

2.1 Alternative definition of a tensor

Show that a two-dimensional matrix, \mathbf{T} , is qualified to be called a second-order tensor if $\mathbf{u}(\mathbf{x}) = \mathbf{T}(\mathbf{x}) \cdot \mathbf{x}$ transforms like a vector.

2.2 Spiral path

A particle is moving in the xy plane on a spiral path described in plane polar coordinates (r, θ) by the equation $r = ae^\theta$, where a is a constant and $\theta = \Theta(t)$. Compute the particle acceleration in terms of the function $\Theta(t)$.

2.3 Fluid parcel properties

Show that for a general scalar, vectorial, or tensorial intensive variable, \mathcal{F} ,

$$\frac{d}{dt} \iiint_{\text{Parcel}} \mathcal{F} \rho \, dV(\mathbf{X}) = \frac{d}{dt} \iiint_{\mathcal{A}} \mathcal{F} \rho \mathcal{J} \, dV(\mathbf{a}) = \iiint_{\text{Parcel}} \rho \frac{D\mathcal{F}}{Dt} \, dV(\mathbf{X}).$$

2.4 Evolution of the unit vector normal to a material surface

Derive an evolution equation for the rate of change of the unit normal vector following a point particle on a material surface.