Problem 1: Consider the following interpolating polynomials. For each sketch the corresponding element, showing shape as generally as possible, locations of nodes, and degrees of freedom:

(a) \( a_0 + a_1 x + a_2 y \)

(b) \( a_0 + a_1 \xi + a_2 \eta + a_3 \xi \eta + a_4 \xi^2 + a_5 \eta^2 + a_6 \xi^2 \eta + a_7 \xi \eta^2 \)

(c) \( a_0 + a_1 x + a_2 x^2 + a_3 x^3 \)

(d) \( a_0 + a_1 x + a_2 y + a_3 z \)
**Problem 2:** For the structure shown below, which consists of a beam element and an axial force element:

(a) Calculate the stiffness matrix \([K_{f1}]\) and write the global force and displacement vectors \([F]\) and \([D]\).

(b) What happens, mathematically, to this matrix as \(A \to 0\). For this part, you may assume that the numerical value of all parameters other than \(A\) is 1.

(c) Explain the physical meaning of your answer to part (b).

\[
\begin{bmatrix}
\frac{12EI_1}{L^3} & \frac{6EI_2}{L^2} & -\frac{12EI_1}{L^3} & \frac{6EI_2}{L^2} \\
\frac{6EI_2}{L^2} & \frac{4EI_3}{L} & -\frac{6EI_2}{L^2} & \frac{2EI_3}{L} \\
-\frac{12EI_1}{L^3} & -\frac{6EI_2}{L^2} & \frac{12EI_1}{L^3} & -\frac{6EI_2}{L^2} \\
\frac{6EI_2}{L^2} & \frac{2EI_3}{L} & -\frac{6EI_2}{L^2} & \frac{4EI_3}{L}
\end{bmatrix}
\]

\([k]\)
a) \[ [K_{ff}] = \begin{bmatrix} \frac{2}{12EI/L^3} & \frac{3}{2EI/L} & \frac{4}{2EI/L^2} \\ \frac{12EI/L^3 + EA/L}{L} & 1 & -\frac{6EI/L^2}{L} \\ \frac{HEI/L}{L} & \frac{HEI/L}{2} & 1 \end{bmatrix} \]

\[
\{F\} = \begin{bmatrix} F_1 \\ 0 \\ 0 \\ 0 \\ F_5 \end{bmatrix}^T
\]

\[
\{D\} = \begin{bmatrix} 0 \\ D_2 \\ D_3 \\ D_4 \\ 0 \end{bmatrix}^T
\]

b) \( \Delta A \rightarrow 0 \) and \( k_{ff} \rightarrow 0 \)

c) The structure becomes \( \Delta M \)
Which is unstable
Problem 3: Shown below are three contours of bending stress obtained from ADINA for a cantilever beam supported at the left end and with a point load applied at the right end. Triangular elements are used.
(a) What kind of elements were used in each analysis?
(b) Has any postprocessing been applied to the stress contours?
(c) Comment on the relative accuracy of the various solutions.
a) CST; CST; LST

b) raw results; averaged results; raw results

c) is most accurate

a), b) may each be more accurate at different parts in the model
Problem 4: You are to integrate the function $a_0 + a_1 \xi^4$ in the interval $[-1, 1]$ using Gauss quadrature.
(a) How many Gauss points should be used to achieve an exact evaluation? Explain your answer.
(b) Perform the integration using two points.
(c) Perform the integration using three points. Compare to the results from (b) and discuss in the context of your answer to part (a).

a) $2n - 1$ must be greater than or equal to the order of the polynomial.
So, we must use 3 points.

b) Evaluate at $\xi = \pm \frac{\sqrt{3}}{2}$ to get, with $W_1 = 1 = W_2$

$$I = 2a_0 + \frac{2a_1}{3}$$

$c)$ Use $\xi = 0, W = \frac{2}{3}$

$$I = 2a_0 + \frac{2a_1}{5}$$

The 3pt. result is in fact the exact result.
**Problem 5:** The structure shown below consists of a rigid bar supported by two springs, and has two degrees of freedom, as shown. Use the direct method to calculate the stiffness coefficients $K_{12}$ and $K_{22}$. Include sketches to illustrate your work.

![Diagram of a structure with a rigid bar supported by two springs](image)

1. Apply $V_2 = 1, V_1 = 0$

2. $S = \frac{1}{2}$

3. \[ \sum F_y = 0 \rightarrow K_{12} - \frac{k_2}{2} + k_{22} = 0 \]

4. \[ \sum M_A = 0 \rightarrow K_{22}(2L) - \frac{k_2}{2}L = 0 \]

5. \[ \frac{K_{22}}{k_2} = \frac{L}{2} \]

6. \[ K_{12} = \frac{k_1}{2} - \frac{k_2}{y} = \frac{k_2}{y} \]
Problem 6: Consider the system below with a block of mass $m$ sliding at velocity $v$ down a ramp with coefficient of sliding friction $\mu$. The ramp has thickness $t$ in the $x$-direction. Calculate the traction exerted by the block on the ramp at point $P$. State any assumptions you are making. The gravitational acceleration is $g$. You should consider only the two dimensional stress state and traction vector.

\[ \{n\} = \frac{1}{12} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \]

\[ \{\Phi\} = \frac{1}{5t} \begin{pmatrix} \sigma_x - 2\sigma_y \\ \sigma_x - 2\sigma_y \end{pmatrix} \]

At $P$, there is a downward force $mg$, and a tangential force $mg(\cos 45^\circ)\mu$.

In stress, there are $\frac{m g}{t}$, $\frac{m g \cos 45^\circ}{t}$.

$L, t =$ dimensions of block.

We need these in global $(x, y)$ coords. and then set equal to traction.

\[ \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_x - 2\sigma_y \\ \sigma_x - 2\sigma_y \end{pmatrix} = \frac{m g}{t} \begin{pmatrix} -1 \\ -1 \end{pmatrix} + \frac{m g \cos 45^\circ}{t} \begin{pmatrix} -1 \\ -1 \end{pmatrix} \frac{1}{\sqrt{2}}. \]