HW2

(I)

(I) DOS

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(1) density of state is given (for 3D device) by

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$$\boxed{g_c(E) = \frac{m_n^* \sqrt{2 m_n^* (E - E_c)}}{\pi^2 \hbar^3} \quad E \geq E_c}$$

- $\boxed{G_c = \int_{E_c}^{E_c + \gamma k_B T} g_c(E)\, dE}$ is giving the number of states available between E_c and $E_c + \gamma k_B T$. (in the conduction band)

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- Similarly for the valence band

$$\boxed{G_v = \int_{E_v - \gamma k_B T}^{E_v} g_v(E)\, dE} \quad \text{with} \quad \boxed{g_v(E) = \frac{m_p^* \sqrt{2 m_p^* (E_v - E)}}{\pi^2 \hbar^3} \quad E \leq E_v}$$

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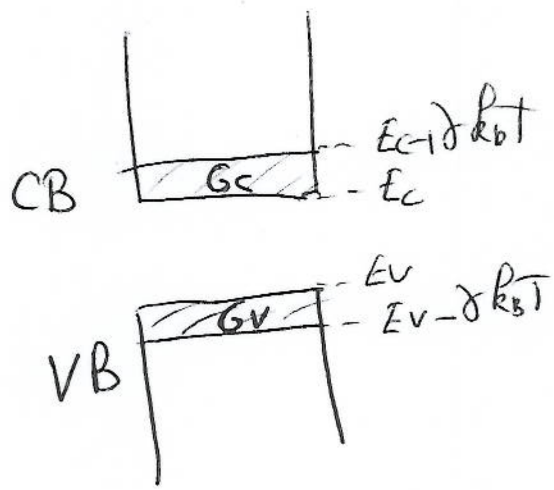

we obtain

$$G_C = \frac{m_n^*\sqrt{2m_n^*}}{\pi^2\hbar^3}\,\frac{2}{3}\,\left(E-E_C\right)^{3/2}\Big|_{E_C}^{E_C+\gamma k_BT}$$

$$\boxed{G_C = \frac{\left(2m_n^*\gamma k_BT\right)^{3/2}}{\pi^2\hbar^3}}$$ 3

$\Longrightarrow$ available state in CB by unit of volume

$$G_V = \frac{m_p^*\sqrt{2m_p^*}}{\pi^2\hbar^3}\left(-\frac{2}{3}\right)\left(E_V-E\right)^{3/2}\Big|_{E_V-\gamma k_BT}^{E_V}$$

$$\boxed{G_V = \frac{\left(2m_p^*\gamma k_BT\right)^{3/2}}{\pi^2\hbar^3}}$$ 3

$$\boxed{3}$$

$$\boxed{2}$$

6/6 <u>for 2D</u>

$$g_{2D}(k)\,dk = 2\frac{S}{(2\pi)^2}\,2\pi k\,dk$$

$$g_{2D}(E) = g_{2D}(k)\left(\frac{dE}{dk}\right)^{-1} \qquad \frac{dE}{dk} = \frac{\hbar^2 k}{m^*}$$

-1 if not specified

$$\boxed{g_{2D}(E) = \frac{S}{\pi}\frac{m^*}{\hbar^2} \quad \text{for } E \geq E_1}$$

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g_{2D} is independent of the energy!

<u>for 1D</u>

$$g_{1D}(k)\,dk = 2\left(\frac{L}{2\pi}\right)2\,dk$$

$$g_{1D}(E) = g_{1D}(k)\left(\frac{dE}{dk}\right)^{-1} = \cancel{2}\left(\frac{L}{\cancel{2}\pi}\right)2\,\frac{m^*}{\hbar^2 k}$$

$$\text{also } k = \sqrt{\frac{2m^*(E-E_1)}{\hbar^2}}$$

-1 if not specified

$$\text{So } \boxed{g_{1D}(E) = \sqrt{\frac{2m^*}{E-E_1}}\left(\frac{L}{\pi\hbar}\right) \quad \text{for } E > E_1}$$

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(4)

(3) ~~[illegible]~~

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3D	$2.41 \cdot 10^{5}\ eV^{-1}$
2D	$4.5 \cdot 10^{4}\ eV^{-1}$
1D	$6.93 \cdot 10^{2}\ eV^{-1}$

(4) for 2D

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$$n_{2D} = \int_{E_1}^{+\infty} \frac{1}{(\pi)} \frac{m^*}{\hbar^2} e^{-\frac{1}{k_BT}(E-E_F)} dE$$

1

$$n_{2D} = \frac{m^*}{\pi \hbar^2} (-k_BT) \cdot e^{-\frac{1}{k_BT}(E-E_F)} \Big|_{E_1}^{+\infty}$$

(+1) if E_1 is here.

$$n_{2D} = \frac{k_BTm^*}{\pi \hbar^2} e^{\frac{1}{k_BT}(E_F - E_1)}$$

$$\boxed{N_{C2D} = \frac{m^* k_BT}{\pi \hbar^2}}$$

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for 1D

$$n_{1D} = \int_{E_1}^{+\infty} \sqrt{\frac{2m^*}{E-E_1}} \left(\frac{1}{\pi\hbar}\right) e^{-\frac{1}{k_BT}(E-E_F)}\, dE$$ /1

$$n_{1D} = \frac{\sqrt{2m^*}}{\pi\hbar} \int_{E_1}^{+\infty} \frac{1}{\sqrt{E-E_1}}\, e^{-\frac{1}{k_BT}(E-E_F)}\, dE$$

$$n_{1D} = \frac{\sqrt{2m^*}}{\pi\hbar}\, e^{\frac{1}{k_BT}(E_F-E_1)} \int_{E_1}^{+\infty} \frac{1}{\sqrt{E-E_1}}\, e^{-\frac{1}{k_BT}(E-E_1)}\, dE$$

$$\eta = \frac{1}{k_BT}(E-E_1)$$

$$d\eta = \frac{1}{k_BT}\, dE$$

$$n_{1D} = \frac{\sqrt{2m^*}\sqrt{k_BT}}{\pi\hbar}\, e^{\frac{1}{k_BT}(E_F-E_1)} \underbrace{\int_0^{+\infty} \eta^{-1/2} e^{-\eta}\, d\eta}_{\Gamma(\frac{1}{2}) = \sqrt{\pi}}$$

$$\boxed{n_{1D} = \sqrt{\frac{2m^* k_BT}{\pi\hbar^2}}\, e^{\frac{1}{k_BT}(E_F-E_1)}}$$

$$\Rightarrow \boxed{N_{C1D} = \sqrt{\frac{2m^* k_BT}{\pi\hbar^2}}}$$ /2

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4/4 $$n_{2D} = \int_{E_1}^{+\infty} \frac{m^*}{\pi \hbar^2} \cdot \frac{1}{1+e^{\beta(E-E_F)}}\, dE$$

$$= \frac{m^*}{\pi \hbar^2} \int_{E_1}^{+\infty} \frac{1}{1+e^{\beta(E-E_F)}}\, dE$$

$$= \frac{m^*}{\pi \hbar^2} \int_{E_1}^{+\infty} \frac{e^{-\beta(E-E_F)}}{e^{-\beta(E-E_F)}+1}\, dE$$

$$= \frac{m^*}{\pi \hbar^2}\left(-\frac{1}{\beta}\right) \ln\left(1+e^{-\beta(E-E_F)}\right)\Bigg|_{E_1}^{+\infty}$$

$$\boxed{n_{2D} = \frac{m^* k_B T}{\pi \hbar^2} \ln\left(1+e^{\frac{1}{k_B T}(E_F-E_1)}\right)}$$ 3

$$\boxed{n_{2D} = N_{C2D} \ln\left(1+e^{\frac{1}{k_B T}(E_F-E_1)}\right)}$$ 1

(−1 if not)

II Distribution function

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(1) <u>Fermi Dirac</u> $f_{FD} = \dfrac{1}{1+e^{\beta(E-E_F)}}$

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<u>Maxwell Boltzmann</u> $f_{MB} = e^{-\beta(E-E_F)}$

	f_{FD}	f_{MB}	Error %
$E-E_F = 2.2\,k_BT$	$\frac{1}{1+e^{2.2}} \simeq 0.1$	$e^{-2.2} \simeq 0.11$	$\sim 10\%$
$E-E_F = 2.9\,k_BT$	$\simeq 0.052$	$\simeq 0.055$	$\sim 5\%$
$E-E_F = 4.6\,k_BT$	$\simeq 0.00995$	$\simeq 0.01005$	$\sim 1\%$

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for non-degenerate SC $\left[(E-E_F) > 3k_BT\right]$

$\Rightarrow f_{FD} \simeq f_{MB}$.

So Fermi-dirac can be approximated by Maxwell-Boltzmann statistics.

1/1

* (2) $T > 0K$

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• $f_{FD}(E) = \frac{1}{1+e^{\beta(E-E_F)}}$ if $E=E_F$ $f_{FD} = \frac{1}{2}$ so $\boxed{50\%}$

1

• if $E_F = E_C$ and $E = E_C + k_BT$

$$\Rightarrow f_{FD}(E) = \frac{1}{1+e^{\beta(\cancel{E_C}+k_BT-\cancel{E_C})}} = \frac{1}{1+e} \simeq 0.269$$

so $\boxed{26.9\%}$

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III Energy band

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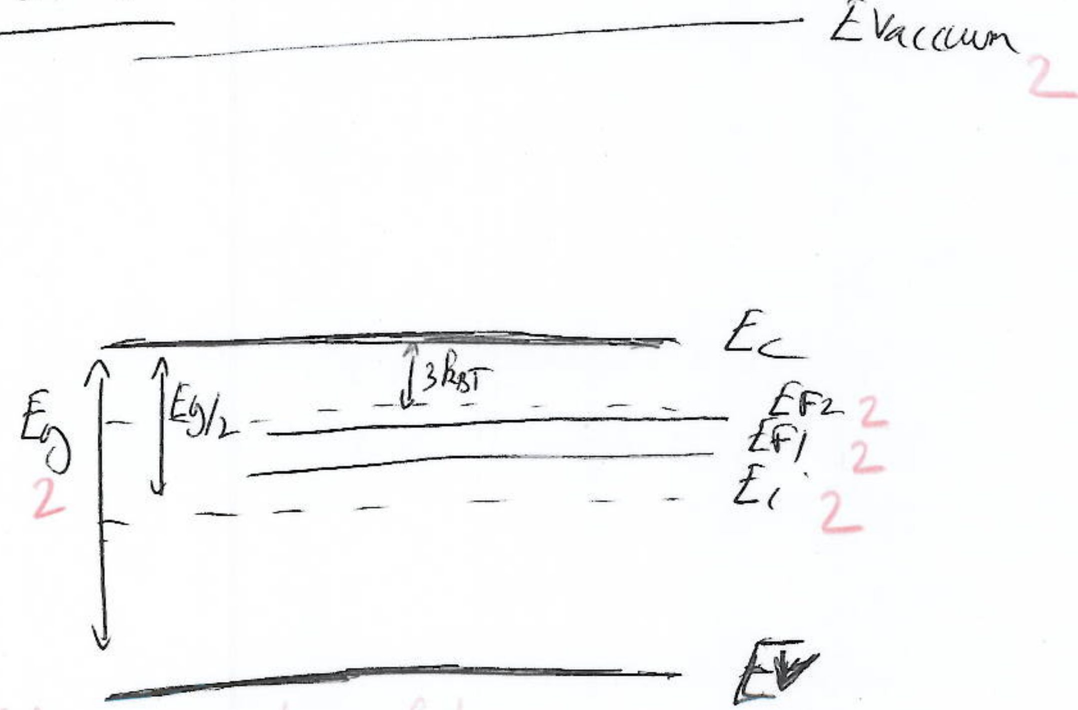

−1 if Eg/2 not specified

−2 if 3kBT not specified