I/ Introduction to transport model

1 Hierarchy of transport models

Decreasing level of Sophistication

Quantum model ⇒
- e^- is a wave
- model is required to describe transport in Dmall size SC.
  ⇒ Nanoelectronics

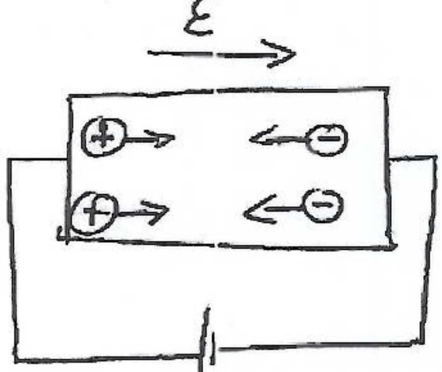
Semi-Classical BTE (Boltzmann Transport equation) ⇒
- e^- is a particle
- Application area: Microelectronics to Nanoelect

Drift-Diffusion. ⇒
- Fluid-type model for
- Long device: Electronicss to microelect

② Carrier transport: basics

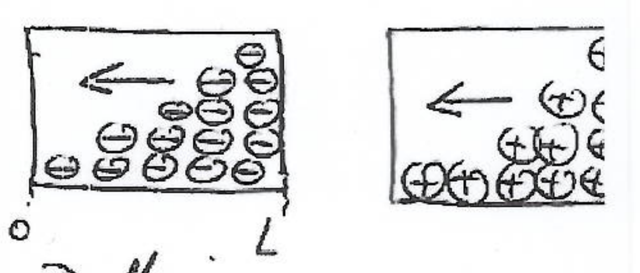
Any motion of free carriers in SC leads to a current.

- Since these carriers are charged particles, this motion can be caused by an electric field $\mathcal{E}$. We refer to this transport mechanism as carrier drift
- Carriers also move from regions where the carrier density is high to regions where it is low. We refer this phenomenon as carrier diffusion



Drift

Fig 3.1a



Diffusion

Fig 3.12

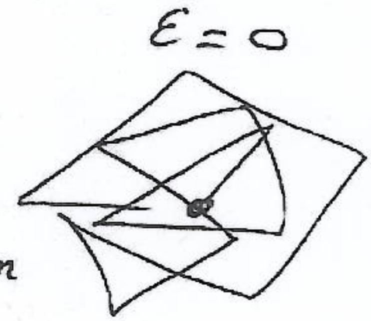
II Drift-Diffusion equation

1 Drift

a Carrier Drift

We Consider 2 cases : (i) w/o electric field
(ii) with electric field

(i) w/o (without electric field).



- the carrier exhibits random motion (can be compared to Brownian motion of fine particles in a liquid)

$\Rightarrow$ the change of direction is due to scattering

- All these small movements average out and the net displacement of e^- is zero.

(ii) with Electric field

- random motion still occurs but, in addition, there is an average of a net motion along the direction of the field.

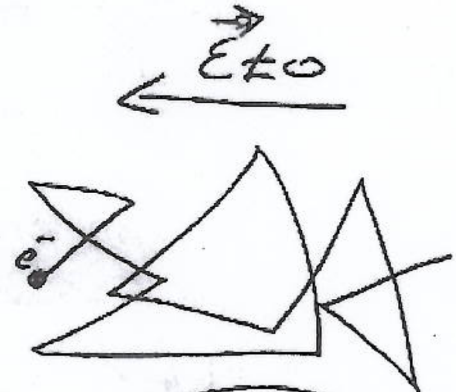


Fig 3.1b

- Due to their electric charge, e^- move on average in the opposite direction of $\vec{E}$; holes move in the same direction.

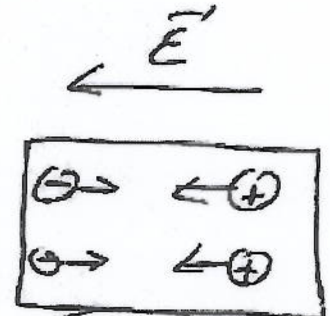
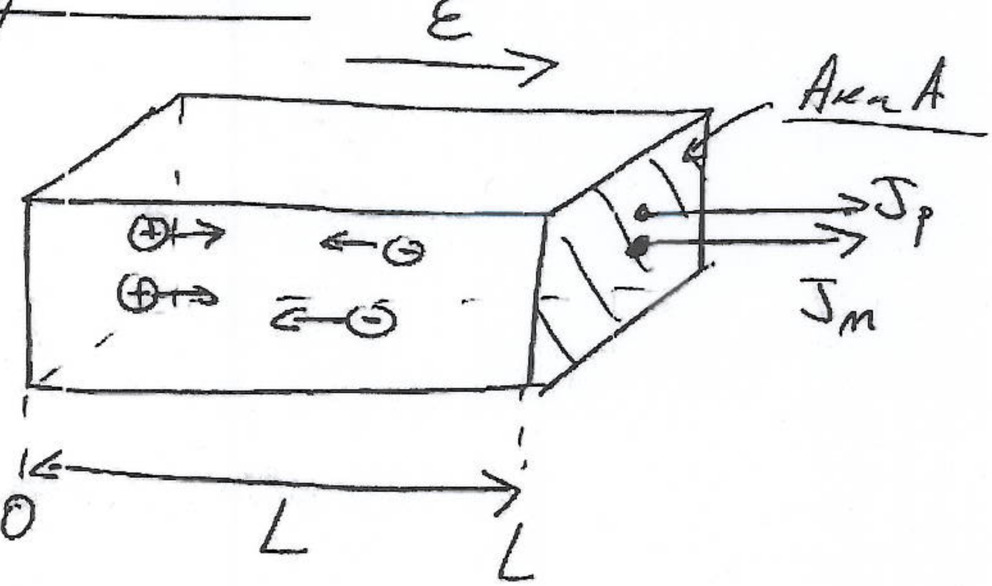


Fig 3.1c

6) Drift current



The current is defined by.

$$I = \frac{Q}{t_r}$$

- Q is the total charge in SC.
- t_r is the transit time of the particle (time needed to go from 0 to L or L to 0)

For e^-

$$\boxed{Q = -qn\,V \qquad t_r = \frac{L}{v_n}}$$

$V = LA$

v_n is the average velocity

For h^+

$$\boxed{\begin{aligned} Q &= q p V \\ t_r &= \frac{L}{v_p} \end{aligned}}$$

It comes

J the current density

$$J = \frac{I}{A}$$ per unit of Area

~~Drift~~

For e^-

$$\boxed{\vec{J}_n = -q n \vec{v}_n}$$

For h^+

$$\boxed{\vec{J}_p = q p \vec{v}_p} \quad (3.2)$$

at low electric field, the velocity is proportional to the electric field

$$\boxed{\vec{v}_n = -\mu_n \vec{E}}$$

μ_n is the electron mobility

$$\boxed{\vec{v}_p = \mu_p \vec{E}}$$

μ_p is the hole mobility

Finally, we get

For e^-

$$\boxed{\vec{J}_n = q \mu_n n \vec{\mathcal{E}}} \quad (3.46)$$

For h^+

$$\boxed{\vec{J}_p = q \mu_p p \vec{\mathcal{E}}} \quad (3.4$$

Rq: For high electric field the velocity saturates and become independent of $\vec{\mathcal{E}}$

$$\vec{v}_n = \frac{-\mu_n \vec{\mathcal{E}}}{\left(1 + \left(\frac{\mu_n \mathcal{E}}{v_{sat_n}}\right)^2\right)^{1/2}}$$

$$\vec{v}_p = \frac{\mu_p \vec{\mathcal{E}}}{\left(1 + \left(\frac{\mu_p \mathcal{E}}{v_{psat}}\right)\right)}$$

if $\vec{\mathcal{E}} \to 0$: $\vec{v}_n \simeq -\mu_n \vec{\mathcal{E}}$ | $\vec{v}_p \simeq \mu_p \vec{\mathcal{E}}$

if $\vec{\mathcal{E}} \to +\infty$: $\vec{v}_n = v_{nsat}$ | $v_p = v_{psat}$

~~Energy band representation~~

c) Mobility

- standard units $cm^2/V.s$
- mobility plays a key role in characterizing the performance of many devices.
- In major semiconductors $\mu_n > \mu_p$
- sample numerical value

Si	GaAs
$N_A = N_D = 10^{14}/cm^3$	$N_A, N_D \leq 10^{15}/cm^3$
at $T = 300K$	at $T = 300K$
$\mu_n = 1360\ cm^2/V.s$	$\mu_n = 8000\ cm^2/V.s$
$\mu_p = 460\ cm^2/V.s$	$\mu_p = 400\ cm^2/V.s$

$\Rightarrow$ The carrier mobility varies inversely with the amount of scattering taking place in the SC.

$\boxed{\text{mobility} \Longleftrightarrow \text{ease to move for the carriers}}$

usually $$\boxed{\mu = \frac{q\tau}{m^*}}$$

m^* is the conduction effective mass $m^* = 0.26 m_0$

- τ is the time between scattering events, "mean free time".
- So μ is expected to be large as $\tau \uparrow$ (means less scattering)

Scattering phenomena include

(i) impurity scattering = such as ionized atoms. (donors and acceptors)

(ii) Lattice scattering = involving collisions with thermally agitated lattice atoms (phonons scattering

(iii) Surface scattering = μ can be much lower at the surface than in the bulk material.

* Doping dependence of μ

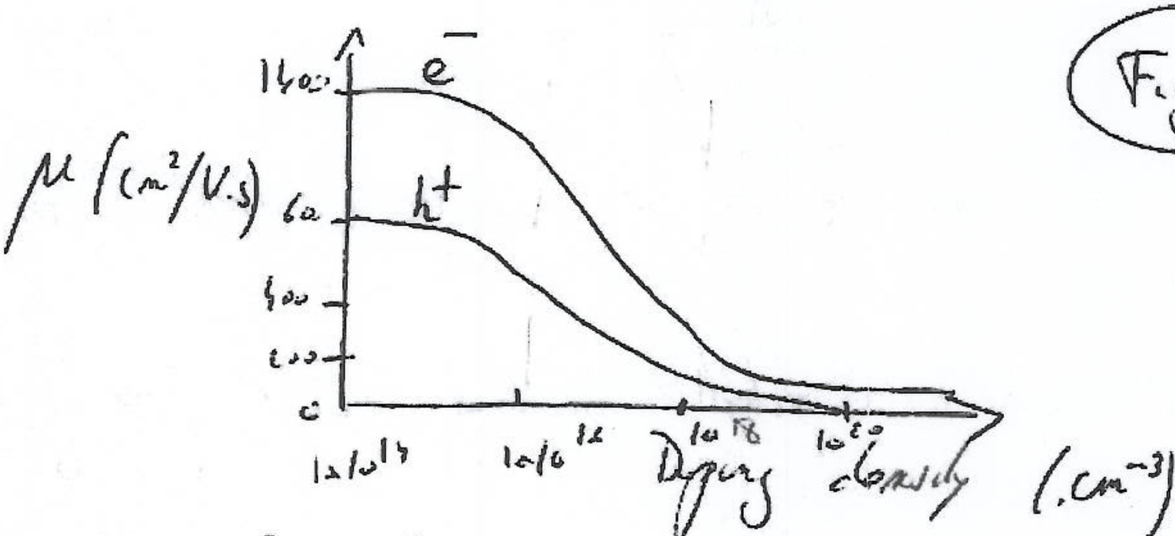


Fig 3.5

- For low doping, μ is almost constant, and is primarily limited by phonon scattering.
- For high doping (i.e. doping increases), mobility μ decreases due to ionized impurity scattering.

(d) Resistivity

The total drift current is

$$\vec{J}_{Drift} = \vec{J}_{N/drift} + \vec{J}_{p/drift}$$

$$\vec{J}_{Drift} = q(\mu_n n + \mu_p p)\,\vec{E}$$

$$\boxed{\vec{J}_{Drift} = \sigma \vec{E}} \quad (3.56)$$

σ is called the conductivity

The resistivity is given by $\rho = \frac{1}{\sigma}$

$$\boxed{\rho = \frac{1}{(q\mu_n n + q\mu_p p)}} \quad (3.7)$$

For N-type SC

$$\boxed{\rho \simeq \frac{1}{q\mu_n N_D}} \quad (3.8a)$$

For P-type SC

$$\boxed{\rho \simeq \frac{1}{q\mu_p N_A}} \quad (3.8b)$$

e) Band bending

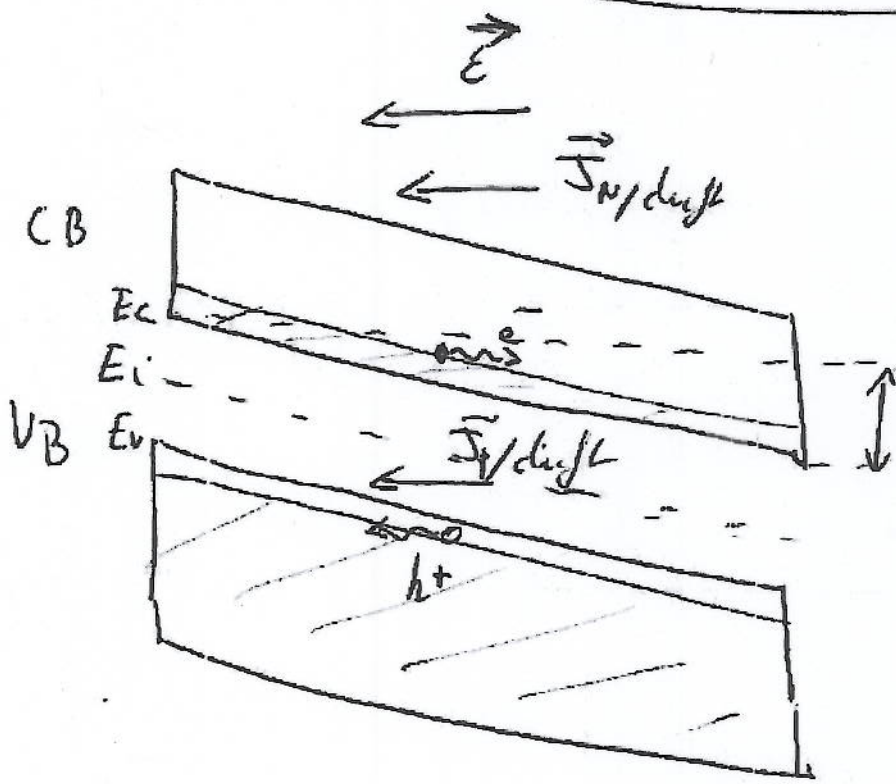
by definition $\mathcal{E} = -\frac{dV}{dx}$.

and $-qV(x)$ is the energy potential created by $\mathcal{E}$ which ~~...~~ is associated to the "band bending".

3.15

Consequently

$$\boxed{\mathcal{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}}$$



e^- drift as going from region of high potential to region of low potential

(2) Diffusion

(a) Carrier diffusion

carriers diffuse from regions where the density is high ~~into~~ to regions where it is low.

Diffusion of carriers can be obtained:

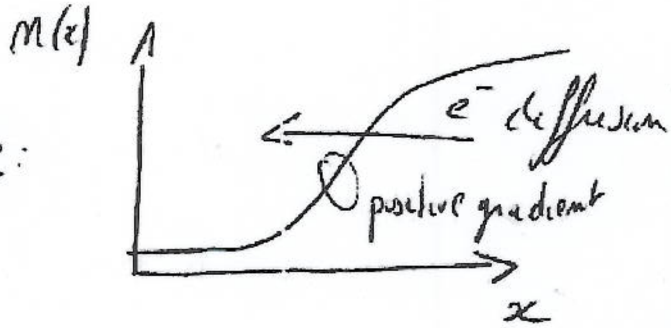
(i) by varying the doping density in a SC.

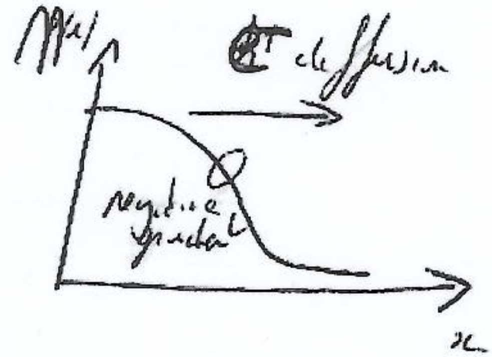
(ii) by applying a thermal gradient.

$\Rightarrow$ The flux of electrons ϕ_n (or holes ϕ_p), resulting from the diffusion process is directly proportional to the $\bar{e}$ concentration gradient

$\frac{dn}{dx}$ (or $\frac{dp}{dx}$).

Examples:





$$\Rightarrow \quad \boxed{\phi_n = -D_n \frac{dn}{dx}} \quad (3.16) \qquad \text{similarly} \quad \boxed{\phi_p = -D_p \frac{dp}{dx}}$$

D_n (D_p) is called the diffusion coefficient for e⁻/h⁺

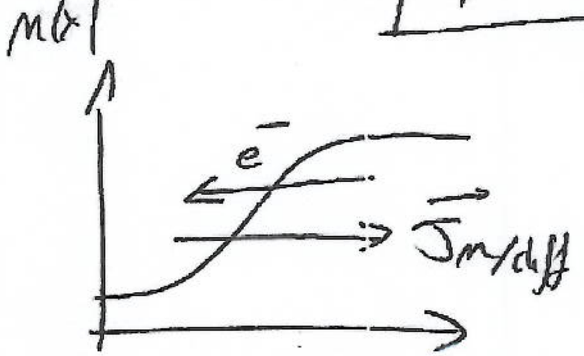
(b) diffusion current

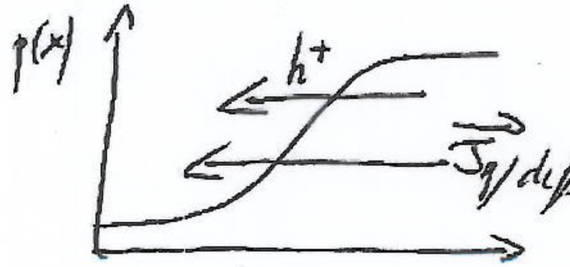
we get

$$\boxed{J_n = -q\,\phi_n = q D_n \frac{dn}{dx}} \quad (3.17)$$

and

$$\boxed{J_p = q\,\phi_p = -q D_p \frac{dp}{dx}}$$





3 Total current

it is obtained by adding the current due to diffusion and to the drift.

3.18

for e^- / for h^+

$$\boxed{\begin{aligned} J_n &= q n \mu_n \mathcal{E} + q D_n \frac{dn}{dx} \\ J_p &= \underbrace{q p \mu_p \mathcal{E}}_{\text{Drift}} - \underbrace{q D_p \frac{dp}{dx}}_{\text{Diffusion}} \end{aligned}}$$

Remark

in 3D

$\frac{d}{dx} \to \vec{\nabla}$

$J_n \to \vec{J_n}$

$\mathcal{E} \to \vec{\mathcal{E}}$

the total current (density current) flowing at any position in the SC is simply:

$$\boxed{\vec{J} = \vec{J_n} + \vec{J_p}} \quad (3.19)$$

4 Einstein Relationships

Relation between μ and D.

we assume.

(i) equilibrium conditions $\Rightarrow$ the net current should be zero.

(3.20) $$J_n = q n \mu_n \mathcal{E} + q D_n \frac{dn}{dx} = 0$$

(ii) non degenerate SC

$$n = n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]$$

$\Rightarrow$ we get

$$\frac{dn}{dx} = \left(\underbrace{\frac{dE_F}{dx}}_{0} - \underbrace{\frac{dE_i}{dx}}_{q\mathcal{E}}\right)\frac{1}{k_B T}\underbrace{n_i \exp\left[\frac{E_F - E_i}{k_B T}\right]}_{n}$$

(3.23) $$\boxed{\frac{dn}{dx} = \frac{-q}{k_B T} n \mathcal{E}}$$

if we replace (3.23) in (3.20)

3.25a $$\boxed{D_n = \frac{k_B T}{q} \mu_n}$$ for e^-

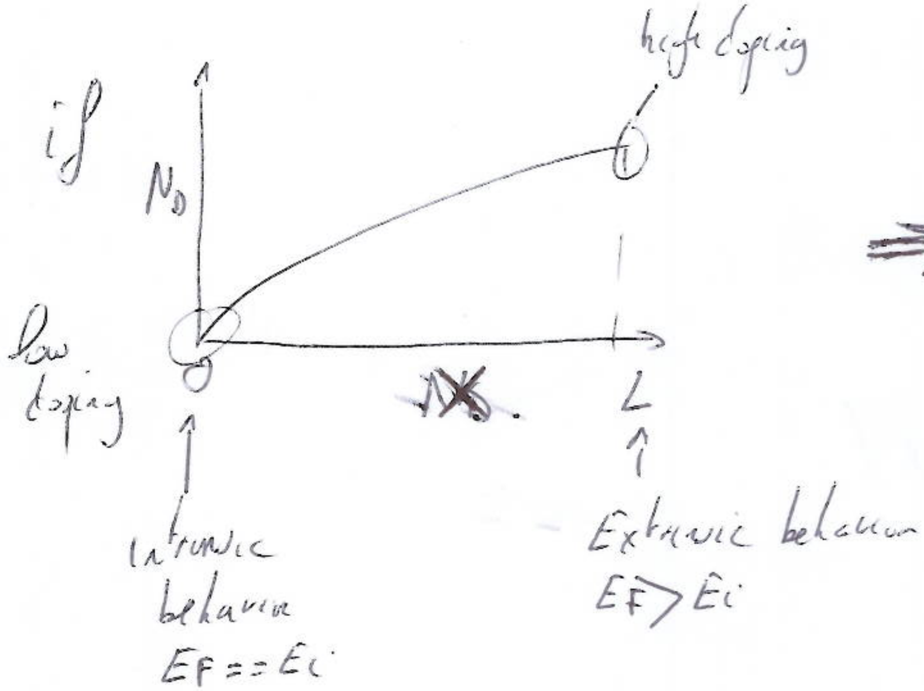
and if we consider h^+ rather than $\bar{e}$

3.25b $$\boxed{D_p = \frac{k_B T}{q} \mu_p}$$ for h^+

Example (Summary).

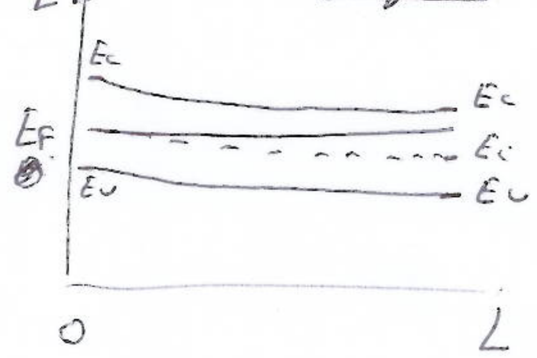
• if E_F is constant and unique $\Longleftrightarrow$ equilibrium conditions.

if

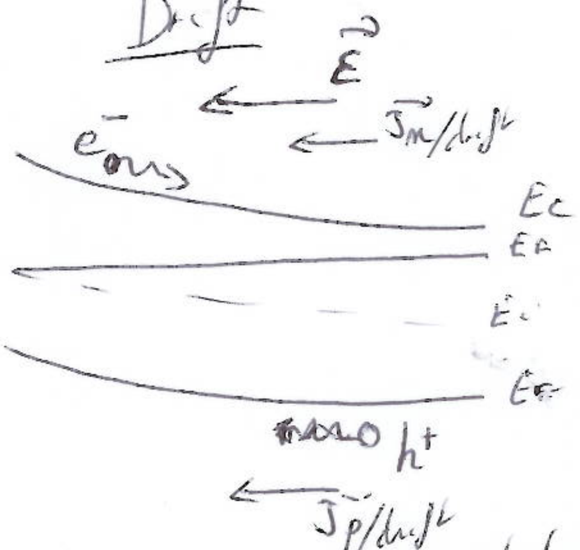


$\Longrightarrow$

Energy band diagram



Drift

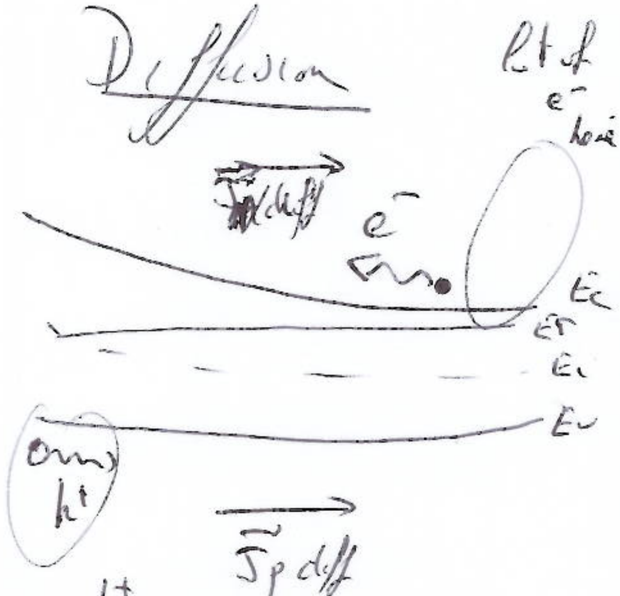


• e^- are going from highest potential to lowest potential

• $\mathcal{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_i}{dx} = \frac{1}{q}\frac{dE_v}{dx}$ (look at the slope)

• $\vec{J}_n$ and $\vec{E}$ ⇒ same direction
$\vec{J}_p$ /drift

Diffusion



more h^+ here than $x = L$