Problem 1: Consider a member loaded such that the only non-zero stresses are $\sigma_x$ (tension) and $\tau_{x\theta}$ (shear, equivalent to $\tau_{xy}$). The material model has $\sigma_{yp} = 50\text{ksi}$, $E = 30000\text{ksi}$ and $G = 10000\text{ksi}$ and Ramberg-Osgood parameters $K = 1.5 \times 10^{11}$, and $n = 5$. The function $F(\tau_{oct})$ needed to calculate the plastic strain increments is

$$F(\tau_{oct}) = \frac{3Kn}{E} \left( \frac{3}{E\sqrt{2}} \right)^{n-1} \tau_{oct}^{n-1}$$

(a) Assume that initial loading is in tension only up to the point of yield. Compute the elastic strain at this point using Hooke’s law.

(b) Now consider stress increments $d\tau_{x\theta} = \sigma_{yp}/10$ and $d\sigma_x = 0$. Calculate the total stress and total strain after this increment is applied (i.e. calculate the elastic strain increment using Hooke’s law and the plastic strain increment using the incremental relations).

(c) Now, instead of applying the post-yield load increment at once, do it in two steps, $d\tau_{x\theta,1} = \sigma_{yp}/20$, and $d\tau_{x\theta,2} = \sigma_{yp}/20$. Calculate the elastic, plastic, and total strain increments, again using Hooke’s law for the elastic part of the strain and the incremental plasticity relations for the plastic part.

(d) Now use 4 steps of magnitude $d\tau_{x\theta,i} = \sigma_{yp}/40$, $i = 1, 2, 3, 4$ to get to the same point in stress space. Calculate total, elastic, and plastic strains.

(e) Calculate the elastic unloading strain.

(f) Plot the paths that this material point follows in strain space ($\varepsilon_x, \gamma_{x\theta}$) and stress space ($\sigma_x, \tau_{x\theta}$). Prepare plots for total strain and plastic strain. Discuss results. Think carefully and spend time preparing a nice graphical presentation of the results. You will be graded on the clarity and quality of the graphical presentation.