Problem 1: Consider a 10 ft long steel wire with cross sectional area 0.1 sq. in. weight density of 500 lb/cubic foot and $E = 30,000$ ksi. A 300 lb weight is hung from the end of the wire.

(a) What is the strain energy in the wire due to the 300lb weight neglecting selfweight?
(b) Due to selfweight, neglecting the applied load?
(c) Due to combined selfweight and applied force?
(d) Comment on the prior results. Are energies additive in this case?
(e) At what velocity would a soccer ball have to travel to have kinetic energy equivalent to the total strain energy in the wire?
Prob 1

a) \[ \sigma = \frac{200 \text{ lb}}{\text{in}^2} \]

\[ U = \frac{\sigma^2 L}{2E \alpha A} = \frac{(200)^2 (60) (12)}{(2) (30,000,000) (1.1)} = 1.8 \]

b) stress \( \sigma \)

\[ \sigma(x) = \rho A x = (500) \frac{1}{12} (0.1) x \]

\[ = 0.2 \sigma(x) \text{ psi} \]

\[ U = \frac{\sigma x}{L E} \]

\[ U = \int_0^{120} \frac{1}{12} \sigma(x) x A \text{ d}x \]

\[ = \frac{(0.2 \sigma(x))^3 (0.1)}{(12) (30,000,000)} \frac{(120)^2}{3} = 1.0 \times 10^{-8} \]

\[ \sigma(x) = 3000 \text{ psi} + 0.2 \sigma(x) \text{ psi} \]

\[ U = \int_0^{120} \frac{(3000 + 0.2 \sigma(x))^2 \text{ psi}}{(2) (30,000,000)} \text{ d}x \]

\[ \text{not additive} \]
Problem 2: Calculate the strain energy in the beam shown below. Neglect possible stress concentration effects at the change in cross section.

\[ U_{0} = \frac{1}{2E} \int_{0}^{h} \int_{0}^{\frac{1}{2} \frac{h}{l}} \frac{P^2}{4A} + 2 \frac{P}{4} \frac{M}{EI} + \frac{M^2}{64EI} \, dy \, dx \]

\[ U_{0} = \frac{1}{2E} \left( \frac{P^2 L}{4A} + \frac{g M^2 L}{64EI} \right) \]

Similar calculation for segment 2 yields:

\[ U_{2} = \frac{1}{2E} \left( \frac{P^2 L}{A} + \frac{M^2 L}{EI} \right) \]

\[ U_{\text{total}} = \frac{1}{2E} \left( 3 \frac{P^2 L}{4A} + \frac{g M^2 L}{8EI} \right) \]
**Problem 3:** Calculate the strain energy in the bar shown below in which the cross section varies linearly from A1 to A2. Comment on the result as A2 approaches zero. Compute the elongation of the bar using equivalence of work and energy. Is the deflection inversely proportional to the average cross sectional area? Comment?
3. \( A(x) = A_i \cdot \left( \frac{A_i - A_f}{L} \right) \) as a function of \( x \)

\[ \sigma_x = \frac{P}{A(x)} = P \left( A_i \cdot \left( \frac{A_i - A_f}{L} \right) \right)^{-1} \]

\[ U_0 = \frac{1}{2} E \sigma_x^2 = \frac{1}{2} E \frac{P^2}{A(x)} \]

\[ U = \int_0^L U_0 A(x) \, dx \]

\[ = \int_0^L \frac{1}{2} E \frac{P^2}{A(x)} \, dx \]

\[ = \frac{P^2}{2E} \int_0^L \frac{x}{A(x)} \, dx \]

\[ = \frac{P^2}{2E} \left[ \frac{1}{A_i - A_f} \cdot \left( \log (A_f) - \log (A_i) \right) \right]_0^L \]

\[ = \frac{P^2}{2E} \frac{1}{A_i - A_f} \left( \log (A_f) - \log (A_i) \right) \]

\[ \frac{1}{2} PD = U \]

\[ 0 = \frac{P L}{E} \frac{1}{A_i - A_f} \left( \log A_f - \log A_i \right) \]

The average cross-section, assumed over the entire length, would not give the correct result.