ECE344 Fall08

HOMEWORK 2

Energy Band Theory and Carrier Densities

1 DOS

1. For a 3D semiconductor, develop an expression for the total number of available states/cm^3 in the conduction band between energies E_c and $E_c + \gamma k_B T$, where γ is an arbitrary constant. Do the same for the valence band between energies $E_v - \gamma k_B T$ and E_v.

2. Using the effective mass approximation, derive the expression of the density of state (DOS) $g(E)$ for a 2D and 1D electron gas. You will use the fact that E_1 is the minimum potential energy for the electrons ($E_1 > E_c$ due to quantization effects in systems with low-dimensionalities) such that

$$E = \frac{\hbar^2 k^2}{2m^*} + E_1.$$

   Also for 2D, we give the following density of state in k-space (per unit of Volume):

$$g(k)dk = 2 * \frac{S}{(2\pi)^2} 2\pi k dk,$$

   and for 1D:

$$g(k)dk = 2 * \frac{L}{(2\pi)} 2dk$$

   S for (2D) Surface and L for (1D) Length.

3. Calculate the number of states per unit of energy in a 3D $100nm$ by $100nm$ by $10nm$ piece of Silicon ($m^* = 1.08m_0$, m_0: mass of the electron) $100meV$ above the conduction band edge. Write the result in units of eV^{-1}. Do the same for a $100nm$ by $100nm$ 2D silicon sheet and $100nm$ 1D silicon wire (we consider $E_1 = 40meV$).

4. For non-degenerate semiconductors, derive analytically the expressions of the effective density of states for a 2D and 1D system (N_c^{2D} and N_c^{1D}).

5. In the general case (Fermi-Dirac distribution), derive analytically the expression of electron density for a 2D electron gas (you will give the expression in function of N_c^{2D}).

2 Distribution function

1. Evaluate the approximation errors if one uses the Maxwell-Boltzmann statistics instead of the Fermi-Dirac statistics, for the following energies: $E - E_F = 2.2k_BT$, $E - E_F = 2.9k_BT$, $E - E_F = 4.6k_BT$. Comment.

2. Under equilibrium conditions and $T > 0K$, what is the probability of an electron state being occupied if it is located at the Fermi level ?, (ii) If E_F is positioned at E_c, determine (numerical answer) the probability of finding an electron at $E_c + k_BT$.

3 Energy band

In the energy band diagram below, indicate the usual positioning of the following energy levels:
(i) E_i, intrinsic Fermi level, (ii) E_{F_1}, Fermi level for N-type non-degenerate semiconductor with $N_D = 10^{15}/cm^3$, (iii) E_{F_2}, Fermi level for N-type non-degenerate semiconductor with $N_D = 10^{17}/cm^3$, (iv) E_{vac}, vacuum energy level, (v) E_G, energy bandgap. Add comments as necessary to forestall any misinterpretation of your graphical answer.