Midterm Exam: One hour and fifteen minutes

Problem 1: Label all degrees of freedom for the structure shown below:
Problem 2: For the system of axial force members shown below and with the stiffness matrix as given, solve for the displacements.

\[ K = \frac{EA}{L} \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 \\
0 & 0 & -1 & 0 & 0 & 2 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\[ K_{ff} = \frac{EA}{L} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} \]

\[ F_f = -\frac{P \sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} \]

\[ D_f = K_{ff}^{-1} F_f \]

\[ K_{ff}^{-1} = \frac{L}{2EA} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ D_f = \frac{L}{2EA} - \frac{P \sqrt{2}}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} \]

\[ D_f = \frac{L}{2EA} - \frac{P \sqrt{2}}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \end{bmatrix} \]
Problem 3: For the two node, 4 degree of freedom beam element, evaluate the given shape function and its derivative at nodes and plot it.

\[ N_2(x) = x - 2 \frac{x^2}{L} + \frac{x^3}{L^2} \]

\[ N_2'(x) = \frac{d}{dx} N_2(x) = 1 - 4 \frac{x}{L} + 3 \frac{x^2}{L^2} \]

Node 1 \( x = 0 \)

\[ N_2(0) = 0 \]
\[ N_2'(0) = 1 \]

Node 2 \( x = L \)

\[ N_2(L) = 0 \]
\[ N_2'(L) = 0 \]
Problem 4: Consider the rectangular finite element with four nodes shown below for the scalar field $\phi(x, y)$. Compute the entries of the matrix $[A]$ that would be used in computing the element shape functions.

\[
[A] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & a & 0 & 0 \\
1 & 0 & b & 0 \\
1 & a & b & ab
\end{bmatrix}
\begin{align*}
\text{node 1: } x = 0, y = 0 \\
\text{node 2: } x = a, y = 0 \\
\text{node 3: } x = 0, y = b \\
\text{node 4: } x = a, y = b
\end{align*}
\]
Problem 5:
(a) Define idealization in the context of the finite element method.
(b) Name at least three steps in the pre-processing phase of conducting a finite element analysis

a) Idealization is the process of converting a physical, real world system into a simplified version of the system that is amenable to mathematical modeling or computational modeling. This process inevitably introduces approximation and error, but is unavoidable.

b) - idealization
   - geometry definition
   - load definition
   - boundary conditions
   - material properties
   - element type
   - element size
   - etc.