Ungrated:

**Problem 1:** Consider a distributed load on an infinite half space as shown below:

(a) Compute the stress field $\sigma_{xx}(x,y)$. The best way to do this is via a computer program to integrate numerically or symbolically.

(b) Generate a contour plot of $\sigma_{xx}(x,y)$
1. The joint density function is
\[ f_{X,Y}(x,y) = \frac{e^{-x}}{\pi} \left( \frac{1}{(x^2 + y^2)^{3/2}} \right) \]

so we can integrate to get
\[ \sigma_x = \left[ \frac{2\pi y}{x^2 + (y^2)^2} \right]_{-\infty}^{\infty} \frac{x^2}{(x^2 + (y^2)^2)^2} \, dx \]

with \( p(u) = 50 + \frac{50}{50} u = 50 + u \)

so
\[ \sigma_x = \int_{-\infty}^{\infty} 2(50 + u) \frac{x^2}{(x^2 + (y^2)^2)^2} \, dx \]

shall show caution after integrating exactly or numerically for roughly \( x \in [0, 200] \)

\( y \in [-100, 100] \)
**Problem 2:** Consider the arrangement of spheres shown below loaded only by the selfweight $W$ of each sphere. Consider the walls to be rigid.

(a) Compute the maximum contact stress at point A.

(b) What is the downward displacement of point B?
Each sphere has weight \( W \).

Therefore, contact force between top and bottom spheres is:

\[ P_{\text{int}} = \frac{W}{2} \left( \sin \Theta \right) \]

The displacement between top and bottom spheres is:

\[ S_1 = 0.77 \left[ P_{\text{int}} \left( \frac{2}{5} \right)^2 \left( \frac{L}{r} \right) \right]^{\frac{1}{2}} \]

And the vertical component is:

\[ S_{1\text{vert}} = S_1 \sin \Theta \]

At A, the contact force is \( \frac{3W}{2} \) and \( r_2 = \infty \quad E_2 = \infty \)

\[ S_{2\text{vert}} = 0.77 \left[ P_{\text{int}} \left( \frac{1}{E} \right)^2 \left( \frac{1}{r} \right) \right] \]

Total vertical displacement is:

\[ S_{\text{vert}} = S_{1\text{vert}} + S_{2\text{vert}} \]
The max contact stress is, at A,

\[ \sigma_c = 1.5 \frac{P_{intz}}{\pi a^2} \]

\[ a = 0.88 \left[ \frac{P_{intz} (E_i + E_z) r_1 r_2}{E_i E_z (r_1 + r_2)} \right]^{1/3} \text{ rigid support} \]

must take limit \( a_0 \) \( E_z, r_2 \to \infty \)

\[ a = 0.88 \left[ \frac{P_{intz} r_1}{E_i} \right]^{1/3} \text{ not } E_z = E_i \]