1.1 *Eigenvalues of a real and symmetric matrix*

Prove that a real and symmetric matrix has three real eigenvalues and three corresponding orthogonal eigenvectors.

1.2 *Similarity transformations*

Consider a square matrix, $\mathbf{A}$, select a nonsingular square matrix, $\mathbf{P}$, whose dimensions match those of $\mathbf{A}$, and compute the new matrix $\mathbf{B} = \mathbf{P}^{-1} \cdot \mathbf{A} \cdot \mathbf{P}$. This operation is called a similarity transformation, and we say that the matrix $\mathbf{B}$ is similar to $\mathbf{A}$. Show that the eigenvalues of the matrix $\mathbf{B}$ are identical to those of $\mathbf{A}$; thus, similarity transformations preserve the eigenvalues.

1.3 *Momentum tensor*

Show that the matrix $\rho\, u_i u_j$ is a second-order tensor, called the momentum tensor.

1.4 *Rigid-body rotation*

Compute the velocity gradient tensor, rate-of-deformation tensor, and vorticity of a two-dimensional flow expressing rigid-body rotation where $u_r = 0$ and $u_\theta = \Omega\, r$, where Ω is a constant.

1.5 *Flow due to the motion of a rigid body*

Consider the flow due to the steady motion of the rigid body translating with velocity $\mathbf{U}$ and rotating about the origin with angular velocity $\mathbf{\Omega}$ in an otherwise quiescent fluid of infinite expanse. In a frame of reference fixed on the body, the flow is steady. Explain why the velocity field must satisfy the equation

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{U} + \mathbf{\Omega} \times \mathbf{x}) \cdot \nabla \mathbf{u}.$$

1.6 *Boundary condition on a propagating wavy material line*

Consider a material line in the xy plane described by the function $y = a \sin[k(x - ct)]$, where k is the wave number and c is the phase velocity. Derive a boundary condition for the velocity.