Example Eigenvalue Problem:

Let \( A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix} \)

the eigenvalue problem is given by

\[
(A - \lambda I) x = 0
\]

in which we require

\[
\det (A - \lambda I) = 0
\]

\[
\begin{vmatrix} 3 - \lambda & 2 \\ 3 & -2 - \lambda \end{vmatrix} = (3-\lambda)(-2-\lambda) - 6 = 0
\]

\[
-6 - \lambda + \lambda^2 - 6 = 0
\]

\[
\lambda^2 - \lambda - 12 = 0
\]

\[
\lambda = \frac{1 \pm \sqrt{1 + 48}}{2} = 4, -3
\]

find the eigenvector corresponding to \( \lambda_1 = 4 \)

(1) \[
\begin{bmatrix} 3 - 4 & 2 \\ 3 & -2 - 4 \end{bmatrix} \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(2) \[
\begin{bmatrix} 3 & -2 - 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(1) \[
-x_1^{(0)} + 2x_2^{(0)} = 0
\]

\[
x_2^{(0)} = \frac{x_1^{(0)}}{2}
\]

\[
x^{(0)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \frac{1}{5}
\]
Find eigenvector corresponding to \( \lambda_2 = -3 \)

(1) \[
\begin{bmatrix}
3 + 3 & 2 \\
3 & -2 + 3
\end{bmatrix}
\begin{bmatrix}
x_1^{(2)} \\
x_2^{(2)}
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(2) \[
6 \ x_1^{(2)} + 2 \ x_2^{(2)} = 0
\]

\[
x_2^{(2)} = -3 \ x_1^{(2)}
\]

\[
x^{(2)} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}^T \frac{1}{\sqrt{10}}
\]