(2) Carrier densities: Complement

(a) Other expressions for n and p

For an intrinsic SC, we have $n = p = n_i$

and let us define $E_i = E_F$ the Fermi-level of an intrinsic SC.

For non-degenerate SC, it comes

$$\begin{cases} n_i = N_c\, e^{(E_i - E_c)/\beta} \Rightarrow \boxed{N_c = n_i\, e^{(E_c - E_i)/\beta}} \\ n_i = N_v\, e^{(E_v - E_i)/\beta} \Rightarrow \boxed{N_v = n_i\, e^{(E_i - E_v)/\beta}} \end{cases}$$

we replace N_c, N_v in the expression for n and p

in general then,

$$\boxed{n = N_c\, e^{(E_F - E_c)/\beta} = n_i\, e^{(E_F - E_i)/\beta}} \quad (2.19a)$$

$$\boxed{p = n_i\, e^{(E_i - E_F)/\beta}} \quad (2.19b)$$

Consequence $\Rightarrow$

$$\boxed{np = n_i^2} \quad (2.22)$$

(b) How to calculate n_i?

$$n_i^2 = N_c N_v e^{-(E_c - E_v)\beta} = N_c N_v e^{-E_g \beta}$$

$$\Rightarrow \boxed{n_i = \sqrt{N_c N_v}\, e^{-\frac{\beta E_g}{2}}} \quad [2.21] \qquad \beta = \frac{1}{k_B T}$$

E_g is the energy bandgap.

$\Rightarrow$ n_i depends on the temperature (see Fig [2.20])

(c) Doped SC (Extrinsic SC)

if the SC is doped by both donors and acceptors, the material is however charge neutral

$$\boxed{n + N_A^- = p + N_D^+} \quad [2.24]$$

$N_A^- \equiv N_A$, $N_D^+ = N_D$ } assume total ionization of dopant atoms.

N_A = density of acceptors

N_D = " " donors.

Let us find a new expression for n and p in function of N_A, N_D, n_i.

$$p = \frac{n_i^2}{n} \Rightarrow \frac{n_i^2}{n} + N_D - n - N_A = 0.$$

$$n^2 - n(N_D - N_A) - n_i^2 = 0 \quad \text{[quadratic equation]}$$

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} \qquad [2.29a]$$

$$p = \frac{n_i^2}{n} = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2} \qquad [2.29b]$$

(d) N-type and P-type SC

- N-type SC has more free e^- than h^+
- P-type SC has more free h^+ than e^-

- a sc containing donor atoms in majority is called N-type sc.
- a sc containing acceptor atoms in majority is called P-type sc.

N-type

$N_A \simeq 0$

$p \ll n$.

$$n \simeq N_D$$

$$p \simeq \frac{n_i^2}{N_D}$$

[2.30]

P-type

$N_D \simeq 0$.

$p \gg n$

$$p \simeq N_A$$

$$n \simeq \frac{n_i^2}{N_A}$$

[2.31]

(e) Where is E_c?

$n = p$ for intrinsic semiconductor.

$$\Rightarrow \quad N_c e^{\beta(E_i - E_c)} = N_v e^{\beta(E_v - E_i)}$$

Solving for E_i

$$E_i = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln\left(\frac{N_v}{N_c}\right)$$

however $\boxed{\frac{N_v}{N_c} = \left(\frac{m^*_p}{m^*_n}\right)^{3/2}}$ [2.35]

at once

$$\Rightarrow \boxed{E_i = \frac{E_c + E_v}{2} + \frac{3}{4} k_B T \ln\left(\frac{m^+_p}{m^+_n}\right)} \quad [2.36].$$

if $T = 0$ or $m^+_p = m^+_n \Rightarrow E_i$ is exactly in the middle of the bandgap.

(in general it will be very close to the middle).

(f) where is E_F?

for extrinsic semiconductor ($E_F \neq E_i$)

from the expression of n and p we get.

[2.37] $$E_F - E_i = k_B T \ln\left(\frac{n}{n_i}\right) = -k_B T \ln\left(\frac{p}{n_i}\right)$$

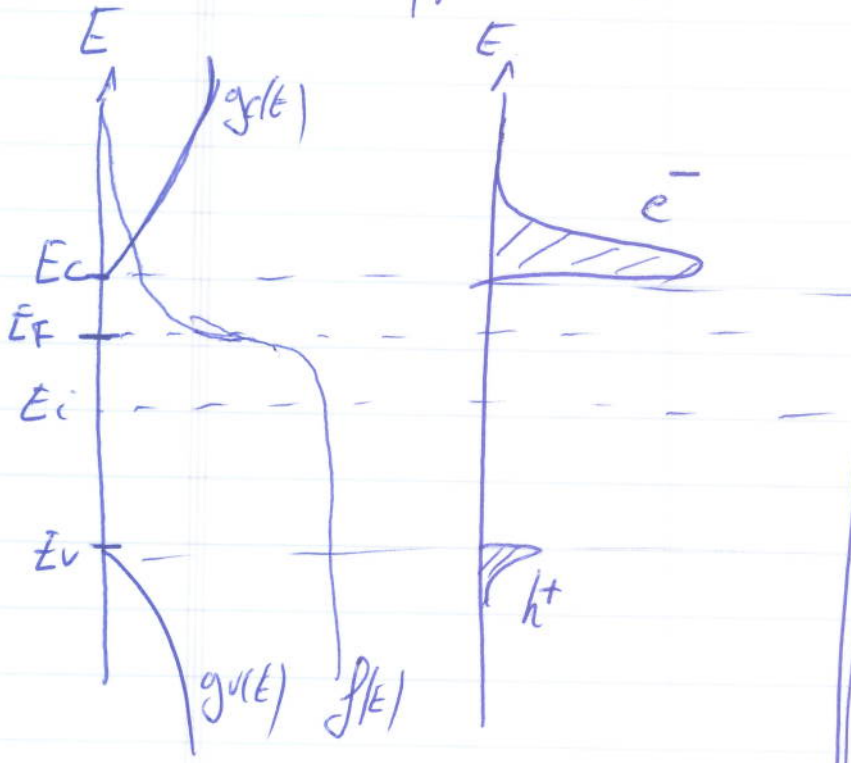
for N-type SC $n \simeq N_d$ $$E_F - E_i = k_B T \ln \frac{N_d}{n_i} \quad [2.38a]$$

for P-type SC $p \simeq N_A$ $$E_F - E_i = -k_B T \ln\left(\frac{N_A}{n_i}\right) \quad [2.38b]$$

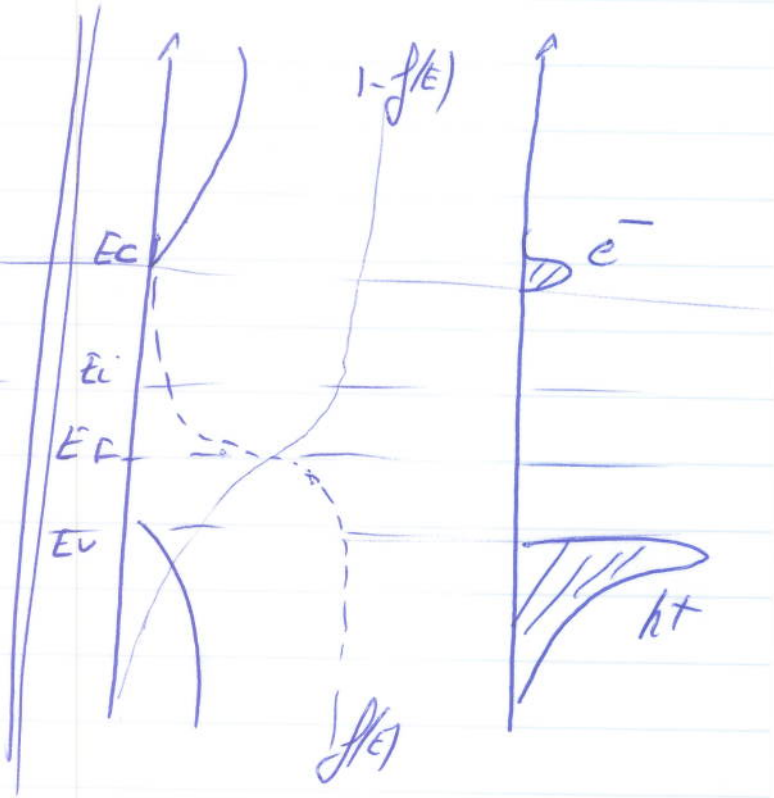
[So $E_F \neq E_i \Rightarrow$ the Fermi-level adjusts itself to ensure charge neutrality in the doped SC.

see Fig 2-21

N-type Fig 2.16a

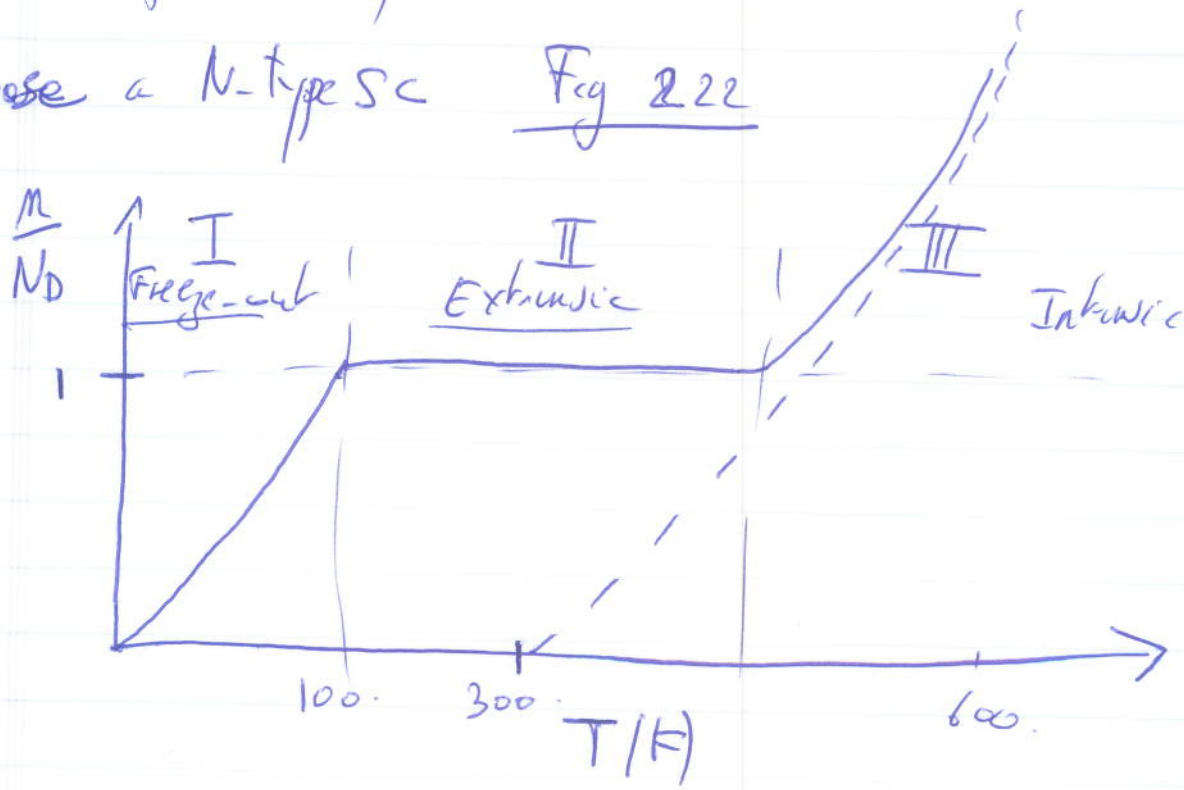


P-type Fig 2.16c



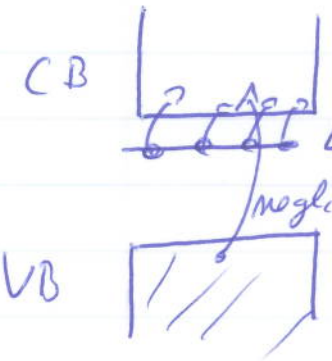
[25.7] (g) temperature dependance

Suppose a N-type SC Fig 2.22



Region I: at low Temperature, the thermal energy is not enough to ionize all the donor impurities.

Some e^- are frozen at the donor energy level.



So $\begin{cases} n < N_D \\ \frac{n}{N_D} < 1 \end{cases}$ $\Rightarrow$ Freeze-out region

Region II at moderate T, the condition of complete ionization is reached $n = N_D$ $\frac{n}{N_D} = 1$.

The number of e^- going from VB to CB is still negligeable as compared to N_D. $\Rightarrow$ Extrinsic region.

Region III if T is very large, the intrinsic carrier concentration n_i (e^- going from VB $\rightarrow$ CB) is comparable to Nd and beyond this point SC becomes intrinsic $\Rightarrow$ intrinsic region

$n > N_D$

$\frac{n}{N_D} > 1$

(h) Band-bending

[Under equilibrium conditions, the Fermi-level E_F inside a structure is constant and unique.

$\Rightarrow$ E_F appears as a horizontal line on equilibrium band diagrams.

However E_c, E_v may vary with position $\Rightarrow$ band bending

Example Non uniformly doped SC

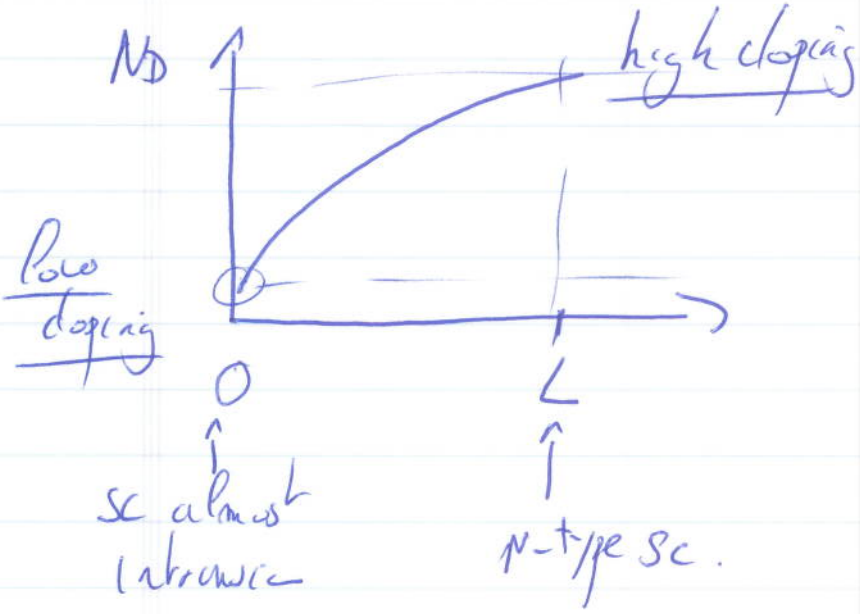


Fig 3.14a

energy band diagram (Recipe)

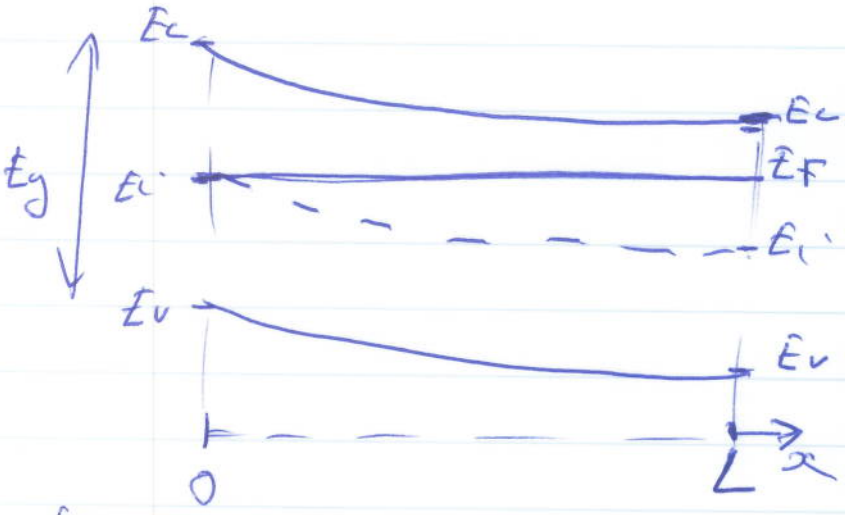
① plot E_F

② if $x \ll L$, SC is almost intrinsic so $E_F \simeq E_i$ in the middle of bandgap

③ if $x \gg 0$, SC is N-Type. So $E_F > E_i$

④ E_g is constant.

Fig 3.14b



⇒ In equilibrium Detailed, E_F is constant and unique; $E_c(x)$, $E_v(x)$, $E_i(x)$ may vary with position x.