344 HW1

I) Photoelectric Effect (24pts)

(12) (1) $W_0 = 4.3eV$ $K_{e^-} = 9$ Joules.

- minimum photon energy $\boxed{E_{ph} = W_0 + K_{e^-}} = 5.39 \simeq \underline{8.5 \cdot 10^{-19} J}$
- $E_{ph} = h\nu$ $\boxed{\nu = \frac{E_{ph}}{h}} = 1.284 \cdot 10^{15} Hz \equiv \boxed{1284 THz}$
- $\boxed{\lambda = \frac{c}{\nu} = \frac{hc}{E_{ph}}} = \frac{6.62 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{8.5 \cdot 10^{-19}} = 2.34 \cdot 10^{-7} m$ $\boxed{= 0.234 \mu m}$
- Planck $\boxed{p = \frac{h}{\lambda}} = 2.83 \cdot 10^{-27} kg\, m/s$ or $\frac{Js}{m}$

2 [12 pts] 2

$W_0 = 2\,eV == 2q$ Joule.

we know that $\boxed{K_{e^-} = h(\nu - \nu_0)}$ (*) if $\nu > \nu_0$ or $\lambda < \lambda_0$

frequency threshold $\boxed{\nu_0 = \frac{W_0}{h}} = \boxed{483.5\ THz}$

wavelength threshold $\boxed{\lambda_0 = \frac{c}{\nu_0}} = \frac{hc}{W_0} = \boxed{620\ nm}$

* if incoming radiation is $\lambda = 610\ nm$ $\left(= \nu = \frac{c}{\lambda} = 491.8\ THz\right)$

$\left.\begin{array}{l}\lambda < \lambda_0 \\ \text{or } \nu > \nu_0\end{array}\right\}$ So one can use (*).

$\boxed{K_{e^-} = h(\nu - \nu_0) = h\nu - W_0} = 0.055 \cdot 10^{-19}\ J == 0.034\ eV$

$== \boxed{34\ meV}$

Since $K_{e^-} = \frac{p^2}{2m}$ $\left(== \frac{1}{2} m v^2\right)$

velocity is $\boxed{v = \sqrt{\frac{2K_{e^-}}{m}}} = \sqrt{\frac{2 \cdot 3.4 \cdot 10^{-3} q}{9.1 \cdot 10^{-31}}} = \boxed{1.09 \cdot 10^5\ m/s}$

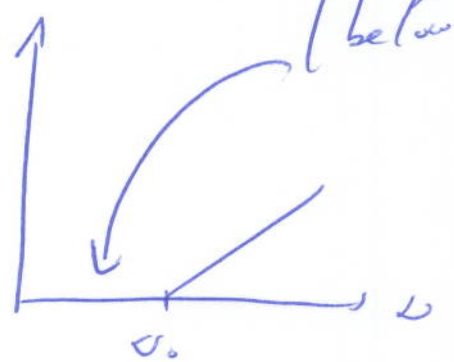
③

a if it is an incoming radiation of $\lambda = 630nm$.

$$\left(\Rightarrow \nu = \frac{c}{\lambda} = 476.2 THz \right)$$

we have $\lambda > \lambda_0$
so $\nu < \nu_0$ } we cannot use equation (*)

The photoelectric effect does not happen (below threshold).



II Fundamentals of QM (26pts).

① for a bullet 0.1kg, $v = 100 m/s$

$$\left[\lambda = \frac{h}{p} = \frac{h}{mv}\right] = \underline{6.63 \cdot 10^{-35} m}.$$

for an e^- at $10^6 m/s$.

$$\left[\lambda = \frac{h}{mv}\right] = \underline{7.2 \cdot 10^{-10} m}$$

The De Broglie wavelength for the bullet is immensurably small. The bullet does not need to be treated as wavelike using Quantum mechanics (unlike electrons).

(2) * if $K_{e^-} = 100\,eV$

$$K_{e^-} = \frac{p^2}{2m} \qquad p = \frac{h}{\lambda}$$

$$K_{e^-} = \frac{h^2}{\lambda^2 2m} \Rightarrow \boxed{\lambda = \sqrt{\frac{h^2}{2mK_{e^-}}}} = \boxed{0.1227\,nm}$$

(3) Infinite square well

$$\Psi_3 = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$$

probability to find e^-

$$P(x) = |\Psi_3(x)|^2 = \frac{2}{L} \sin^2\left(\frac{3\pi x}{L}\right)$$

probability is maximum if $\sin(\cdot) = 1 \Rightarrow \frac{3\pi x}{L} = \frac{\pi}{2} + k\pi$ $k = 0, 1, 2 \ldots$

$$\Rightarrow x = \frac{L}{6} + \frac{kL}{3}$$

Solution possible are $\left\{\frac{L}{6}; \frac{L}{2}; \frac{5L}{6}\right\}$

* one can also do

$P(x)$ max if $\frac{dP(x)}{dx} = 0$.

$$\Rightarrow 2\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi x}{L}\right) = 0.$$

if $\sin\left(\frac{3\pi x}{L}\right) = 0$

Solution

$$\frac{3\pi x}{L} = k\pi \Rightarrow \left\{0, \frac{L}{3}, \frac{2L}{3}, L\right\}.$$

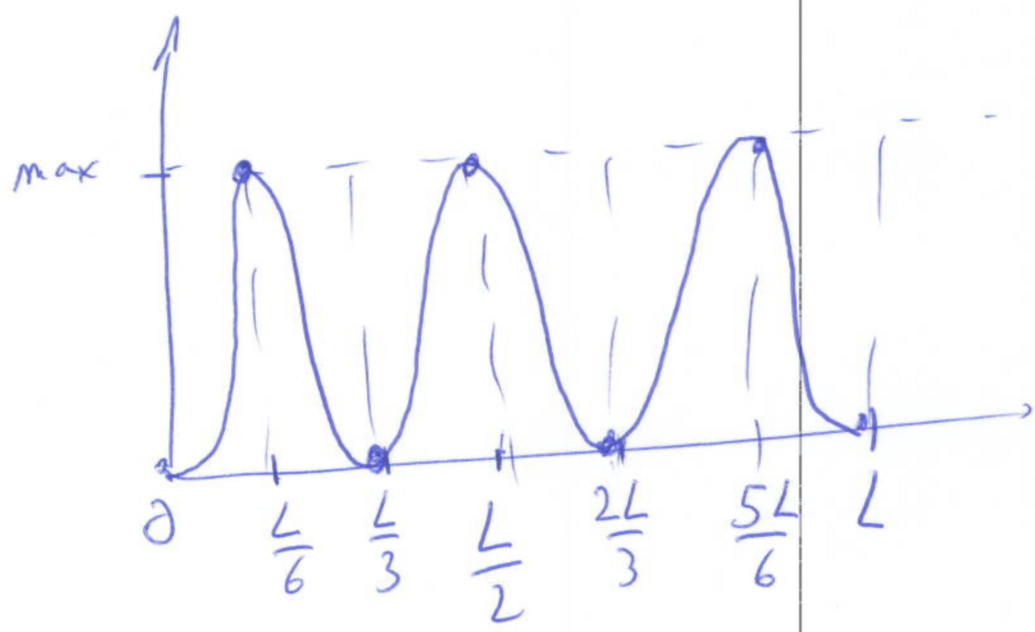
however these extrema are minima since $\psi_3 = 0$ with $\sin\frac{3\pi x}{L} = 0$

if $\cos\left(\frac{3\pi x}{L}\right) = 0$

$$\frac{3\pi x}{L} = \frac{\pi}{2} + k\pi$$

$$\Rightarrow \underbrace{\left\{\frac{L}{6}, \frac{L}{2}, \frac{5L}{6}\right\}}$$

These extrema are maxima



(4) $E < 1\,\mathrm{eV}$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2} < q.$$

$$n^2 < \frac{q\,2mL^2}{\hbar^2 \pi^2}$$

$$\boxed{n < \sqrt{2qm}\left(\frac{L}{\hbar\pi}\right)}$$

Example if $L = 1.10^{-6}\,\mathrm{m}$. ($== 1\mu\mathrm{m}$).

$1 \leq n \leq 1629 \Rightarrow$ 1629 states are available.