than N region /2

Part III – 10 pts

1pt What is the differences between an intrinsic and an extrinsic semiconductor ?

intrinsic → pure SC

extrinsic → SC doped with impurities /1

3pts List three assumptions that are made when deriving equation $n \equiv N_d$?

- Charge neutrality /1
- complete ionization /1
- N-type SC /1

3pts List three generation recombination processes

- Band to Band /1
- SRH mechanism (trap-assisted) /1
- Auger recombination /1

3pts Describe the continuity equation in words

The continuity equation describes the evolution of carrier concentration with time. /1
This change of carrier concentration with time is due to the difference between incoming and outgoing flux of carriers plus the generation and minus the recombination.
/2

2 Practice Exercises [30pts]

5pts The photoelectric effect can occur not just for metal but for any substance, including living tissue. Ionization of DNA molecules can cause cancer or birth defects. If the energy required to ionize DNA is on the same order of magnitude as the energy required to produce the photoelectric effect in a metal, let us suppose a work function $W_0 = 5.3eV$ which of these types of electromagnetic waves might pose such a hazard? Explain.

- 60 Hz waves from power lines
- 100 MHz FM radio
- microwaves from a microwave oven with 2.4GHz
- visible light at 540THz
- ultra violet light at 5000THz
- x-rays at 50000THz /2

photo-electric effect: if $W_0 = 5.3\ eV$ minimum photon energy $E_{ph} = W_0 = h\nu$

$\Rightarrow \nu_c = \frac{E_{ph}}{h} = 1284\ THz$ (frequency threshold) /2

if $\nu > \nu_c$ /1 You may be in trouble after long exposure

5pts For the two semiconductors sample specified below, determine the position of E_F by finding $E_F - E_i$, and draw a (Silicon) energy band diagram. Sample (1) N-type $N_D = 10^{15}/cm^3$; Sample (2) P-type $N_A = 10^{16}/cm^3$.

(1) $N_D = 10^{15}/cm^3$

$n \approx N_D = 10^{15}/cm^3$

$p = \frac{n_i^2}{N_D} = 10^5/cm^3$ /1

$E_F - E_i = \frac{k_B T}{q} \ln\left(\frac{n}{n_i}\right) = 0.3eV$ /1

for eV

You can use bulk formula

0.3 — E_c, E_F, E_i, E_v /0.5

(II) $p = N_A = 10^{16}/cm^3$

$n = \frac{n_i^2}{p} = 10^4/cm^3$ /1

$E_i - E_F = \frac{k_B T}{q} \ln\left(\frac{p}{n_i}\right) = 0.36eV$ /1

~~$E_F - E_i = 0.3eV$~~

E_c

0.36 eV — E_i, E_F, E_v /0.5

12pts A N-type Silicon material (a) is uniformly doped with N_d donors and maintained under equilibrium conditions at $T = 300K$. Calculate the different parameter values in the following table knowing that $E_F = E_c - E_g/6$.

Parameter	Value	Units	Explanations (give analytical formula)
n 1.82×10^{16}	$\sim 1.8\,10^{16}$	cm^{-3}	$n_i\, e^{qE_g/3}$
p 5.49×10^{3}	$5.5\,10^{3}$	cm^{-3}	$\frac{n_i^2}{n} = n_i\, e^{-qE_g/3}$
N_d	$18\,10^{16}$	cm^{-3}	$N_d \sim n$.
Total charge density Q	0	Coulomb	$Q = q(-n + p + N_d - N_A)$ $\Rightarrow$ Charge neutrality $= 0$.
Ratio Resistivity between pure Silicon and Silicon (a) ($\mu_n = 1360cm^2/V.s$) $\mu_p = 460 cm^2/V.s$	1.12×10^{6}	N/A	pure $\rho_0 = \frac{1}{q n_i(\mu_n + \mu_p)}$ here $\rho = \frac{1}{q \mu_n N_D}$; $\frac{\rho_0}{\rho_a} = \frac{\mu_n N_D}{n_i(\mu_n + \mu_p)}$
Diffusion coefficient D_n ~~35.2~~	35.2	cm^2/s	$D_n = \left(\frac{q}{k_0T}\right)^{-1} \mu_n$

/2 /2 /2 /2 /2 /2

8pts In a non-degenerate Germanium at room temperature, we give $n_i = 10^{13}/cm^3$, $n = 10^4 p$ and $N_a = 0$. Determine the following values.

Parameter	Value	Units	Explanations (give analytical formula)
n	10^{15}	cm^3	$n = \frac{n_i^2}{p} = \frac{10^4 n_i^2}{n} \Rightarrow \boxed{n = 10^2 n_i}$
p	10^{11}	cm^3	$p = \frac{n}{10^4}$
N_d	$\sim 10^{15}$	cm^3	$n = p + N_d$; $N_d = n - p$.
$E_F - E_i$	119	meV	$E_F - E_i = k_0T \ln\left(\frac{n}{n_i}\right) = k_0T \ln(100)$

/2 /2 /2 /2

3 Problem [20pts]

Part I – 14pts

Using the continuity equation and the definition and the definition of the hole current density derive an equation for $p(x)$ such that:

$$a\frac{\partial p(x)}{\partial t} = \alpha\frac{\partial^2 p(x)}{\partial x^2} - \beta\frac{\partial p(x)}{\partial x} - \gamma p(x) + \eta$$

Give the expressions of a, α, β, γ, η;

$$\frac{\partial p(x)}{\partial t} = D_p\frac{\partial^2 p(x)}{\partial x^2} - p(x)\mu_p\frac{\partial E}{\partial x} - \mu_p E\frac{\partial p(x)}{\partial x} + G - U$$

$a = 1$ /4

$\alpha = D_p$ /2

$\beta = +\mu_p \mathcal{E}$ /2

$\gamma = +\mu_p \frac{\partial \mathcal{E}}{\partial x}$ /2

$\eta = G - U$ /1

In the following table, we perform a series of assumptions, give the new expressions of a, α, β, γ, or η, if they are affected by these assumptions.

Assumptions	a	α	β	γ	η	
No extrinsic R/G	—	—	—	—	$-U$	/1
SRH processes for intrinsic R/G	—	—	—	—	$-\left(\frac{p-p_0}{\tau_p}\right)$	/1
No Electric field	—	—	0	0	—	/1
Steady state	0	—	—	—	—	/1

Write the new resulting equations for $p(x)$

$$0 = D_p\frac{\partial^2 p(x)}{\partial x^2} - \frac{(p(x) - p_0)}{\tau_p}$$ /2

Part II – 6pts

A uniform electric field is assumed which causes a constant gradient of the bands (see Figure 1). What are the directions of the electrons and holes drift currents in the VALENCE band ?

electrons ← holes ← /2

The total current density due to the electrons in the valence band can be written as:

$$\overrightarrow{J_{VB}} = \frac{1}{V} \sum_{\text{filled sates}} (-q)\overrightarrow{v_i},$$

where V is the volume of the semiconductor, q is the positive electric charge, and v_i is the velocity for a given particle in the state i. From the above equation, show that J_{VB} can be rewritten in terms of positive charge particles (holes–missing electrons).

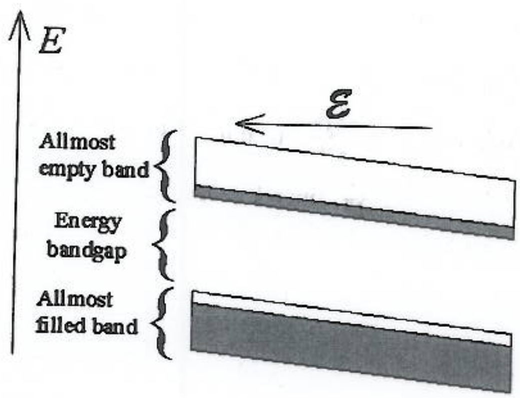

Figure 1: Energy band diagram in the presence of a uniform electric field.

$$\overrightarrow{J_{VB}} = \frac{1}{V}\underbrace{\left(\sum_{\text{all states}} (-q)\vec{v_i}\right)}_{} - \frac{1}{V}\sum_{\text{empty states}} (-q)\vec{v_i}$$ /2

= 0 does not contribute to current if VB is completely filled.

$$\Rightarrow \boxed{\overrightarrow{J_{VB}} = \frac{1}{V}\sum_{\text{empty states}} (q)\overrightarrow{v_i}} \Rightarrow$$ virtual positive charged particle moving uphill.

/2

Equation summary and some parameter values

Fundamental constant:

$m_0 = 9.1 * 10^{-31} kg$	$h = 6.63 * 10^{-34} J.s$	$\hbar = 1.054 * 10^{-34} J.s$
$c = 3.0 * 10^8 m/s$	$q = 1.6 * 10^{-19} C$	$k_B = 1.38 * 10^{-23} J/K$
$\epsilon_0 = 8.8e * 10^{-12} F/m$	$\epsilon_{ox} = 3.9\epsilon_0$	

Semiconductor properties at $T = 300K$:

	$E_g(eV)$	$n_i(cm^{-3})$	$N_c(cm^{-3})$	$N_v(cm^{-3})$	m_e^*	m_p^*	ϵ_r
Si	1.12	$1.0 * 10^{10}$	$3.2 * 10^{19}$	$1.8 * 10^{19}$	$1.18m_0$	$0.81m_0$	11.8
$GaAs$	1.42	$2 * 10^6$	$4.3 * 10^{17}$	$9.4 * 10^{18}$	$0.066m_0$	$0.52m_0$	13.2

Other formula:

- Photon energy: $E = h\nu$ with $\nu = c/\lambda$
- Free electron dispersion relation: $E = \frac{\hbar^2 k^2}{2m}$, since $p = \hbar k$
- For infinite quantum well $E_n = n^2 \frac{\hbar^2 \pi^2}{2mL^2}$, since $k = n\pi/L$