a) \[ \sigma_r = \frac{1}{r} \frac{\partial}{\partial r} \left( r \phi \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \phi, \theta \]

\[ = 0 + 0 \]

\[ \sigma_r = 0 \]

\[ \tau_\theta = \frac{\partial}{\partial r} \phi, r \]

\[ \tau_\theta = -\left( \frac{1}{r} \phi, \theta \right), r \]

\[ \tau_\theta = \frac{c}{r^2} \]

b) In a ring \( a < r < b \)

The moment generated at \( r = b \) is clockwise

\[ M_b = \tau_\theta \frac{2\pi}{b} \]

\[ = \frac{c}{b} \frac{2\pi}{b} = \frac{2\pi c}{b} \]

The moment at \( r = a \) is

\[ M_a = -\tau_\theta \frac{2\pi}{a} \]

\[ = -\frac{c}{a^2} \frac{2\pi}{a} = \frac{-2\pi c}{a^2} \]

The signs result from the normal vectors at \( r = a \) and \( r = b \) pointing in different directions.
these moments are in static equilibrium

Note that these moments are independent of \(a, b\). Therefore, despite the fact that \(a_0 \to b \to \infty\)
\(T \to 0\), and as \(a \to 0\) \(T \to \infty\), the moments remain in equilibrium.

at \(r=0\) in the infinite plate we have a singularity in the stress.

\[ \gamma_{r\theta} = \frac{T_{r\theta}}{G} = \frac{C}{r^2 G} \]

\[ \varepsilon_r = \varepsilon_\theta = 0 \]

Since \(\varepsilon_r = u_r, r = 0\)

\[ u = \varepsilon_\theta f(\theta) \]

but this problem is axisymmetric, so that all parts of the solution must be independent of \(\theta\). We conclude that \(f(\theta) = 0\)

and \(u = 0\)

Now, \(\varepsilon_\theta = 0 = \frac{V_r \theta}{r} + \frac{u_r \theta}{r}\)

so \(V_r \theta = 0\)

\[ V = f(r) \]

Also, \(\gamma_{r\theta} = \frac{C}{r^2 G} = V_r + u_\theta - \frac{V}{r}\)

\[ \alpha \quad \frac{V_r - \frac{V}{r}}{r} = \frac{C}{r^2 G} \]

Since we know from above that \(V = f(r)\), we can write \(V_r\) as \(V'\).
we must solve
\[ r^2 V' - rv = \frac{C}{G} \]

**Homogeneous part**

Let \( V_h = C_1 r \)

\[ V' = C_1 \]

\[ r^2 (C_1) - r (C_1 r) = 0 \]

**Particular part**

Let \( V_p = C_2 r^{-2} \)

\[ V' = -C_2 r^{-3} \]

\[ r^2 \left( \frac{-C_2}{r^2} \right) - r \left( \frac{C_2}{r} \right) = \frac{C}{G} \]

\[ -2 C_2 = \frac{C}{G} \]

\[ C_2 = -\frac{C}{2G} \]

\[ V = V_h + V_p \]

\[ = C_1 r + \left( -\frac{C}{2G} \frac{1}{r} \right) = C_1 r - \frac{C}{2Gr} \]

b.c. \( V(a) = 0 \)

\[ C_1 a - \frac{C}{2Ga} = 0 \]

\[ C_1 = \frac{C}{2Ga^2} \]

\[ V = \frac{Cr}{2Ga^2} - \frac{C}{2Gr} \]
\[ \phi = C \left[ r^2(\alpha - \theta) + r^2 \sin \theta \cos \theta - r^2 \cos \theta + \tan \alpha \right] \]

a) \[ J_r = 2C (\alpha - \theta - \sin \theta \cos \theta - \sin \theta \tan \alpha) \]

\[ J_\theta = 2C (\alpha - \theta + \sin \theta \cos \theta - \cos \theta + \tan \alpha) \]

\[ \tau_{\rho} = C - C \cos(2\theta) - C \tan \alpha \sin(2\theta) \]

b.c: \[ \alpha = \alpha \]

\[ J_\theta = \tau_{\rho} = 0 \]

\[ J_\theta (\alpha) = 2C (\sin \alpha \cos \alpha - \cos \alpha \tan \alpha) = 0 \checkmark \]

\[ \tau_{\rho} (\alpha) = C - C \cos 2\alpha - C \tan \alpha \sin 2\alpha = 0 \checkmark \]

\[ \tau_{\rho} = C - C = 0 = 0 \checkmark \]

\[ \tau_{\rho} = 2C (\alpha + \theta - \tan \alpha) = q \]

\[ = 2C (\alpha - \tan \alpha) \]

\[ C = \frac{q}{2(\alpha - \tan \alpha)} \]
b/c \[ \alpha = 20^\circ \]

Find a section \( mn \) and draw stresses.

\[
\begin{align*}
\tan \theta &= \frac{y}{x} \\
x &= c_2 \\
y &= c_0 + \theta \\
x &= c_0 + x \tan \theta
\end{align*}
\]

\[
r = \sqrt{x^2 + y^2}
\]

\[
r = c_2 + y^2
\]

\[
\phi = r \sin \theta
\]

\[
r = \sqrt{c_2^2 + r^2 \sin^2 \theta}
\]

\[
r^2 = c_2^2 + r^2 \sin^2 \theta
\]

\[
r^2 (1 - \sin^2 \theta) = c_2^2
\]

\[
r^2 = \frac{c_2^2}{\cos^2 \theta}
\]

\[
r = \frac{c_2}{\cos \theta}
\]

get \( \sigma_x, \tau_{xy} \) intermed \( \sigma_x, \tau_{xy} \)
\[ \sigma_x = \sigma_r \cos^2 \theta + \sigma_0 \sin^2 \theta - 2 \tau_{\theta \phi} \sin \theta \cos \theta \]

\[ \tau_{xy} = (\sigma_r - \sigma_0) \sin \theta \cos \theta + \tau_{\theta \phi} (\cos^2 \theta - \sin^2 \theta) \]

\[ \sigma_x = \frac{2 \xi \alpha - 2 \xi \theta - \xi \sin (2 \theta)}{2 \alpha - 2 \tan \alpha} \]

\[ \tau_{xy} = \frac{\xi \cos 2 \theta - \xi}{2 \alpha - 2 \tan \alpha} \]

Both independent of \( r \)!!
Note $\tau_{xy} \neq 0$ at the bottom surface due to change from polar to Cartesian.

Note: Beam theory solution is based on $\tau = \frac{M_y}{I}$

$\tau = \frac{VQ}{I_t}$