3.1 *Boundary condition on a propagating wavy material line*

A material line in the xy plane is described by the function $y = a\sin[k(x - ct)]$, where k is the wave number, and c is the phase velocity. Derive a boundary condition for the velocity.

3.2 *Beltrami and complex lamellar flows*

Explain why a two-dimensional or axisymmetric flow cannot be a Beltrami flow where the vorticity is parallel to the velocity, but is necessarily a complex lamellar flow where the velocity is perpendicular to its curl.

3.3 *Drawing a streamline*

(a) Write a computer program that computes the streamline passing through a specified point in a given two-dimensional flow. The integration should be carried out using the modified Euler method. The size of the step, Δt, should be selected so that the integration proceeds approximately by a preset distance at every step.

(b) Write a computer program that returns the velocity at a specified point inside a rectangular domain of flow in the xy plane confined between $a_x \le x \le b_x$ and $a_y \le y \le b_y$, where a_x, b_x, a_y, and b_y are four specified constants.

The input should include the two components of a two-dimensional velocity field at the nodes of an $N_x \times N_y$ rectangular grid with grid points located at $x_i = a_x + (i-1)\Delta x$ and $y_j = a_y + (j-1)\Delta y$, for $i = 1, \ldots, N_x + 1$ and $j = 1, \ldots, N_y + 1$, where $\Delta x = (b_x - a_x)/N_x$ and $\Delta y = (b_y - a_y)/N_y$ are the grid spacings. The velocity between grid points should be computed by bilinear interpolation.

(c) Combine the programs into a program that draws streamlines in a rectangular domain. Run the program to draw the streamline pattern in a square box, $0 < x < 1$ and $0 < y < 2$, with $N_x = 16$ and $N_y = 32$ divisions. The x and y velocity components at the grid points are

$$u_{ij} = \exp(i\pi\Delta x) - i\pi\Delta x\cos(j\pi\Delta y), \qquad v_{ij} = \sin(j\pi\Delta y) - j\pi\Delta y\exp(i\pi\Delta x).$$

Is this velocity field solenoidal?