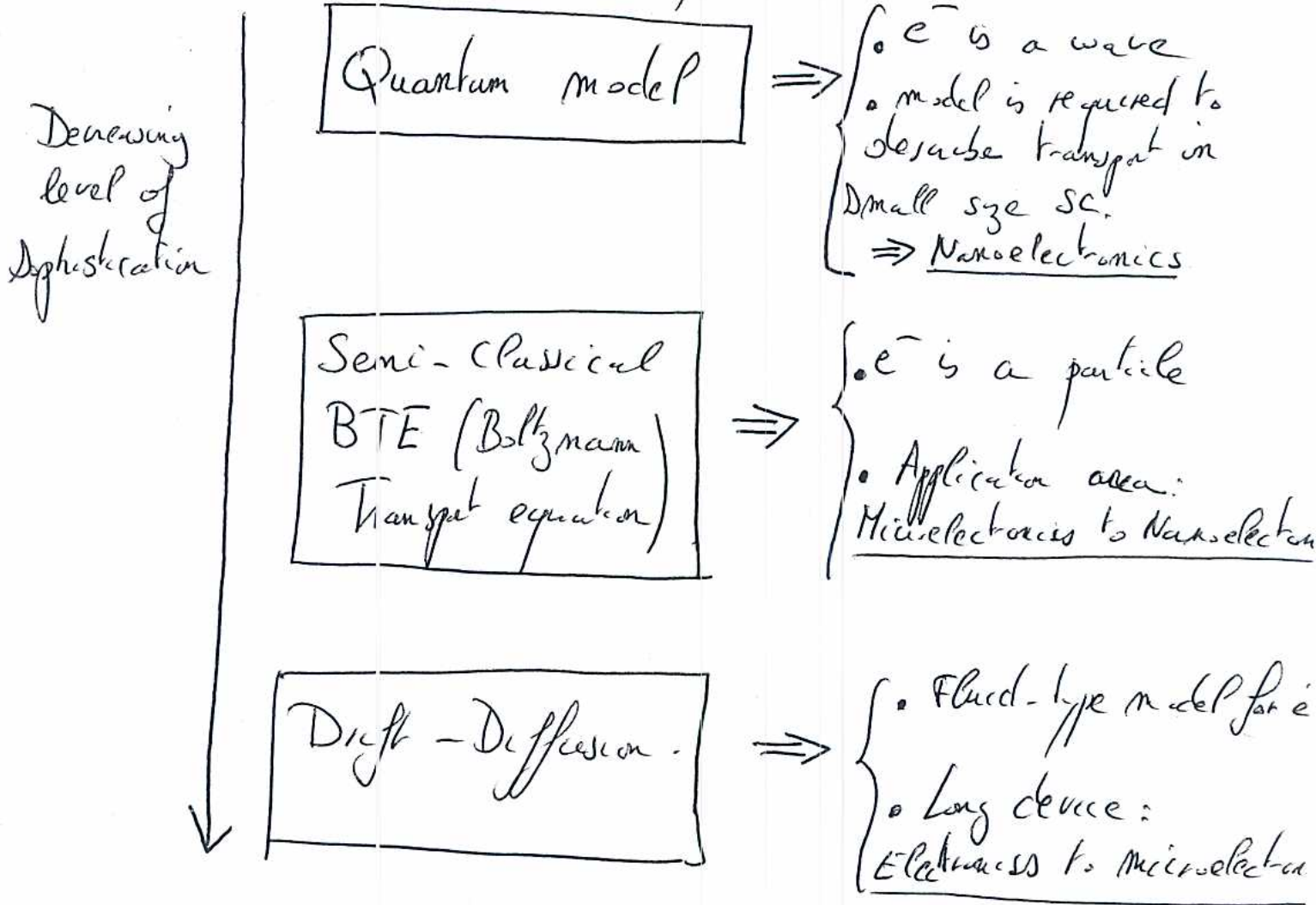


I Introduction to transport models

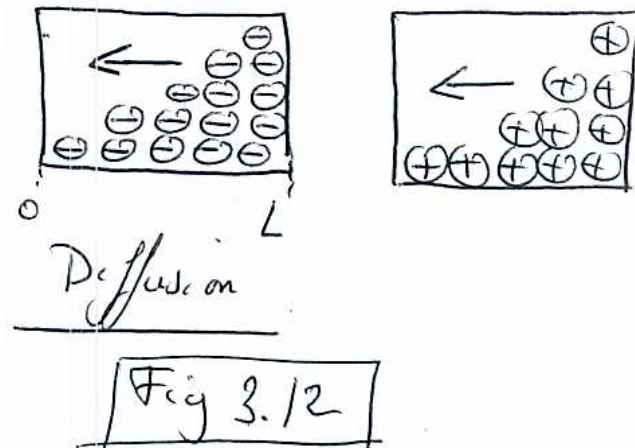
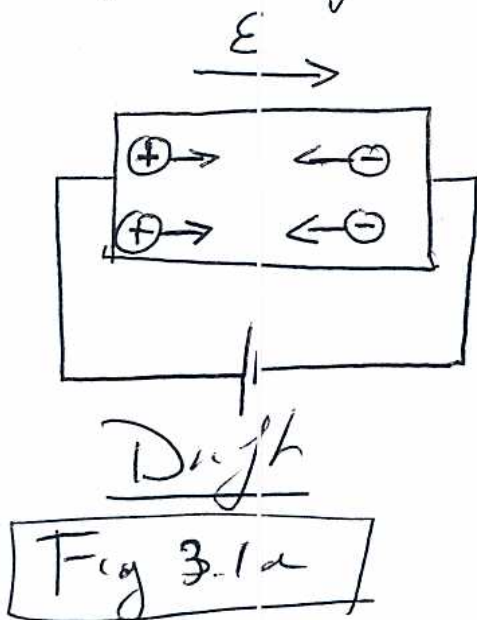
① Hierarchy of transport models



(2) Carrier transport = basics

Any motion of free carriers in SC leads to a current.

- Since these carriers are charged particles, this motion can be caused by an electric field E .
we refer to this transport mechanism as carrier drift.
- Carriers also move from regions where the carrier density is high to regions where it is low.
we refer this phenomenon as carrier diffusion.



II Drift-Diffusion equation

① Drift

② Carrier Drift

~~③ Drift current~~

We consider 2 cases: (i) w/o electric field
(ii) with electric field

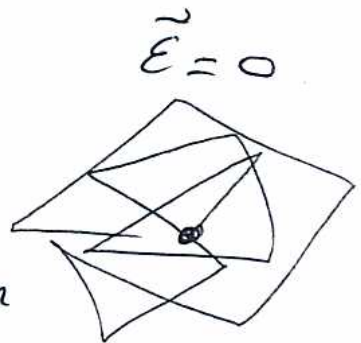
(i) w/o (without electric field).

• The carrier exhibits random motion (can be compared to Brownian

motion of fine particles in a liquid)

⇒ the change of direction is due to scattering

• All these small movements average out and the net displacement of e^- is zero.



(ii) with Electric field

• Random motion still occurs but, in addition, there is an average of a net motion along the direction of the field.

• Due to their electric charge,

e^- move on average in the opposite direction of \vec{E} ;

holes move in the same direction.



Fig 3.1b

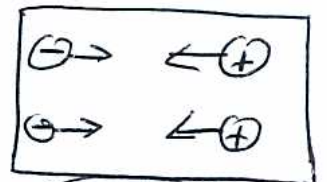
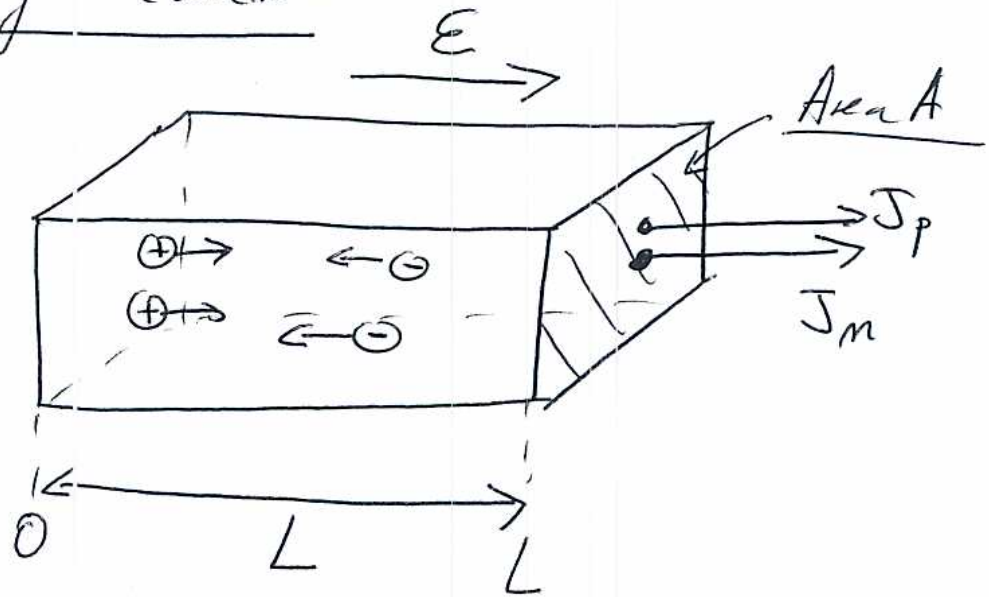


Fig 3.1c

⑥ Drift current



The current is defined by.

$$I = \frac{Q}{t_r}$$

- Q is the total charge in SC.
- t_r is the transit time of the particle (time needed to go from 0 to L or L to 0)

For e^-

$$Q = -q_n V$$

$$t_r = \frac{L}{v_m}$$

$$V = LA$$

v_m is the average velocity

For h^+

$$Q = q_p V$$
$$I = \frac{Q}{t} = \frac{q_p V}{t}$$

If $C = 12.5$

J the current density

$$J = \frac{I}{A} \text{ per unit of Area}$$

~~current density~~

For e^-

$$\vec{J}_n = -q n \vec{v}_n$$

For h^+

$$\vec{J}_p = q p \vec{v}_p \quad (3.2)$$

at low electric field, the velocity is proportional to the electric field

$$\vec{v}_n = -\mu_n \vec{E}$$

μ_n is the electron mobility

$$\vec{v}_p = \mu_p \vec{E}$$

μ_p is the hole mobility

Finally, we get

For e^-

$$\vec{J}_m = g \mu_m M \vec{E} \quad (3.4b)$$

For h^+

$$\vec{J}_p = g \mu_p P \vec{E} \quad (3.4c)$$

Rq: For high electric field the velocity saturates and become independent of \vec{E}

$$\vec{v}_m = \frac{-\mu_m \vec{E}}{\left(1 + \left(\frac{\mu_m E}{v_{salm}}\right)^2\right)^{1/2}}$$

$$\vec{v}_p = \frac{\mu_p \vec{E}}{\left(1 + \left(\frac{\mu_p E}{v_{psal}}\right)^2\right)^{1/2}}$$

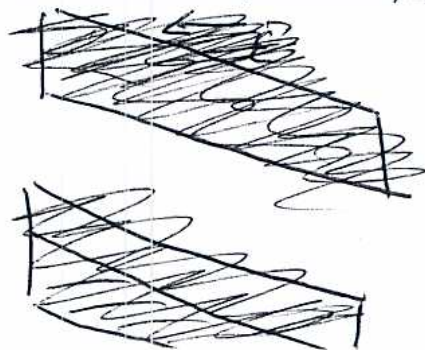
if $E \rightarrow 0$ $\vec{v}_m \approx -\mu_m \vec{E}$

$\vec{v}_p \approx \mu_p \vec{E}$

if $E \rightarrow +\infty$ $v_m = v_{salm}$

$v_p = v_{psal}$

~~Energy band representation~~



(E) Mobility

- standard units $\text{cm}^2/\text{V}\cdot\text{s}$
- mobility plays a key role in characterizing the performance of many devices.
- In major semiconductors $\mu_n > \mu_p$.
- sample numerical value.

$$\begin{array}{l} \text{Si} \\ N_A = N_D = 10^{14}/\text{cm}^3 \\ \text{at } T = 300\text{K} \end{array}$$

$$\mu_n = 1360 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$\mu_p = 460 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$\begin{array}{l} \text{GaAs} \\ N_A, N_D \leq 10^{17}/\text{cm}^3 \\ \text{at } T = 300\text{K} \end{array}$$

$$\mu_n = 8000 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$\mu_p = 400 \text{ cm}^2/\text{V}\cdot\text{s}$$

\Rightarrow The carrier mobility varies inversely with the amount of scattering taking place in the sc.

Mobility \Leftrightarrow ease to move for the carriers.

usually

$$\mu = \frac{qZ}{m^*}$$

m^* is the conductivity effective mass for $m^* = 0.26m_0$ Si

- Z is the time between scattering events, "mean free time".
- So μ is expected to be large as $Z \uparrow$ (means less scattering)

Scattering phenomena include

- (i) impurity scattering = such as ionized atoms (donors and acceptors)
- (ii) Lattice scattering = involving collisions with thermally agitated lattice atoms (phonons scattering)
- (iii) Surface scattering = μ can be much lower at the surface than in the bulk material.

Doping dependence of μ

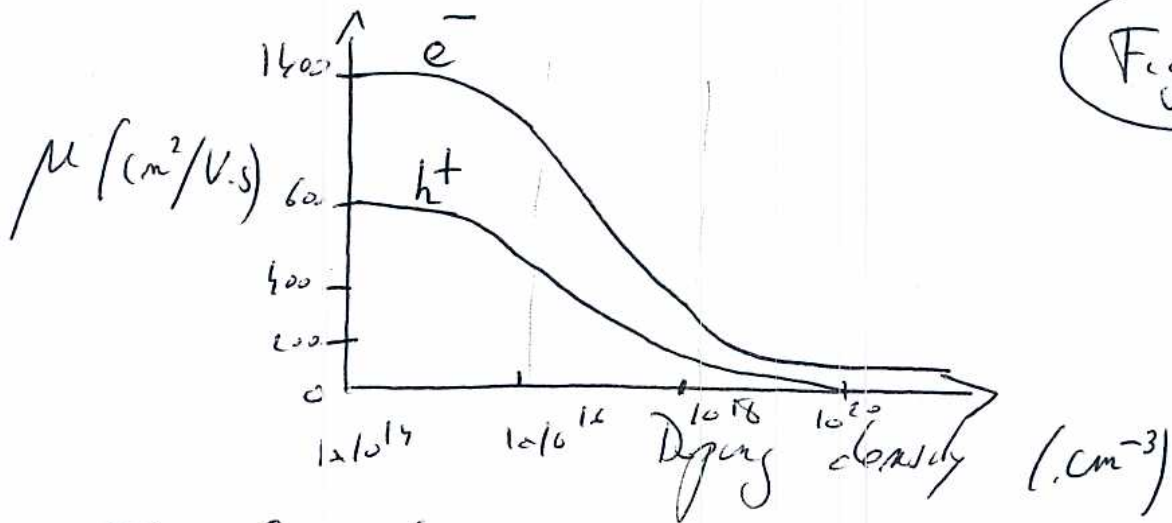


Fig 3-5

- For low doping, μ is almost constant, and is primarily limited by phonon scattering.
- For high doping (if doping increases), mobility μ decreases due to ionized impurity scattering.

(d) Resistivity

The total drift current is

$$\vec{J}_{\text{Drift}} = \vec{J}_{n/\text{drift}} + \vec{J}_{p/\text{drift}}$$

$$\vec{J}_{\text{Drift}} = q(\mu_n n + \mu_p p) \vec{E}$$

$$\boxed{\vec{J}_{\text{Drift}} = \sigma \vec{E}} \quad (3.56)$$

σ is called the conductivity

The resistivity is given by $\rho = \frac{1}{\sigma}$

$$\boxed{\rho = \frac{1}{q(\mu_n n + \mu_p p)}} \quad (3.7)$$

For N-type SC

$$\boxed{\rho \approx \frac{1}{q \mu_n n_D}} \quad (3.8a)$$

For P-type SC

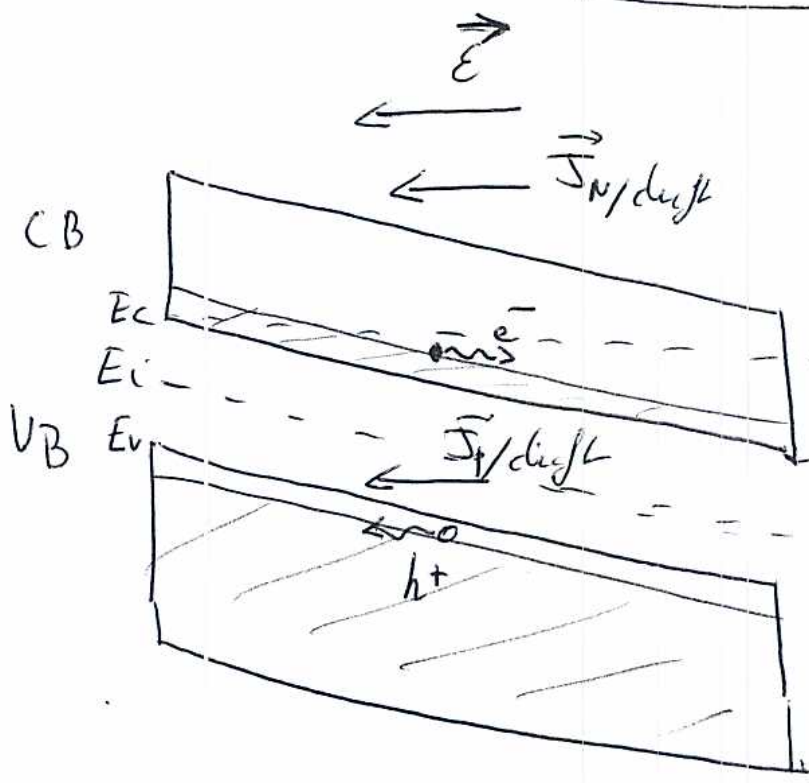
$$\boxed{\rho \approx \frac{1}{q \mu_p n_A}} \quad (3.8b)$$

② Band bending

by definition $E = - \frac{dV}{dx}$

and $-qV(x)$ is the energy potential created by \bar{E} , which ~~is~~ is associated to the "band bending".

consequently
$$E = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx} = \frac{1}{q} \frac{dE_i}{dx}$$



e^- drift are going from region of high potential to region of low potential.

(2) Diffusion

(a) Carrier diffusion
 carriers diffuse from regions where the density is high ~~into~~ to regions where it is low.

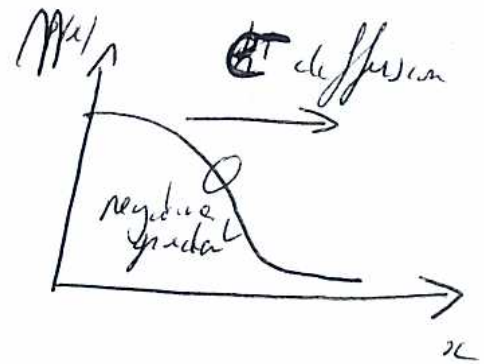
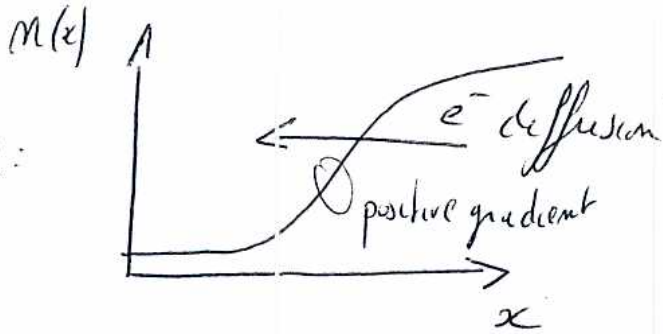
Diffusion of carriers can be obtained:

- (i) by varying the doping density in a SC.
- (ii) by applying a thermal gradient.

⇒ The flux of electrons (ϕ_n for holes ϕ_p) resulting from the diffusion process is directly proportional to the e^- concentration gradient

$$\frac{dn}{dx} \quad \left(\text{or } \frac{dp}{dx} \right).$$

Examples:



\Rightarrow $\boxed{\phi_m = -D_n \frac{dn}{dx}}$ (3.16) similarly $\boxed{\phi_p = -D_p \frac{dp}{dx}}$

D_n (D_p) is called the diffusion coefficient for e^- (h^+).

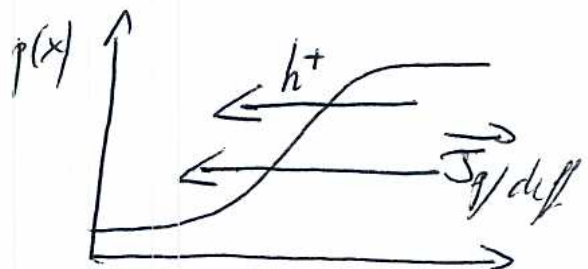
(b) diffusion current

we get

$$\boxed{J_n = -q \phi_n = q D_n \frac{dn}{dx}} \quad (3.17)$$

and

$$\boxed{J_p = q \phi_p = -q D_p \frac{dp}{dx}}$$



(3) Total current

it is obtained by adding the current due to drift ^{and} to the diffusion.

3.18

for e^-	$\vec{J}_n = q n \mu_n E + q D_n \frac{dn}{dx}$ $\vec{J}_p = q p \mu_p E - q D_p \frac{dp}{dx}$	<u>Remark</u> in 3D $\frac{d}{dx} \rightarrow \vec{\nabla}$ $J_n \rightarrow \vec{J}_n$ $E \rightarrow \vec{E}$
for h^+		

Drift
Diffusion

The total current (density current) flowing at any position in the SC is simply:

$\vec{J} = \vec{J}_n + \vec{J}_p$	3.19
-----------------------------------	------

④ Einstein Relationship

Relation between μ and D .

We assume.

(i) equilibrium conditions \Rightarrow the net current should be zero.

3.20
$$J_m = q n \mu_n E + q D_n \frac{dn}{dx} = 0$$

(ii) Non degenerate SC

$$n = n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]$$

\Rightarrow we get

$$\frac{dn}{dx} = \left(\frac{dE_F}{dx} - \frac{dE_i}{dx} \right) \frac{1}{k_B T} \underbrace{n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]}_n$$

\swarrow 0 \downarrow qE

3.23

$$\frac{dn}{dx} = -\frac{q n E}{k_B T}$$

if we replace (3.23) in (3.20)

3.25a $D_n = \frac{k_B T}{q} \mu_n$ for e^-

and if we consider h^+ rather than e^-

3.25b $D_p = \frac{k_B T}{q} \mu_p$ for h^+

III Transport equations

In addition to the drift-diffusion equation, the transport model is also defined by the carrier density equation, the Poisson equation and the continuity equation.

(1) Carrier density and Poisson equation

In order to calculate J_n and J_p , one needs to know n , p , E .

E is obtained from the electrostatic potential V .

$$E = -\frac{dV}{dx}$$

19
From Maxwell equation, one can derive the Poisson equation.

$$\boxed{-\frac{d^2 V(x)}{dx^2} = \frac{q}{\epsilon} (-n + p + N_d - N_a)}$$

$\epsilon = \text{permittivity}$; $N_d(x)$ and $N_a(x)$ are known.

$\Rightarrow \boxed{V(x) \text{ depend then on } n(x) \text{ and } p(x)}$

~~However~~

* For non-degenerate SC

At equilibrium

$$n(x) = n_i \exp\left(\frac{E_F - E_c(x)}{k_B T}\right)$$

$$p(x) = n_i \exp\left(\frac{-E_F + E_v(x)}{k_B T}\right)$$

Because of the electric field in the SC and the band bending phenomenon $E_c(x) = E_{c0} - qV(x)$.
(E_{c0} is a reference)

$\Rightarrow n, p$ depends on $V(x)$.

\Rightarrow self-consistent (coupled) problem.

* Under non-equilibrium conditions (device with contacts)

E_F is not unique for e^- and h^+ ,
due to the contacts (carrier injections).

we introduce the notion of quasi-Fermi level.

F_N for e^- and F_p for h^+

$$n(x) = n_i \exp\left(\frac{F_N - E_i(x)}{k_B T}\right)$$

$$p(x) = n_i \exp\left(\frac{-F_p + E_i(x)}{k_B T}\right)$$

3.72

Quasi-Fermi levels are "imaginary reference" \Rightarrow imtef
(Remark imtef spelled backward is fermi :-)

② Continuity equations

So far, we assumed a steady state (no time dependence for transport).

* continuity equation describe the evolution of the carrier concentrations n and p with time $\Rightarrow \partial n / \partial t$ and $\partial p / \partial t$

The local carrier density may vary for the following reasons:

- (i) because of contact = carriers can be injected or removed from the device.
- (ii) An external source of energy can increase the h^+ and e^- concentration (example = light) e^- can jump from VB to CB \Rightarrow a free $e^- - h^+$ pair is then created (extrinsic generation process).

→ The generation rates, G_n for e^- , G_p for h^+ , due to this external source is called extrinsic generation rate. In the textbook.

$$G_n = \frac{\partial n}{\partial t} \Big|_{\text{other processes (light, etc.)}}$$

$$G_p = \frac{\partial p}{\partial t} \Big|_{\text{other processes (light, etc.)}}$$

(iii) in the absence of any outside influence, free e^- and h^+ can be created or annihilated within a region of the semiconductor.

~~in order~~ (in order to restore the equilibrium situation which has been perturbed by external force)

For example:

→ If the width of the bandgap is small enough, e^- can jump from VB to CB or CB to VB.

→ they can also jump into a permitted energy ~~bands~~ located inside the bandgap (created by impurities or defects).

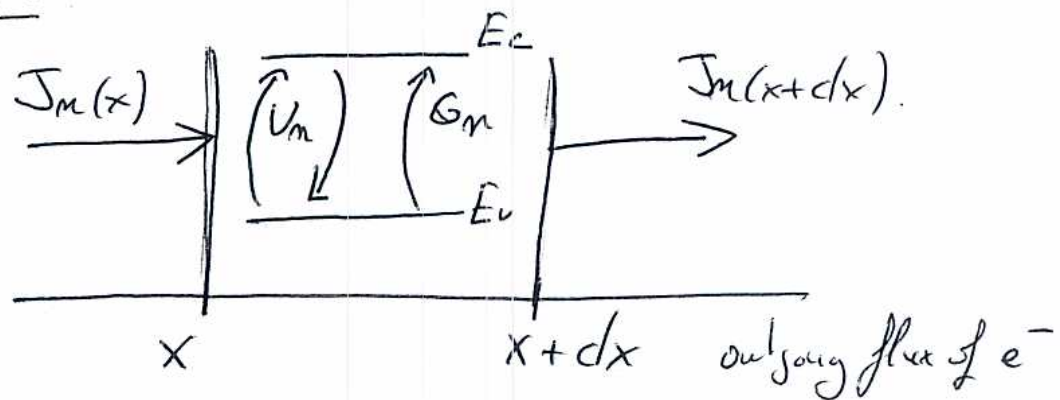
if free e^- and h^+ are created → intrinsic generation process
 if free e^- and h^+ are lost → intrinsic recombination process.

The net intrinsic generation/recombination (R-G) rates are noted U_n for e^- and U_p for h^+ in the textbook =

$$\begin{cases} U_n = -\frac{\partial n}{\partial t} \Big|_{\text{thermal}} \Big|_{R-G} \\ U_p = -\frac{\partial p}{\partial t} \Big|_{\text{thermal}} \Big|_{R-G} \end{cases}$$

The intrinsic R-G rates will be analyzed in detail in the next section.

So finally let us derive the expression
of $\frac{\partial n}{\partial t}$.



$$\underbrace{\frac{\partial n}{\partial t} dx}_{\text{variation of number of free } e^- \text{ in the volume } dx \text{ in function of time}} = \left(\underbrace{\frac{J_n(x)}{-q}}_{\substack{\text{incoming} \\ \text{flux of } e^- \\ \text{entering} \\ \text{the volume}}} - \underbrace{\frac{J_n(x+dx)}{-q}}_{\substack{\text{outgoing} \\ \text{flux of } e^-}} \right) + \underbrace{(G_n - U_n)}_{\text{G/R processes}}$$

Using Taylor expansion $J_n(x+dx) = J_n(x) + \frac{\partial J_n(x)}{\partial x} dx$

⇒

3.46

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n(x)}{\partial x} + (G_n - U_n) \quad \text{for } e^-$$

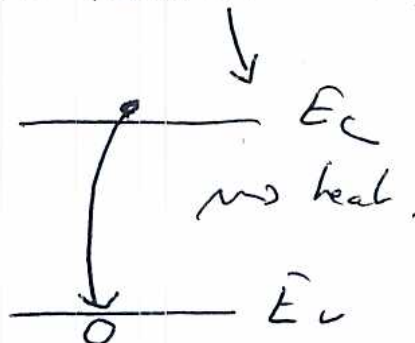
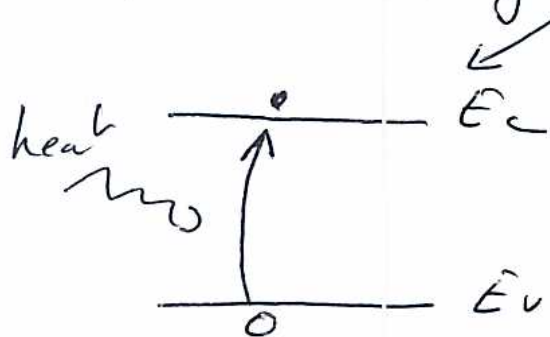
and also

$$\frac{\partial p}{\partial t} = \frac{1}{q} \frac{\partial J_p(x)}{\partial x} + (G_p - U_p) \quad \text{for } h^+$$

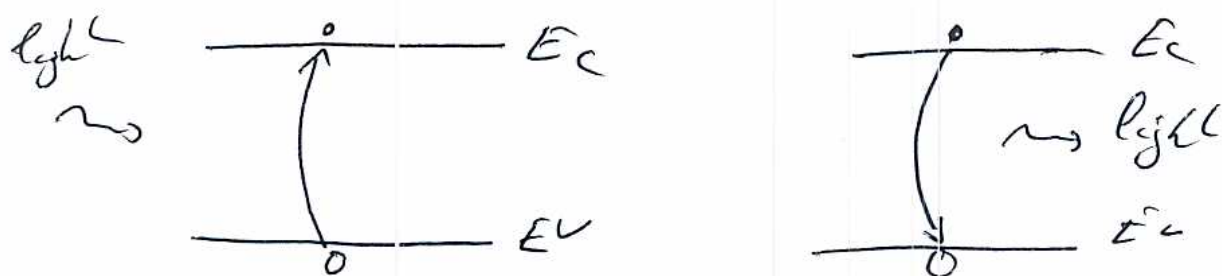
3 Carriers generation and recombination

(a) band to band process (Fig 3-15)

Direct thermal generation or recombination.



• Photogeneration or radiative recombination.



this process depends on the density of available e^- and hole. The G/R rate is expected to be $U = r - g = v_n (n_p - n_i^2)$. Also in thermal equilibrium $U = 0$ since there is no net current.

we note that $\boxed{U = r - g}$

\uparrow intrinsic R rate \downarrow intrinsic G rate.

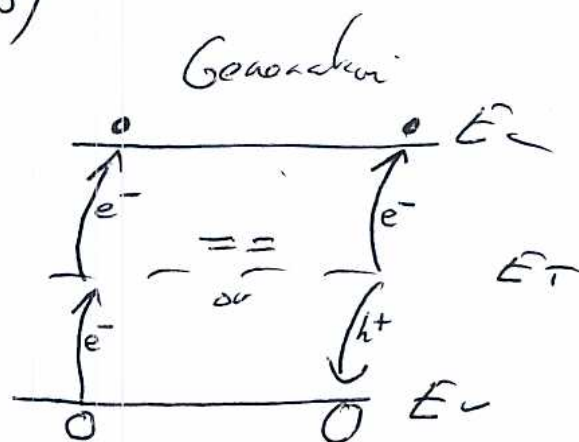
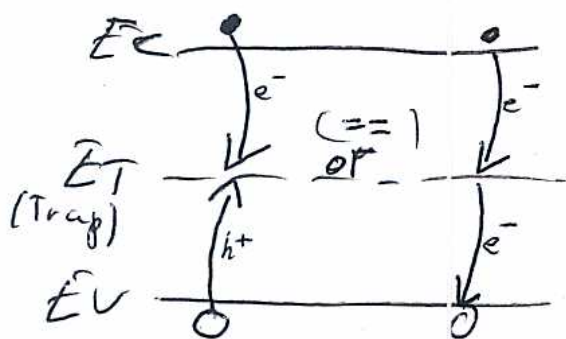
$\Rightarrow U = 0$ if $p_n = n_i^2$ equilibrium
 $\Rightarrow U > 0$, Recombination dominates, $p_n > n_i^2$
 $\Rightarrow U < 0$, generation dominates, $p_n < n_i^2$

we finally get $U = b(m_p - m_e^2)$ with b the molecular constant. U acts like a "force" to bring p_m to its equilibrium value m_e^2 .

(b) R-G Center Recombination

It is a trap assisted process (also called SRH mechanism)

$\left\{ \begin{array}{l} e^- \text{ and } h^+ \text{ can jump into a permitted energy } E_T \\ \text{located inside the bandgap (created by impurities} \\ \text{or defects).} \end{array} \right.$ (Fig 3.15)



this expression can be further simplified
for

P-type semiconductor ($p \gg n$).

$$U = \frac{n - n_0}{Z_n}$$

3.34b

N-type semiconductor $n \gg p$

$$U = \frac{p - p_0}{Z_p}$$

3.34a

n_0 and p_0 are the density at equilibrium.

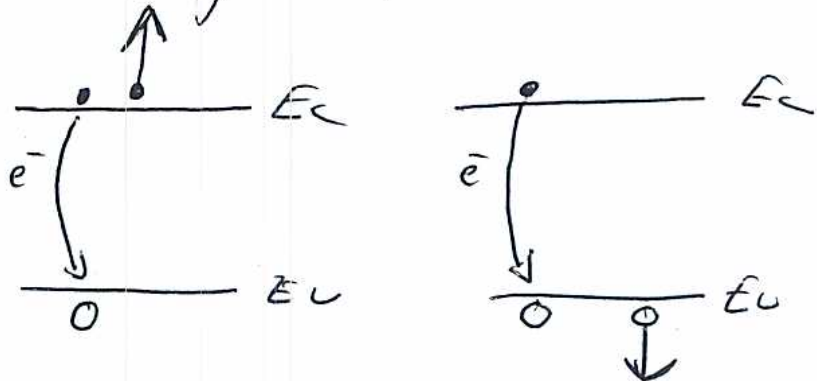
$\Delta n = n - n_0$ and $\Delta p = p - p_0$ are the excess carrier density.

Rq: In (3.34), the result depends on the minority carrier concentration.

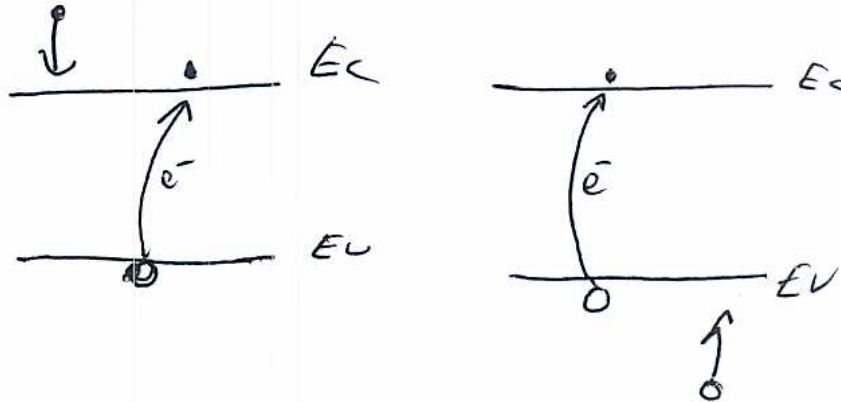
(c) Auger Recombination

Fig 3.15c (involves 3 particles)

Recombination
(carriers near the band edge)



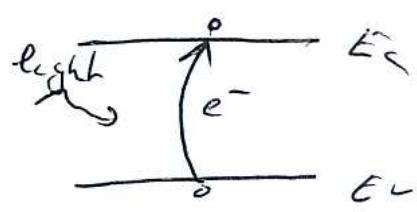
Generation
(energetic carriers involved)



(d) Photogeneration

of each photon creates an e^-/h^+ pair.

$$G_{p, light} = G_m, light = \alpha \frac{P_{ph}(x)}{E_{ph} A}$$



α is the absorption coefficient of the material at the energy of the incoming photon E_{ph} .

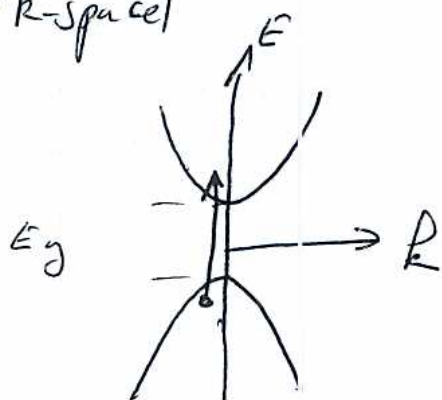
P_{opt} is the optical power.

$E_{ph} > E_g \Rightarrow$ necessary condition.

Fig
3.18

* Direct bandgap

(k-space)



$$p_f = p_i \quad (\text{since } k_f = k_i)$$

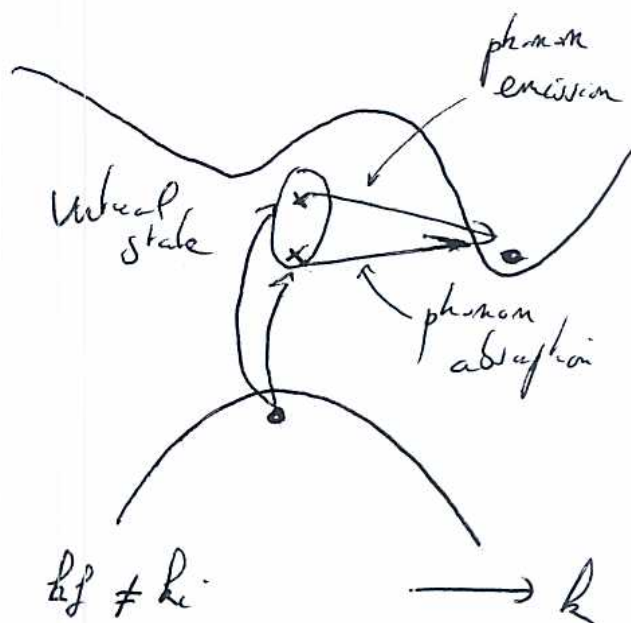
$$E_f = E_i + E_{ph}$$

\nearrow final \downarrow initial \downarrow phonon

$$\alpha \propto (h\nu - E_g)^\delta$$

with $\delta = \frac{1}{2}, \frac{1}{3}$

* Indirect bandgap



$$k_f \neq k_i$$

$$p_f = p_i \pm q_s$$

$$E_f = E_i \mp E_s \rightarrow \text{phonon}$$

$$\delta = 2$$