University of Massachusetts, Amherst
CEE/MIE 605: Finite element analysis

Midterm exam: 1 hour and 15 minutes

Problem 1: Quick answers:
i) Which of the following is not part of the process of idealization (circle one):
   Definition of support conditions
   Definition of loads
   Entering nodal coordinates to software
   Error checking of solution
   Model in 1D, 2D or 3D.
   - either ok

   ii) Name as many of the groups of governing equations for solid mechanics / elasticity problems as possible.
       Equilibrium, compatibility, strain-displacement, constitutive

   iii) The finite element method converts governing partial differential equations to (circle one):
       Ordinary differential equations
       Algebraic equations
       Trigonometric equations

   iv) Which of the following should be viewed as generally increasing accuracy in a finite element solution (circle as many as apply):
       Use higher order elements
       Use a computer with greater CPU speed/power
       Refine the mesh to use smaller elements
Problem 2: Compute the stiffness coefficient $k_{11}$ for an axial force element as shown below (two different materials concentrically arranged). Show and define clearly the displacements applied to compute $k_{11}$ and the resulting reaction forces/stiffness coefficients.

\[ k_{11} \rightarrow \rightarrow k_{21} \]

\[ u_1 = 1 \]

In inner core \( F = \frac{E_1 A_1}{L} \)

In outer ring \( F = \frac{E_2 A_2}{L} \)

Total force \( k_{11} = \frac{E_1 A_1}{L} + \frac{E_2 A_2}{L} \)
Problem 3: Compute the unknown displacements (using matrix finite element methods) for the following system composed of two beam elements. In the process of doing so you should carefully define $F_r$, $F_s$, $D_r$, $D_s$, $K_{ff}$ in addition to the element matrices.

\[ E = 2.00 \times 10^9 \text{ Pa} \]
\[ I = 5.2 \times 10^{-7} \text{ m}^4 \]

If \( A = \begin{bmatrix} a & c \\ a & b \end{bmatrix} \) \( A' = \frac{1}{ab - cd} \begin{bmatrix} b & -c \\ -a & a \end{bmatrix} \)

\[
K^{(1)} = 1.04 \times 10^5 \begin{bmatrix}
1 & 12 & -12 & 6 & 4 \\
12 & 6 & 2 & 12 & -6 \\
-12 & 6 & 2 & 12 & -6 \\
6 & -6 & 12 & 4 & 6 \\
4 & 6 & 2 & 12 & -6
\end{bmatrix}
\]

\[
K^{(2)} = 1.04 \times 10^5 \begin{bmatrix}
3 & 4 & 5 & 6 \\
12 & 6 & -12 & 6 \\
-12 & 6 & -12 & 6 \\
4 & 6 & 2 & 12 \\
12 & -6 & -6 & 12
\end{bmatrix}
\]

\[
K_{ff} = 1.04 \times 10^5 \begin{bmatrix}
2 & 3 & 4 & 6 \\
4 & 6 & 2 & 0 \\
12 & -24 & 0 & 6 \\
2 & 0 & 8 & 2 \\
0 & 6 & 2 & 4
\end{bmatrix}
\]
NAME =

\[ \mathbf{K}_{fs} = \begin{bmatrix} 1 & 5 \\ 6 & 0 \\ -12 & -12 \\ 6 & 6 \\ 0 & -6 \end{bmatrix} \]

\[ \mathbf{K}_{sf} = \begin{bmatrix} 2 & 3 & 4 & 4 & 6 \\ 6 & -12 & 6 & 6 & 0 \\ 0 & -12 & 6 & 6 & 0 \end{bmatrix} \]

\[ \mathbf{K}_{ss} = \begin{bmatrix} 1 & 5 \\ 12 & 0 \\ 0 & 12 \end{bmatrix} \]

\[ \mathbf{F}_{f} = \begin{bmatrix} 0 \\ -p \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

Sohe \[ \mathbf{R}_{f} = \mathbf{K}_{sf}^{-1} \mathbf{F}_{f} \]
Problem 4: For the linear triangle shown below:

i) Write down the vector $X$ and the matrix $A$

ii) Compute the nodal load vector generated by the applied traction shown

\[ N_1(x, y) = 1 - x - y \]
\[ N_2(x, y) = x \]
\[ N_3(x, y) = y \]

\[ \begin{bmatrix} N \end{bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \]

\[ [A] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \]

\[ f = \int \begin{bmatrix} N_1 & 0 & N_1 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{bmatrix} P_0 \\ 0 \\ 0 \end{bmatrix} \, dx \, dy \]

\[ \int_{0}^{1} N_1 \, dy = y - \frac{y^2}{2} - xy \bigg|_{y=0, x=0}^{y=1} = 1 - \frac{1}{2} = \frac{1}{2} \]

\[ \int_{0}^{1} N_2 \, dy = xy \bigg|_{y=0, x=0}^{y=1} = 0 \]

\[ \int_{0}^{1} N_3 \, dy = \frac{y^2}{2} \bigg|_{y=0, x=0}^{y=1} = \frac{1}{2} \]
\[ \langle \mathbf{f} \rangle = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \{ \rho \} + \\
= \frac{1}{2} \rho_{\text{tot}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]