Problem 1: Consider the state of stress defined below:

\[
\sigma = \begin{bmatrix}
-2 & 0 & -\sqrt{\frac{14}{4}} \\
0 & 1 & 0 \\
-\sqrt{\frac{14}{4}} & 0 & 4
\end{bmatrix}
\]

(a) Sketch the non-zero stresses on a cubic material element. Make sure to indicate a coordinate system and take care of signs.
(b) Find the principal stresses.
(c) Find one principal direction. If you did not find the principal direction using eigen-analysis, show that the direction you found corresponds to an eigenvector of the stress matrix.
(d) For what value of the Poisson’s ratio \(\nu\) does this state of stress represent the solution to a plane strain problem.

Problem 2: The structural element shown in figure (a) below is a corbel, used to transfer vertical loads from a beam end or arch springing into a column or wall. Figure (b) shows a gross idealization of such a structural element.

(a) Would a plane stress or plane strain model of a corbel be more accurate in your opinion? What would affect this accuracy?
(b) Write down as complete a set of traction boundary conditions as possible for the corbel, including along \(x = 0\). State clearly any assumptions you are making, and evaluate whether they are reasonable or not. Your answer may be in terms of the constants \(a, b_1, b_2\).
Problem 3: Consider the stress function for 2D elasticity

\[ \Phi(x, y) = c_1 x^6 y^2 + c_2 x^3 y^3 + c_3 x^3 y \]

(a) Determine as much as you can about the constants \( c_1, c_2, c_3 \) by enforcing compatibility.
(b) Calculate the stresses resulting from the stress function after substitution of results from (a).
(c) Calculate the tractions on the boundary of a plate of dimension \( a \times b \) (see figure below), and sketch these tractions on a drawing of the plate.

Problem 4: Consider the circular cross section shown below. Assume that a membrane inflated over a circular area has height of the form \( z = c_1 \frac{r^2}{2} (r^2 - c_2^2) \), where \( p \) is the pressure, \( s \) is the membrane stress, and \( c_1, c_2 \) are constants.

(a) Determine the constants \( c_1 \) and \( c_2 \) using boundary conditions and the governing equation \( \nabla^2 z = -\frac{p}{s} \). Note: \( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \).
(b) Show using the membrane analogy for torsion that \( J = \frac{\pi a^4}{2} \) for a circular cross section.