Problem 1:

Cross-sectional area of the bar shown varies linearly from \(A_0\) at the left end to \(\gamma A_0\) at the right end, where \(\gamma\) is a constant. Determine the consistent mass matrix that operates on axial d.o.f. \(u_1\) and \(u_2\).

Problem 2:

(a) Let the following matrices be applicable to a certain problem of axial vibration with two d.o.f.:

\[
[K] = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \quad [M] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

Exact eigenvalues and eigenvectors are \(\lambda_1 = 1\) and \(\{\mathbf{D}\}_1 = [ 2 \ 1 ]^T\) for mode 1 and \(\lambda_2 = 6\) and \(\{\mathbf{D}\}_2 = [ 1 \ -2 ]^T\) for mode 2. Consider the approximate eigenvectors \([ 1.7 \ 1.0 ]^T\) and \([ 1.2 \ -2.0 ]^T\), and show that the Rayleigh quotient provides good estimates of \(\lambda_1\) and \(\lambda_2\) (in Eq. 11.4-13, \(\lambda_i = \omega_i^2\)).

Problem 3: Write a program (C, java, matlab, ...) to perform explicit integration of the equation of motion

\[m \ddot{x} + c \dot{x} + kx = f(t)\]

of a single degree of freedom system with \(m = 1, k = 400, c = 0.05\sqrt{mk}\), with initial condition \(x(0) = \dot{x}(0) = \ddot{x}(0) = 0\), and

\[
f(t) = \begin{cases} 
0 & 0 \leq t < 2 \\
10 & 2 \leq t < 5 \\
0 & 5 \leq t
\end{cases}
\]

(a) calculate the maximum time step that can be used to obtain a stable solution.
(b) show the resulting displacement time history for several values of \(\Delta t\) and discuss convergence and accuracy.