Compressive Sensing in Noise
- Algorithms require setting a sensitivity parameter
- Ideal performance requires knowledge of noise characteristics
- LARS, Homotopy, OMP solve for all parameter values

Reconstruction from noisy measurements

Basis Pursuit Denoising
\[ \hat{x} = \arg \min_x \| x \|_1 + \lambda \| y - \Phi x \|_2 \]

Lasso
\[ \hat{x} = \arg \min_x \| y - \Phi x \|_2 \text{ s.t. } \| x \|_1 < \delta \]

Basis Pursuit with Inequality Constrain
\[ \hat{x} = \arg \min_x \| x \|_1 \text{ s.t. } \| y - \Phi x \|_2 < \epsilon \]

Greedy Algorithms
\[ \hat{x}(i) = \text{arg max}_k |c_k(i)|, \]
\[ x(i) = x(i-1) + \hat{x}(i) \phi_k, \]
\[ \hat{r}(i) = \| x(i) - r(i) \|_2 \]

Iterate if \( \| r(i) \|_2 < \epsilon \)

Parameter value \((\lambda, \delta, \epsilon)\) depends on magnitude of measurement noise

Solvers that require fixed parameter value
- Gradient Projection for Sparse Reconstruction
- \( \ell_1 \) Regularized Least Squares
- Complexity Regularization
- Log Barrier Method

Solvers that obtain solutions for all parameter values
- Least Angle Regression
- Homotopy Continuation
- Orthogonal Matching Pursuit

Solution can overfit noise or lack detail

- Small \( \lambda \)
- Ideal \( \lambda \)
- Large \( \lambda \)

Cross Validation for CS Reconstruction
- Split measurement set into reconstruction measurements and cross validation measurements
- Validate reconstruction algorithm using performance in cross validation

Recover \( \hat{x} \) from \( y \) using reconstruction algorithm for a parameter value \( \lambda \)

Validate parameter value \( \lambda \) using performance in cross validation measurements \( y_{CV} \)

Problem restricted to the support of \( \Phi x \)

Debiasing (DB) solves least squares problem restricted to the support of the solution
- Homotopy (HT) improves with debiasing
- For large enough \( M_{CV} \), cross validation outperforms residual criterion
- OMP outperforms Homotopy

Residual-based termination
- Cross validation termination
- Oracle termination

Normalized MSE, dB

Normalized MSE, dB

Number of cross validation measurements

Signal sparsity

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