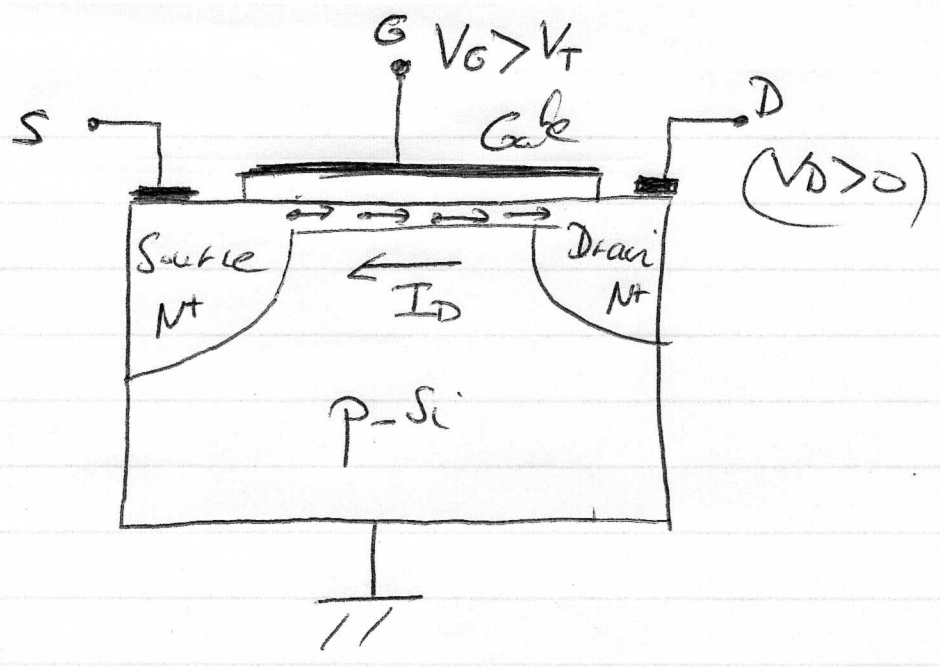




See Fig 17.1

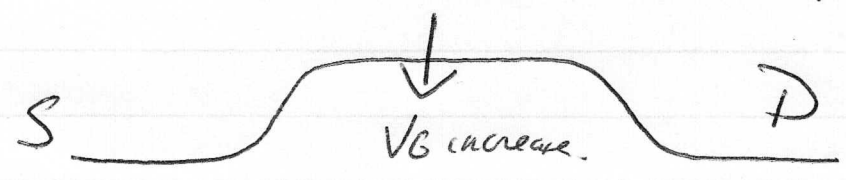


# ① Introduction

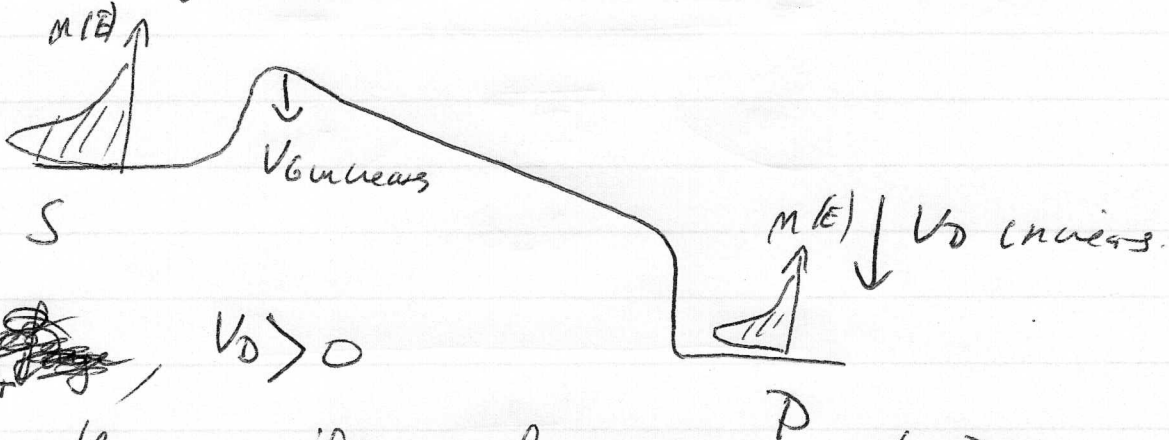
## ① The role of the gate electrode.

Positive gate voltage does two things.

- (i) Invert the surface (if  $V_G > V_T$ ) and increases the conductivity of the channel.
- (ii) Reduces the potential energy barrier seen by the  $e^-$  from the source to the Drain regions. ~~from source to drain~~   
 ~~through the channel~~



(b) The role of the Drain electrode



if  $V_0 > V_T$ ,  $V_0 > 0$

$e^-$  can then flow from Source to Drain.

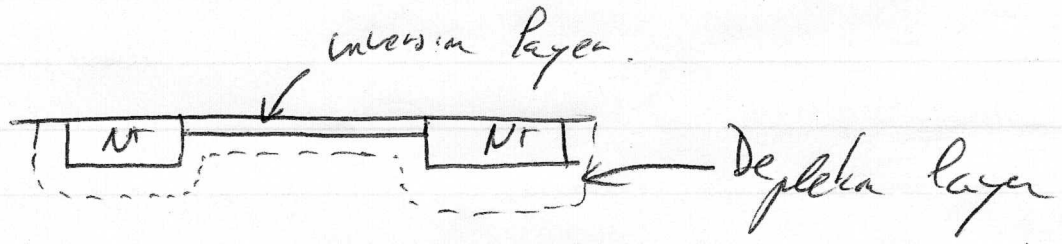
(c) MOSFET operations Fig. 17.2

$V_G > V_T$

(a)  $V_0 > 0$  (small)

Variation of the electron density along the channel

is small  $I_D \propto V_D \Rightarrow$  linear regime.



(b)  $V_D > 0$  (larger)

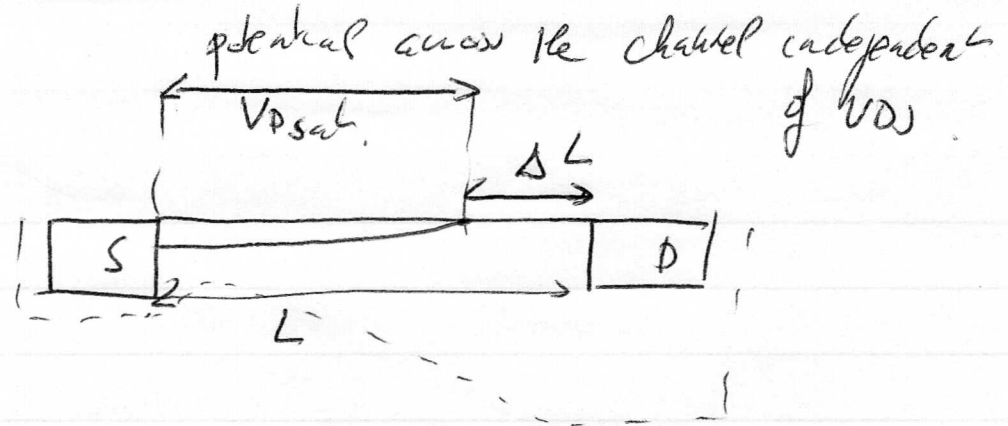
increase in the drain current is reduced to the reduced conductivity of the channel at the drain end.



(c)  $V_D = V_{Dsat}$  Drain saturation voltage.

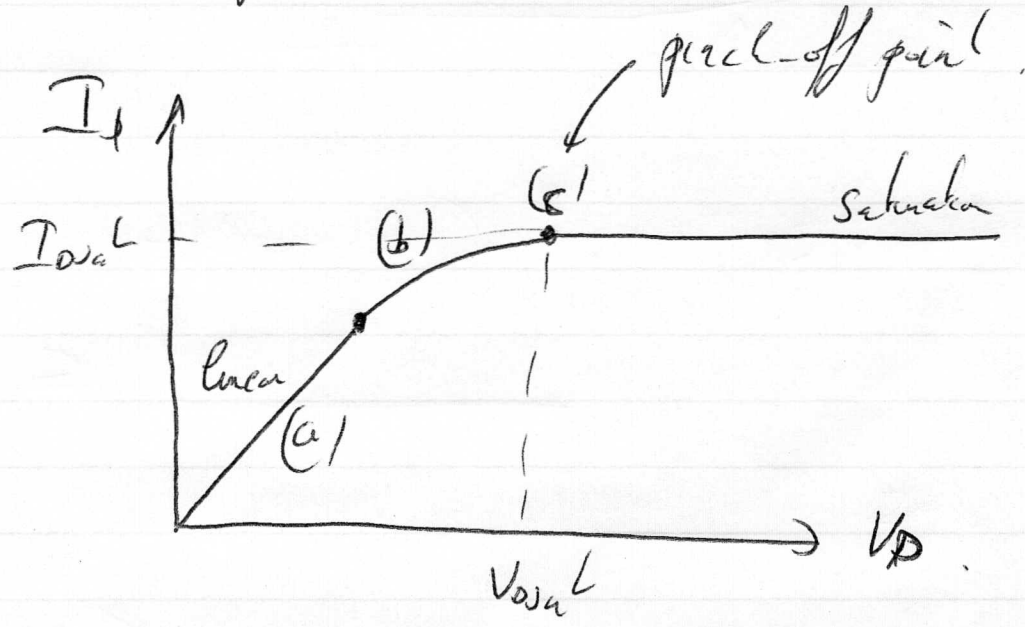
$\Rightarrow$  Pinch off point :  $e^-$  density at the end channel is (ideally) zero

(d)  $V_D > V_{Dsat}$   $\Rightarrow$  Post pinch-off characteristic. The excess drain voltage is dropped across the highly resistive pinch-off region denoted by  $\Delta L$ .



The current saturates since it is fixed by the potential dropped across the channel  $V_{psat}$ .

Fig 17.3





## ② MOSFET analysis

### ① The linear model

it describes the behavior of a MOSFET biased with a small drain to source voltage.

MOSFET acts as a linear device  $\Rightarrow$  can be modeled as a linear resistor whose resistance is modulated by the gate voltage.

general expression for the drain current.

$$I_D = - \frac{Q_{inv} W L}{t_r}$$

$Q_{inv}$  = inversion layer charge per unit of area.

$W$  = gate width.

$L$  = gate length

$t_r$  = transit time

$$t_r = \frac{L}{v}$$

$v$  = velocity.

$$\boxed{v = \mu E = \mu \frac{V_{DS}}{L}}$$

$\nearrow$  mobility       $\uparrow$  electric field

$$\Rightarrow \boxed{I_D = -\mu Q_{inv} \frac{W}{L} V_{DS}}$$

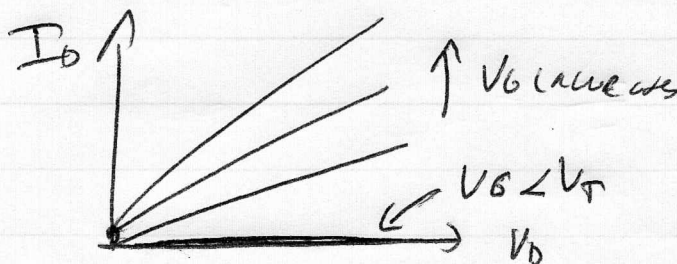
we draw in the MOS capacitor section that

$$V_G = V_T - \frac{Q_{inv}}{C_{ox}} \quad \text{if } V_G > V_T$$

$$\Rightarrow \boxed{Q_{inv} = -C_{ox} (V_G - V_T)}$$

Finally  $\Rightarrow \boxed{I_D = \mu C_{ox} \frac{W}{L} (V_G - V_T) V_{DS}}$

$\left\{ \begin{array}{l} \times \text{ if } V_{DS} \text{ is small } |V_{DS}| \ll |V_G - V_T| \Rightarrow \mu, E \text{ and } Q_{inv} \text{ are expected constant between source and drain} \\ \times \text{ also } I_D = 0 \text{ if } V_G < V_T \end{array} \right.$



① The quadratic model. (Square-Law theory)

the model includes the voltage variation between Source and Drain for the ~~charge~~-inversion layer charge -

$$Q_{inv}(x) \approx -C_{ox}(V_G - V_T - V(x))$$
 17-16

$V(x)$  is the electrostatic potential in the channel due to the Source-Drain voltage.

$$V(x) = \begin{cases} 0 & \text{if } x=0 \\ V_D & \text{if } x=L \end{cases}$$

In this model  $V_T$  is independent of  $x$ , this means that the variation of the depletion charge is ignored.

⇒ most commonly used model

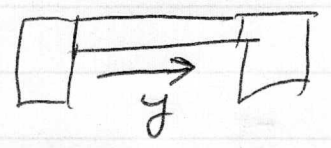


in the linear model, we obtained

$$I_D = \mu C_{ox} \frac{W}{L} (V_G - V_T) V_D \quad \underline{V_G > V_T}$$

Let us consider that the linear model is valid locally.

$$\begin{cases} L \rightarrow dy \\ V_D \rightarrow dV \end{cases}$$



$$\text{also } V_G - V_T \Rightarrow V_G - V_T - V$$

$$\Rightarrow \boxed{I_D = \mu C_{ox} \frac{W}{dy} (V_G - V_T - V) dV}$$

Integration from source to Drain.

$$\int_0^L I_D dy = \mu C_{ox} W \int_0^{V_D} (V_G - V_T - V) dV$$
  
$$\Rightarrow L I_D \quad \left( (V_G - V_T) V_D - \frac{V_D^2}{2} \right)$$

Since  $I_D$  is constant along the channel.

we obtain

$$V_G > V_T \quad \boxed{I_D = \mu C_{ox} \frac{W}{L} \left[ (V_G - V_T) V_D - \frac{V_D^2}{2} \right]} \quad \boxed{\frac{17.17}{(W=2Z)}}$$

The Drain current increases linearly, with the applied  $V_D$  potential, but then reaches a maximum value.

$$\textcircled{17.21} \quad \boxed{V_D = V_G - V_T \equiv V_{D,sat}}$$

↑  
Drain saturation voltage.

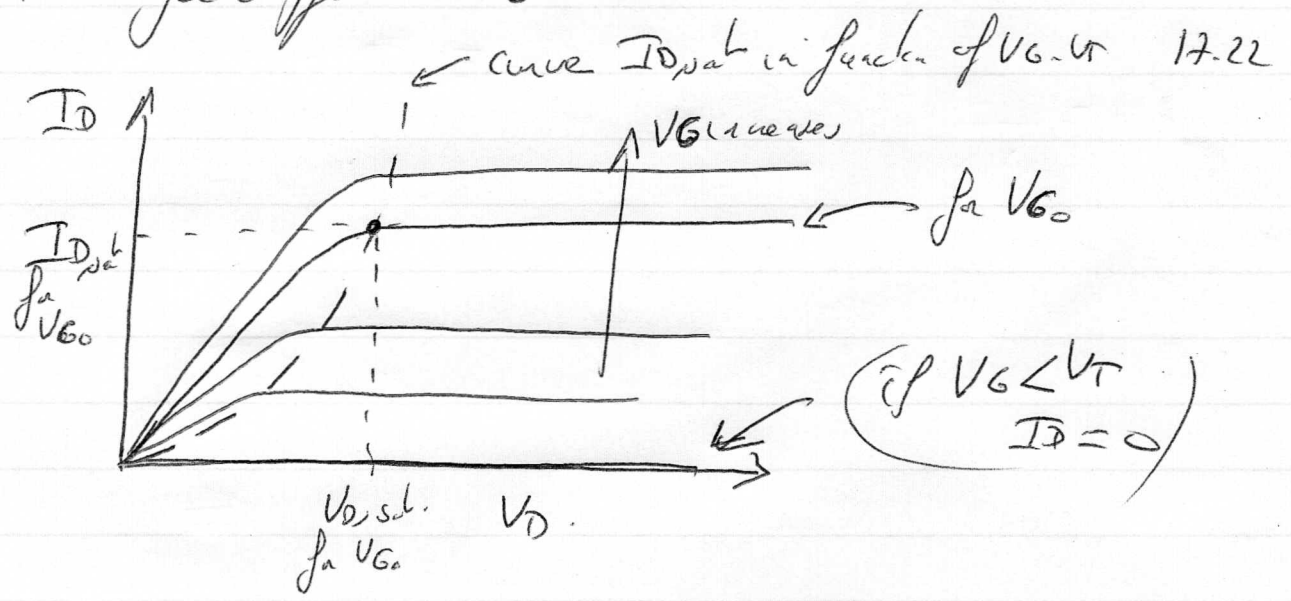
obtain using  
 $\left( \frac{dI_D}{dV_D} = 0 \right)$

we call  $I_{D,sat}$ , the value of the saturated Drain current  $\Rightarrow$  pinch-off point at  $V_{D,sat}$ .

$$\boxed{\frac{I_{D,sat}}{I_D} = \mu C_{ox} \frac{W}{L} \left( \frac{V_G - V_T}{2} \right)^2} \quad \boxed{(17.22)}$$

The equation (17.17) is evaluated beyond the pinch-off point  
if  $(V_{DS} > V_G - V_T) \Rightarrow$  The current maintain the value  $I_{D,sat}$ .

The quadratic model explains the typical  $I-V$  characteristics of a MOSFET, which are normally plotted for different  $V_G$



~~\* we should~~

c) the variable depletion layer - (Bulk-Charge layer)

in the quadratic model, we have.

$$Q_{inv(x)} = -C_{ox} (V_G - V_T - V(x)) \quad V_G > V_T$$

where  $V_T = V_{FB} + V_S + \frac{\sqrt{2\epsilon q N_A V_S}}{C_{ox}}$   $V_S = 2V_F$

due to depletion layer charge.

in this model we account for the variation of the depletion layer

layer  $\Rightarrow V_{T(x)} = \frac{V_{FB} + V_S + \sqrt{2\epsilon q N_A (V_S + V(x))}}{C_{ox}}$   $V_S = 2V_F$

finally

$$|Q_{inv} = -C_{ox} (V_G - V_T(x) - V(x))$$

(17.23)   
 (this is an approximation)

$\Rightarrow$  after calculation (integration).

17.28   
 (this is an approximation)

$$I_D = \mu C_{ox} \frac{W}{L} (V_G - V_{FB} - 2V_F - \frac{V_D}{2}) V_D - \frac{2}{3} \mu \frac{W}{L} \sqrt{2\epsilon q N_A} \left[ (2V_F + V_D)^{3/2} - (2V_F)^{3/2} \right]$$



we will use

$$I_D = \mu C_{ox} \frac{W}{L} \left\{ \left( V_G - V_{FB} - 2V_F - \frac{V_D}{2} \right) V_D - \frac{2}{3} \delta \left[ (2V_F + V_D)^{3/2} - (2V_F)^{3/2} \right] \right\}$$

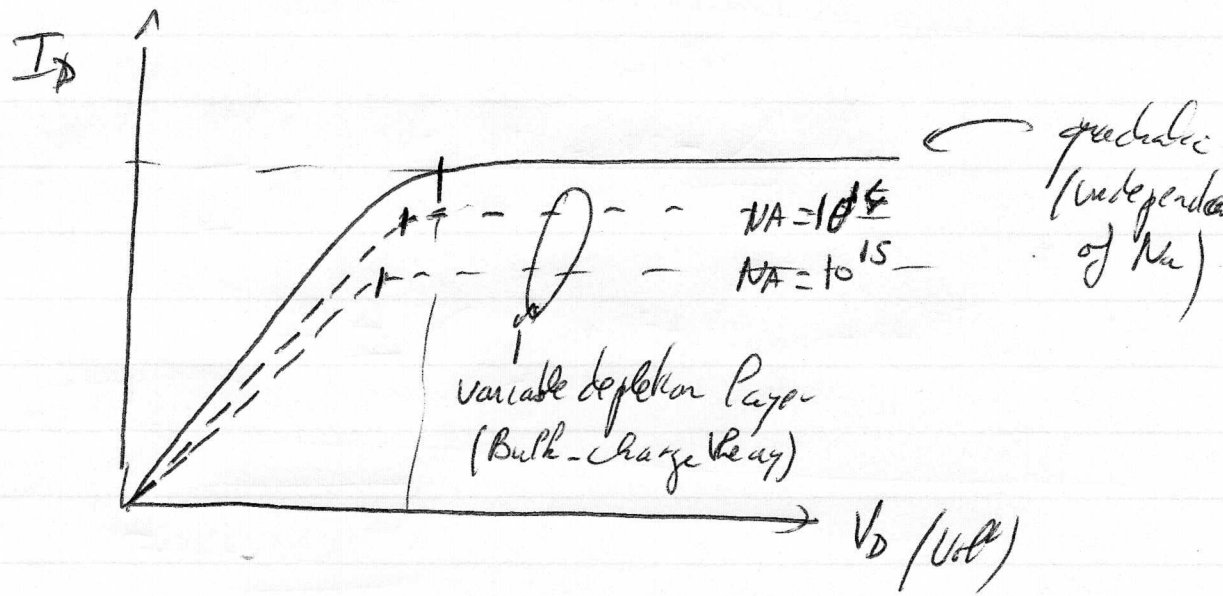
with  $\delta = \frac{\sqrt{2q\epsilon_0 N_A}}{C_{ox}}$

Again this expression is not valid after the pinch-off point for describing the saturation regime.

$$V_{Dsat} = V_G - V_{FB} - 2V_{FB} + \frac{\delta^2}{2} - \delta \sqrt{V_G - V_{FB} + \frac{\delta^2}{4}}$$

⇒ the quadratic model yields to larger current compared to more accurate derivation like the variable depletion layer model.

See Fig 17.9



(d) Conductance and transconductance

Transconductance  $g_m$ .  $g_m = \left. \frac{\partial I_D}{\partial V_G} \right|_{V_D \text{ constant}}$

if  $g_m \nearrow \Rightarrow$  speed of the device  $\nearrow$

quadratic model (square-law theory)

Variable depletion model (bulk-charge theory)

$$g_m = \mu C_{ox} \frac{W}{L} V_D$$

$$g_m = \mu C_{ox} \frac{W}{L} V_D \quad \underline{\underline{\text{still}}}$$

in saturation

in saturation

$$g_{m, \text{sat}} = \mu C_{ox} \frac{W}{L} (V_G - V_T)$$

$$g_{m, \text{sat}} = \mu C_{ox} \frac{W}{L} V_{D, \text{sat}}$$

↑  
almost linear with  $V_G$

conductance  $g_d$  is given by:

$$g_d = \left. \frac{\partial I_D}{\partial V_D} \right|_{V_G \text{ constant}} \Rightarrow \left\{ \begin{array}{l} \text{ease with which an} \\ \text{electric current flows} \\ \text{through a material} \\ \rightarrow \text{reciprocal of resistance} \end{array} \right.$$

quadratic model

$$g_d = \mu C_{ox} \frac{W}{L} (V_G - V_T - V_D)$$

$g_d \searrow$  if  $V_D \nearrow$

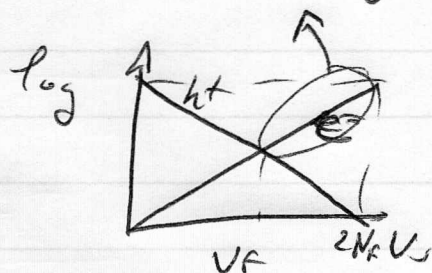


Also  $g_d \rightarrow 0$  as the device operates in the saturated region  $g_d, sat = 0$

### 3 Subthreshold current

So far, we assumed that  $I_D = 0$  if  $V_G < V_T$ .  
Actually there is a significant amount of  $e^-$  near the surface if the device operates below strong inversion.

inversion.  $V_F < V_S < 2V_F$   
↑  
Surface potential.



⇒ Experimentally we observe that the Drain current below threshold "subthreshold current" is independent of the Drain voltage  $V_D$

⇒ This means that the subthreshold current is caused by diffusion rather than Drift.



After a bit of calculation, we get

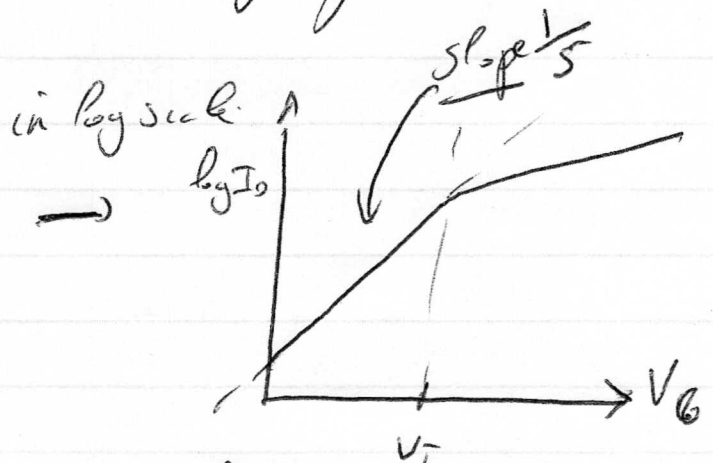
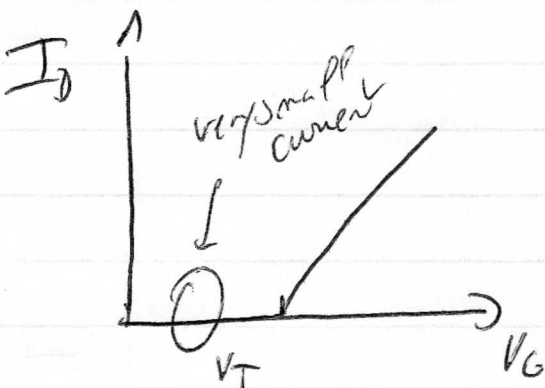
$$I_0 = \mu_n \frac{W}{L} q \left( \frac{k_B T}{q} \right)^2 \frac{m_c^2}{N_a} \left[ 1 - \exp\left( \frac{qV_0}{k_B T} \right) \right] \frac{\exp\left( \frac{qV_0}{k_B T} \right)}{E_s}$$

electric field at  
the surface  $E_s = -\frac{dV_s}{dx}$

$$E_s = \frac{qNA}{\epsilon_0}$$

$\Rightarrow I_0$  is independent of  $V_0$  as long as  $V_0$  is larger than a few  $\frac{k_B T}{q}$  (exponential term goes to zero).

$\Rightarrow I_0$  also increases exp. with the Duffe potential.



$$\left[ S = \frac{k_B T}{q} \ln(10) \left( 1 + \frac{C_0}{C_{ox}} \right) \right]$$