APPLICATION OF THE TURBULENT POTENTIAL MODEL TO COMPLEX FLOWS

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ABSTRACT

The turbulent potential model is a RANS model that avoids modeling the Reynolds stress tensor. As a result it has the ability to obtain the physical accuracy of Reynolds stress transport equation models at a cost and complexity comparable to popular two equation models. The model's ability to predict channel flow, free-shear layers, homogeneous shear flow, stagnation point flow, backward facing step flows, and boundary layers with and without strong adverse pressure gradients has been demonstrated previously. In the present study, the performance of the turbulent potential model is evaluated in a series of complex non-equilibrium turbulent flows. These include three-dimensional boundary layers, unsteady vortex shedding, rotating turbulent flows and boundary layer transition.

KEYWORDS

Turbulence, Modeling, Transition, Unsteady, Three-dimensional, Rotation, RANS, Boundary layers.

INTRODUCTION

Typical Reynolds Averaged Navier-Stokes (RANS) turbulence models are ultimately concerned with modeling the Reynolds stress tensor. Solution of the partial differential equations governing the evolution of the Reynolds stress tensor (Reynolds stress transport equation models) can be complex to implement, and expensive to solve numerically due to inherent stiffness and coupling of the equations. Simpler and less expensive RANS models, such as two equation models, algebraic Reynolds stress tensor and the mean flow variables. Such a relation assumes that some sort of equilibrium exists between the turbulence and the driving mean flow. However, in complex turbulent flows involving large-scale unsteadiness, rotation, rapid geometrical changes, transition, and other complex phenomena the assumption of turbulent equilibrium may not be well founded.

The turbulent potential model is a compromise between these two approaches. It makes no equilibrium assumptions, but has a computational cost and implementation complexity comparable to modern low Reynolds number two-equation models. This compromise is possible because the model

no longer attempts to model the Reynolds stress tensor. Only the turbulent body force vector (the divergence of the Reynolds stress tensor) is required to predict the mean flow evolution, and the turbulent potential model solves modeled evolution equations for the scalar and vector potentials of this turbulent body force vector. In mathematical terms the potentials are prescribed by the expressions $\nabla \cdot \mathbf{R} = \nabla \phi + \nabla \times \psi$ and $\nabla \cdot \psi = 0$. The latter expression is a gauge restriction, and the divergence free gauge was chosen for its simplicity. The potentials are assumed to be turbulent quantities that vanish when the turbulence vanishes, such as at walls or in the laminar free-stream.

The model focuses on the potentials rather than the body force vector itself because momentum conservation is then guaranteed, and because the potentials have a physical interpretation which can be used to guide the model development (Perot, 1996). The scalar potential can be viewed as a turbulent pressure, and in an incompressible flow this quantity could be absorbed directly into the pressure. However, we continue to carry it as an independent quantity in the model because it is an excellent indicator of turbulence anisotropy. It behaves similarly to the v² variable used in the elliptic relation models of Durbin (1995). Despite the similarity in function, the evolution equation for the scalar potential is very different from the evolution equation for v². The turbulent potential modeling framework has the ability to include ellipticity, but does not currently do so at this time. Rather than using ellipticity to obtain correct near wall behavior, the current model uses asymptotically exact near wall dissipation and pressure strain models. The vector potential acts as a generalization of the Reynolds shear stress. In two-dimensional flows, the vector potential points out of the plane of interest (like vorticity) and only has one non-zero component. If one assumes an eddy viscosity hypothesis (which we do not), then it can be shown that the vector potential is directly proportional to the mean vorticity, $\Psi = v_T \omega$.

The efficacy of the turbulent potential modeling approach was demonstrated in Perot (1999) on very fundamental turbulent flows, including boundary layers, free-shear layers, adverse pressure gradient boundary layers, and the backward facing step. In this work, we have tested the model in more physically demanding turbulent flows, in order to demonstrate it ability to predict flows in which turbulent non-equilibrium is present.

TURBULENT POTENTIAL MODEL

The transport equations that constitute the turbulent potential more are summarized below. Comments about the derivation of specific terms in these equations can be found in Perot (1999).

$$\begin{split} \frac{D\phi}{Dt} &= \nabla \cdot \left[(\upsilon + \upsilon_{t}\sigma_{\phi})\nabla\phi \right] - \left(2\alpha\frac{\hat{\varepsilon}}{k} + \frac{2\upsilon(\nabla\phi^{1/2} \cdot \nabla\phi^{1/2})}{\phi} \right) \phi + C_{t} \left(\frac{\upsilon_{T}}{\upsilon} \right)^{1/2} P \\ &+ C_{p1}\frac{\hat{\varepsilon}}{k} (2k/3 - \phi) + C_{p2}\frac{\phi}{k} P - (\upsilon + 2\upsilon_{t}) \left[\nabla \mathbf{k} \cdot \nabla(\phi/\mathbf{k}) \right] + C_{p3} \left(\frac{\psi \cdot \psi}{\upsilon_{T} (1 + 25/\operatorname{Re})} - P \right) \frac{\phi}{k} \\ \frac{D\psi}{Dt} &= \nabla \cdot \left[(\upsilon + \upsilon_{t})\nabla\psi \right] + \phi \mathbf{s} - \left(\alpha\frac{\hat{\varepsilon}}{k} + \frac{2\upsilon(\nabla k^{1/2} \cdot \nabla\phi^{1/2})}{(k\phi)^{1/2}} \right) \psi + C_{t} \left(\frac{\upsilon_{T}}{\upsilon} \right)^{1/2} k \mathbf{s} \\ &- C_{p1}\frac{\hat{\varepsilon}}{k}\psi + C_{p2} \left(\frac{\psi}{k} P - \phi \mathbf{s} \right) - (\upsilon + 2\upsilon_{t}) \left[\nabla \mathbf{k} \cdot \nabla(\psi/\mathbf{k}) \right] + C_{\mu} (2\alpha - 1)\phi\hat{\mathbf{\omega}} - 2\alpha\frac{\psi^{2}\mathbf{s}}{k} \\ \frac{Dk}{Dt} &= \nabla \cdot \left[(\upsilon + \upsilon_{t}\sigma_{k1})\nabla k \right] + P - \varepsilon \\ \frac{D\varepsilon}{Dt} &= \nabla \cdot \left[(\upsilon + \upsilon_{t}\sigma_{\varepsilon})\nabla\varepsilon \right] + C_{r}\frac{\hat{\varepsilon}}{k} (C_{\varepsilon 1}P - C_{\varepsilon 2}\varepsilon + C_{\varepsilon 3} \right] \psi \times \hat{\mathbf{\omega}} \end{split}$$

where Ω is the external frame rotation rate,

$$P = \mathbf{\Psi} \cdot \mathbf{s} , \ \mathbf{v}_t = C_{\mu} \frac{\phi k}{\varepsilon}, \ \alpha = 1 / \left(1 + 1.5 \frac{\phi}{\kappa} \right), \ \hat{\varepsilon} = \varepsilon / \left(1 + 10 \upsilon \nabla \sqrt{k} / k \right), \ \hat{\omega} = \omega + 2\Omega$$

and the model constants are given by:

$$C_{\mu} = 0.21, C_{p1} = 1 + 2.0 \frac{v_t}{v_t + 10v} (1 - \alpha), C_{p2} = 3/5, C_{p3} = 3/5 + 6/7, C_t = .0033, \sigma_{\phi} = 0.33$$

The k- ε constants are close to the standard values and are given by,

$$C_{\varepsilon 1} = 1.45, \ C_{\varepsilon 2} = 1.83 - 0.16 \exp(-k^2/4\nu\varepsilon),$$
$$C_{\rm r} = \frac{(1+0.105\xi)}{(1+0.06\xi)}, \ \xi = |\hat{\omega}| \frac{k}{\hat{\varepsilon}}, \ \sigma_{k} = 0.33 + 0.67 \ P/\hat{\varepsilon}, \ \sigma_{\varepsilon} = 0.33 + 0.5 \ P/\hat{\varepsilon}$$

Despite the inclusion of k and ε transport equations, this should *not* be viewed as a variation of the k- ε modeling approach. These extra variables are only used to model the source terms in the turbulent potential transport equations, they are not used to predict the Reynolds stresses or calculate the influence of the turbulence on the mean flow evolution. Only the vector potential directly influences the mean flow evolution in the flows studied herein. The vector **s** in the production term is defined by the equation $\nabla^2(\phi \mathbf{s}) = \nabla \times \{\nabla \cdot (\mathbf{S}\phi)\}$, where **S** is the rate of strain tensor. In flows with a single inhomogeneous direction (e.g. boundary layer flows), it can be shown that **s** is equal to the vorticity.

THREE-DIMENSIONAL BOUNDARY LAYERS

Three-dimensional boundary layers are an example of a type of flow in which the turbulence does not have time to reach equilibrium with the mean flow. When two-equation models are applied to 3D boundary layers, they usually yield poor results (Fannelop *et. al.* 1975, Bradshaw *et. al.* 1996). It is nearly impossible to model all the Reynolds shear-stresses correctly with only a single scalar eddy viscosity. The study conducted by Ölçmen & Simpson (1993) shows that using more complicated algebraic (or nonlinear) eddy-viscosity models improves the performance of the models on slightly. The models that accounted for the anisotropy of the eddy viscosity in general performed better but the anisotropic constants had to be changed for different flows. The turbulent potential model has been tested on spanwise driven channel flow and the flow over an infinite swept bump.

The spanwise driven channel flow is regular fully-developed channel flow suddenly subjected to a large spanwise pressure gradient at time zero. A DNS simulation of this flow at a bulk velocity Reynolds number of 3300 was performed by Moin et al. (1990). It has been modeled by Durbin (1993) using an elliptic Reynolds stress transport equation model. The turbulent potential model predictions for the mean velocity and Reynolds shear stresses are shown in Figure 1. The DNS results and the model show almost no change in the streamwise velocity (Figure 1a) over the time of the simulations. The mean spanwise velocity (Figure 1b) increases monotonically in time and the model predictions (lines) closely match the DNS results (symbols) at non-dimensional times (tu_r/h) of 0.3, 0.6 and 0.9. Note that the spanwise boundary layer is essentially a laminar developing layer at these early times. A simple two equation model would show excessive growth of this layer because it would apply the fully turbulent eddy viscosity of the streamwise flow to the spanwise boundary layer development. The Reynolds shear stresses are shown in Figure 1c. The upper set correspond to the shear-stress for the streamwise velocity and the lower set of curves are the shear-stress corresponding to the spanwise velocity. The unique configuration of this flow allows us to directly determine some of the Reynolds stress from the turbulent potentials, but in general the model actively avoids solving for Reynolds stresses, and Reynolds stresses are not easily recovered from the turbulent potentials.



Figure 1: Spanwise driven channel flow at Re=180. Turbulent potential model (lines) compared with the DNS data of Moin at non-dimensional times 0.0, 0.3, 0.6, 0.9.

Another test of the turbulent potential model is the flow over an infinite swept bump (Webster et. al, 1996). In this flow the boundary layer height is the same order of magnitude as the bump so the boundary layer is highly distorted and non-equilibrium in nature as it is subjected to both streamwise pressure gradients and changes in curvature as in goes over the swept bump. Wu & Squires(1998) have modeled this flow using LES and the elliptic relaxation model (Durbin, 1995). An unstructured grid with high near wall resolution was used to calculate the potential model predictions for this flow. An inlet condition of a fully developed zero pressure gradient two-dimensional boundary layer at $Re_{\theta} = 1400$ is used as the upstream condition and was introduced at a half bump chord length upstream of the leading edge of the bump. A slip boundary condition is applied at the top wall. The streamwise velocity profiles are shown as function of the distance from the channel floor in Figure 2a. The results are shown at various positions downstream of leading edge of the bump. The model predictions are the symbols and the experimental results are the lines. The streamwise component of velocity increases and reaches a maximum value at the apex of the bump. At trailing edge, the flow is very close to separation because of the adverse pressure gradient caused by the flow expansion. The flow relaxes to a two-dimensional boundary layer as it moves downstream of the bump and is close to inlet velocity profile.



Figure 2: Mean flow predictions of flow over a swept bump at $Re_{\theta} = 1400$. (a) Streamwise velocity (b) Spanwise velocity. Symbols are model predictions and lines are the experimental data.

The spanwise velocity profiles are shown in Figure 2b. The spanwise component of the velocity is negative on top of the bump and reaches a maximum positive value at the end of the bump. This change in sign of the spanwise velocity is because the spanwise pressure gradient switches sign once it reaches the top of the bump. Downstream of the bump the model seems to recover a little too quickly compared to the experiment. The model was also tested for an inlet boundary layer of $\text{Re}_{\theta} = 3260$, and mesh resolution studies were performed. Additional details of these calculations and the predictions of the three-dimensional boundary layers formed on a prolate spheroid at an angle of attack can be found in Are, Zhang & Perot (2002).

UNSTEADY VORTEX SHEDDING

To test this model's ability to predict unsteady non-equilibrium turbulent flow and large-scale unsteadiness, the problem of vortex shedding behind a 2D triangular cylinder was chosen. This flow is inherently unsteady. This geometry is slightly easier to simulate than the circular cylinder, since the separation points are fixed. The computational domain consisting of approximately 25,000 adaptively located triangles was used in this simulation (Zhang & Perot, 2000). The inlet mean streamwise velocity is set to a constant and the vertical velocity is set to zero. For turbulent kinetic energy and dissipation rate, we use the same boundary conditions as described in Johnasson *et al.* (1993).

$$U_{in} = 17.0 \text{ m/s}; \quad k_{in} = (0.05U_{in})^2; \quad \varepsilon_{in} = \frac{0.16k_{in}^{3/2}}{0.2\ell}$$

where ℓ is the height of the duct (which is 3 times of the height of the triangle). The total mass flow was $\dot{m}_1=0.6 \text{ kgs}^{-1}$ in their experiment, and the inlet velocity is evaluated based on that value. A zero gradient boundary condition is used for all the variables at the outlet. Slip-wall boundary conditions are used for the duct wall. The Reynolds number of this simulation is Re=U_{in}d/v=45,000, where *d* is the height of the triangle.

It was observed that an almost perfect periodicity existed when the stream function of a point about one triangle height behind the triangle near the centerline was studied. The shedding frequency is 109.3 (s⁻¹). The corresponding Strouhal number defined by $Sr=fd/U_{in}$ is 0.257, which should be compared with experimental value of 0.25 (Sjunnesson *et al.* 1991) and the computed value of 0.27 in Johnnasson *et al.* (1993). Although the instantaneous flow is asymmetric, the time-averaged fields are always symmetric or anti-symmetric. Figure 3 shows the mean streamwise velocity at the centerline. The length of recirculation zone is accurately predicted, while the location of the maximum negative velocity is slightly upstream compared with the experiments. The magnitude of the maximum negative velocity is also a little lower than the experiment data.



Figure 3: Time averaged streamwise velocity on the centerline behind the triangular cylinder.

Figure 4 shows the streamwise velocity at different cross sections behind the triangle. The calculated velocity profiles are in reasonable agreement with the experimental data. It is hypothesized that due to mesh size restrictions the shear layer leaving the triangle is not fully resolved. The computed shear layer is thicker than the real one, thus close to the back of the triangle, the fluid is slowed down and driven backwards more than it should be.



Figure 4: Time averaged stream-wise velocity behind the triangle: □, calculations; *, experiments. (a) 15mm, (b) 38mm, (c) 150mm, (d) 376mm.

ROTATING FLOWS

Rotation redistributes energy but does not create or destroy it. The k equation is therefore insensitive to rotation, and consequently two-equation models have considerable difficulty capturing the effects of rotation. Even the non-linear k- ε models, such as the model developed by Speziale (1987), only have a weak dependence on rotation rate. Reynolds stress transport equation closures do a much better job of responding to rotation because the exact equations for the Reynolds stresses involve Coriolis terms. Frame indifference was enabled by Speziale (1989) in Reynolds stress closures by ensuring that the Reynolds stresses depend on the rotation rate solely through a quantity called the intrinsic vorticity.

Like Reynolds stress transport equation closures, the turbulent potential model has explicit Coriolis terms due to rotation. However, unlike the stress transport closures these Coriolis terms must be modeled (since they contain unknown Reynolds stresses) and are therefore only exact in certain limits. The turbulent potential model uses the intrinsic vorticity to guarantee frame indifference, and a detailed analysis of the frame invariance of the turbulent potentials and frame consistency of the model has been performed by Bhattacharya (2002).

Rotating channel flow is a good test case for estimating the Coriolis term. The model has been compared to the DNS results of Kristoffersen & Andersson (1993). The DNS has a turbulent Reynolds number $\text{Re}_t = u_{\tau}h/\upsilon$ of 194 where h is the channel half width and $u_{\tau} = \sqrt{\upsilon d\overline{\nu}/dy}_{wall}$ is the shear velocity. The pressure gradient is equal to 1. The Rossby number Ro for this case is defined as $2h |\Omega|/u_{\tau}$. From Figure 5(a) we can see that even for zero rotation, the mean velocity is a little over predicted (by about 5%), and the reason for this is known to be due to the low Reynolds number. Reynolds numbers of 395 and 590 give very good agreement with the mean flow and the low order turbulent statistics when the flow is not rotating. This small overshoot is present in both Figure 5(b) and Figure 5(c) where the Rossby numbers are 0.15 and 0.5 respectively. The model gets the slope of the mean velocity right, as well as the slope at the wall.



Figure 5: Mean velocity profile for $\text{Re}_{\tau} = 194$. (a) Ro =0.0 (b) Ro = 0.15 (c) Ro = 0.5. Symbols are DNS data and lines are the model predictions.

In the case of swirling pipe flow the rotation is along an axis aligned with the mean flow rather than perpendicular to it as in the rotating channel flow. The model has been compared to the experiments of Imao & Itoh (1996). The calculations have been done with respect to a non-rotating frame, so that Coriolis terms are not present in the calculations. The radius of the pipe is R. and the turbulent Reynolds number $u_{\tau}R/v$ for this case is 572. The rotation rate is defined as $N = \overline{v}_{\theta}|_{wall} / \overline{v}_{z}|_{mean}$, and the two rotation rates tried out are 0, and 1.0. The non-dimensional pressure gradients applied for the two cases are 1.0 and 0.6 respectively. Figures 6a and 6b show the angular and axial velocity profiles for the two rotation rates, N=0 & N=1.0. The predictions for the angular velocity are very accurate, while the axial velocity seems to be slightly underpredicted in the low rotation number case.



Figure 6: Axial and angular velocity profiles for N = 0 & N = 1, compared to experiments of Imao & Itoh (1996).

PREDICTING TRANSITION IN BOUNDARY LAYERS

Laminar to turbulent transition is a critical feature of many flows. The transition location can have a profound affect on macroscopic variables such as total drag or heat transfer. Correlations for transition are often difficult to implement and rarely apply to complex flow situations. In Wang & Perot (2002) is argued that well formulated non-equilibrium turbulence models should also be able to model the behavior of small amplitude disturbances and hence predict transition. Speziale *et al.* (1995) proposes a similar argument. Transition, is a highly non-equilibrium process where the turbulence grows exponentially within a quasi-steady mean flow and is therefore a stringent test of the turbulent potential model.

The model predicts flat plate boundary layer transition with free stream turbulence intensities ranging from 6% to 0.03% (natural transition). The present study also shows the ability of the model to predict the effect of noise levels on natural transition and the effect pressure gradient, both strong & adverse, on transition.

The process of transition is studied by looking at the evolution of the friction coefficient on a flat plate boundary layer as a function of the downstream distance. The model predictions are compared to experimental data with different initial turbulence intensities. The mean velocity is initially uniform flow for all cases and the initial values of velocity U_0 , turbulence Reynolds number $\text{Re}_T = k^2/(v\varepsilon)$, turbulence intensity level $Tu = (\frac{2}{3}k)^{\frac{1}{2}}/U_{\infty}$ for five experimental cases with four different turbulence intensities are given in Table 1. The initial potentials ϕ and ψ are set to $\frac{2}{3} \text{ k}$ and zero respectively. All experiments were performed in air so $v = 1.55 \times 10^{-5}$ was used in every case.

 TABLE 1

 INITIAL FLOW PARAMETERS FOR THE TEST CASES IN FIGURE 7

U_0 (m/s)	Re _T	Ти	Source
24.4	100	0.03%	Schubauer & Klebanoff
22	250	1.25%	Abu -Ghannam & Shaw
14.42	250	1.3%	Dhawan & Narasimha
5.4	200	3.0%	ERCOFTAC, T3A
9.4	200	6.0%	ERCOFTAC, T3B

The friction coefficients are plotted against Re_x , in Figure 7. The friction coefficients for laminar and turbulence flows, i.e. $C_f = 0.664 \text{Re}_x^{-1/2}$ and $C_f = 0.027 \text{Re}_x^{-1/7}$ respectively, are also plotted for comparison. The computations agree well with the experiments. The excessive overshoot at the end of the transition may be a result of using the boundary layer approximation to compute the results. The boundary layer approximation (small streamwise derivatives) is not well founded when the flow transitions and the boundary layer grows very suddenly.



Figure 7: Transition in zero pressure gradient boundary layer at various initial turbulence intensities. The symbols represent experiment data and the lines are the model predictions.

The case of Tu = 0.03% is remarkable in that it represents natural transition, a first for a RANS model. The difference of the present result from experimental data of Schubauer *et al.* (1955) is actually expected. Our result assumes a noisy initial condition (relatively large value for the scalar potential), and is comparable to classical predictive theories such as e^9 rule which predicts a value of 2.0×10^6 (Warsi, 1999). Wang & Perot (2002) show that a smaller scalar potential (less noise) can delay the transition location to 2.8×10^6 or even 5.0×10^6 which is the even lower noise value found by Wells (1967).

The transition behavior of two variable pressure gradient boundary layers is shown in Figure 8. These ERCOFTAC data sets (Coupland, 1990) have both favorable and adverse pressure gradients designed to have some similarity to a turbine blade. It is seen that the model predicts the delayed transition locations well.



Figure 8: Transition in a variable pressure gradient boundary layer at 3% and 6.6% initial turbulence intensities, ERCOFTAC T3C test cases.

CONCLUSIONS

This work has demonstrated the advantages of using a non-equilibrium turbulence model when attempting to predict complex turbulent flow phenomena. We have been able to accurately predict three-dimensional boundary layers, unsteady flows, the effects of rotation, and boundary layer transition with different pressure gradients, turbulence levels, and noise levels. In the past, true non-equilibrium modeling required solving the coupled Reynolds stress transport equations. We demonstrate in this work, a computationally simpler and faster approach. The turbulent potential is fully non-equilibrium and makes no algebraic assumptions about how the turbulence is related to the mean flow. However, it can be implemented and computed with a cost and complexity comparable to popular two-equation models.

Acknowledgements

This work was sponsored by the Air Force Office of Scientific Research, USAF, under grant number F49620-00-1-0033, and the Office of Naval Research under grant number N00014-99-1-0194.

REFERENCES

- Are S., Zhang X. and Perot B. (2002). Application of the turbulent potential model to unsteady flows and three-dimensional boundary layers. Accepted to the *Int. J. of Turbomachinery*.
- Bhattacharya A. (2002). Application of the turbulent potential model to rotating flows. Masters Thesis, University of Massachusetts, Amherst, MA.
- Bradshaw P., Launder B. E. and Lumley J. L. (1996). Collaborative testing of turbulence models, *Journal of Fluid Eng.*, **118**, 243-247.
- Coupland J. (1990). ERCOFTAC Special interest group on laminar to turbulent transition and retransition T3C test cases.
- Durbin P. A. (1993). On modeling three-dimensional turbulent wall layers, Phys. Fluids A, 5, 1231.
- Durbin P. A. (1995). Separated flow computations with $k \varepsilon v^2$ model, AIAA Journal, 33:4, 659.
- Fannelop T. K. and Krogstad P. A. (1975). Three-dimensional turbulent boundary layers in external flows: a report on Euromech60, *Journal of Fluid Mechanics*, **71**, 815-826.
- Imao S. & Itoh M. (1996). Turbulent characteristics of the flow in an axially rotating pipe. *Int. J. Heat and Fluid Flow*, **17**, 444-451.
- Johansson S. H., Davidson L. and Olsson E. (1993). Numerical Simulation of Vortex Shedding Past Triangular Cylinders at High Reynolds Number Using a k-ε Turbulence Model, *International Journal For Numerical Methods In Fluids*, **16**, 859-878.
- Kristoffersen, R., & Andersson, H.I. (1993). Direct simulations of low-Reynolds-number turbulent flow in a rotating channel. J. Fluid Mech., 256, 163-197.
- Moin P., Shih T.-H., Driver D. and Mansour N. (1990). Direct Numerical simulation of threedimensional turbulent boundary layer, *Phys. Fluids A*, **2**, 1846-1853.
- Ölcmen M. S. and Simpson R. L. (1993). Evaluation of algebraic eddy-viscosity models in threedimensional turbulent boundary layers, *AIAA Journal*, **31**, 1545-1554.
- Perot B. (1996). A new approach to turbulence modeling, *Proceedings of the Center for Turbulence Research Summer Program*, Stanford University.
- Perot B. (1999). Turbulence modeling using body force potentials, Phys. Fluids, 11:9, 2645-2656.
- Perot B. and Wang H. (1999). Modeling separation and reattachment using the turbulent potential model, *The 4th International Symposium on Engineering Turbulence Modeling and Measurements*, Corsica, France.
- Schubauer G. B. abd Klebanoff P. S. (1955). Contribution to the mechanism of boundary layer transition, *NACA TR-3489*.
- Speziale C. G. (1987). On non-linear K-l and K-E models of turbulence. J. Fluid Mech., 178, 459.
- Speziale C. G. (1989). Turbulence modeling in non-inertial frames of reference. *Theoret. Comput. Fluid Dynamics*, 1, 3-19.
- Speziale, C.G., Abid R. and Blaisdell G.A. (1995). On the consistency of Reynolds stress turbulence closures with hydrodynamic stability theory, *ICASE Report TR-95-46*.
- Sjunnesson A., Nelsons C. and Max E. (1991). LDA measurement of velocities and turbulence in a bluff body stabilized flame, *Volvo Flygmotor AB*, Trollhattan.
- Wang C. & Perot B. (2002). Prediction of turbulent transition in boundary layers using the turbulent potential model, Submitted to *J. of Turbulence*.
- Warsi Z.U.A. (1999). *Fluid Dynamics Theoretical and Computational Approaches*, CRC Press LLC, Florida.
- Webster D. R., DeGraff D. B. and Eaton J. K. (1996). Turbulence characteristics of a boundary layer over a swept bump, *Journal of Fluid Mechanics*, **323**, 1-22.
- Wells C. S. (1967). Effects of free-stream turbulence on boundary layer transition, AIAA J., 5:1, 172.
- Wu X. and Squires K. D. (1998). Prediction of the three-dimensional boundary layer over a swept bump, *AIAA Journal*, **36**, 55-514.
- Zhang X. and Perot B. (2000). Potential Model Prediction of Flow Past a Triangular Cylinder Using an Unstructured Staggered Mesh Method, *CFD 2000*, June 11-13, Montreal, Canada.