A Self-Adapting Turbulence Model for Hybrid RANS/LES

J. Gadebusch & J. B. Perot

Department of Mechanical and Industrial Engineering, University of Massachusetts
Amherst, MA - 01003

Email: *perot@ecs.umass.edu*

ABSTRACT

A two-equation transport model is used to model turbulence at any mesh resolution, from RANS to LES, to DNS. The two-equation model used is a slight variation of the standard k/ε model that allows the backscatter of energy to the resolved scales. The model automatically adapts to the mesh resolution provided and no user interaction is necessary. This model is tested against moderately high Reynolds number isotropic decaying turbulence and gives very good predictions at any mesh resolution. A detailed analysis shows that at LES resolutions the solution remains fully unsteady and three-dimensional and the solution does not approach a RANS like solution. At DNS resolutions, it is shown that the model automatically becomes negligible.

1. INTRODUCTION

Recent technological advances in supercomputing are paving the way for large eddy simulations (LES) to becoming an increasingly useful tool for engineering predictions of many turbulent flows. However, classical LES modeling requires advanced knowledge of the turbulence (the integral length scale must lie in the inertial range) that is usually unavailable. In addition LES models are not really necessary in large portions of the flow domain and therefore waste resources considerably. Because of this, there has been considerable interest in hybrid turbulence models that can perform either RANS modeling or LES. With such a model, one can obtain initial predictions with coarse meshes (RANS) or more accuracy can be obtained with finer meshes (LES). In theory one can even perform LES and RANS within the same problem domain. The self-adapting model proposed in this paper is fundamentally different from prior LES models and current hybrid models in that it achieves a completely natural evolution from RANS to LES, and with enough mesh resolution to DNS.

The transition of how much turbulent kinetic energy is represented by the model compared to how much turbulent kinetic energy is computed via first principles is how most turbulence models are classified. The range of applicable turbulence models are shown in Figure 1 in relation to a turbulent energy spectrum. Each model tries to represent the energy in the spectrum to the right of the model's name.

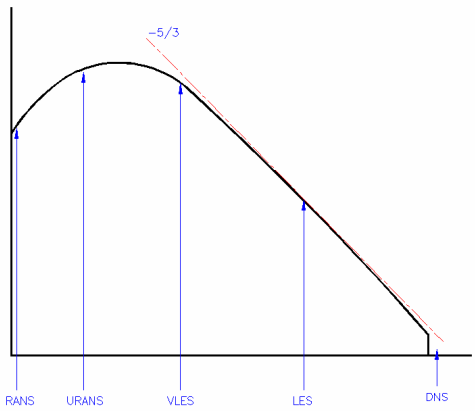

Figure 1. Illustration of Energy Spectra

RANS models contain most of the turbulent energy in the model. LES computes considerably more of the turbulent energy via first principles and DNS resolves all of the turbulence directly and models none. To introduce a more detailed terminology, in between RANS and LES lies URANS (unsteady RANS) and VLES (very large eddy simulation).

Recent developments in hybrid modeling approaches try to transition from URANS to LES depending on the situation. However many of these models require user intervention and usually some form of a

"blending function" to compute the turbulent viscosity. The self-adapting modeling approach proposed will be shown to work at any mesh resolution and over the entire spectrum. It can therefore do, RANS, URANS, VLES, LES and even DNS. Most importantly, the character of the model is not set by the user (or the geometric location), but adapts to whatever the mesh can support. The proposed approach therefore models only as much turbulent kinetic energy as necessary (for that mesh) and resolves as much of the energy using first principals as possible. It is probably not correct to consider the proposed approach to be a hybrid model in the classic sense (though it has many similarities to those models) because it does not blend an LES and a RANS model together. The proposed model is closer to DES in that it is a single set of transport equations that changes its character (RANS, LES or DNS) depending on the flow situation. However, in contrast with other models, this change does not depend on geometric considerations but rather on the available mesh resolution.

2. MATHEMATICAL BACKGROUND

The classic mathematical theory behind RANS and LES makes these two modeling approaches look fundamentally different. RANS is based on ensemble averages and LES on filtering. However, a closer examination by Germano [1] revealed some very important insights. Most importantly, the exact but unclosed governing equations for RANS and LES are mathematically identical. While the RANS equations can be derived from the assumption of ensemble averaging and the LES equations from filtering operations, these assumptions are overly restrictive and neither system *must* be derived with those assumptions. The only required assumption is that the velocity field can be split into two parts and that this splitting operation commutes with differentiation. With this assumption the equations for turbulence evolution are

$$\overline{u}_{i,t} + (\overline{u}_i \overline{u}_j)_{,j} = -\overline{p}_{,i} + \nu \overline{u}_{i,jj} - R_{ij,j} \qquad (1a)$$

where $\overline{u}_i$ and $\overline{p}$ and the computed velocity and pressure and $R_{ij} \equiv \overline{u_i u_j} - \overline{u}_i \overline{u}_j$ is the unknown turbulent stress tensor. The exact (but unclosed) evolution equation for this stress tensor is

$$\begin{aligned} R_{ij,t} + \overline{u}_k R_{ij,k} = \nu R_{ij,kk} - (R_{jk}\overline{u}_{i,k} + R_{ik}\overline{u}_{j,k}) \\ - T_{ijk\ ,k} - (< p_{,i}, u_j > + < p_{,j}, u_i >) \\ - 2\nu < u_{i,k}, u_{j,k} > \end{aligned} \qquad (1b)$$

where the double bracket is given by $< a_i, b_j > \equiv \overline{a_i b_j} - \overline{a}_i \overline{b}_j$ and the turbulent transport is

$$T_{ijk} \equiv \overline{u_i u_j u_k} - \overline{u}_i R_{jk} - \overline{u}_j R_{ik} - \overline{u}_k R_{ij} - \overline{u}_i \overline{u}_j \overline{u}_k .$$

The bracketed terms require a model to close the system. In RANS the overbar might denote an ensemble average, for LES it might be an explicit filtering operation. However, it can also be an implicit operation because, in practice, the overbar operation is never actually performed. In this paper, it is assumed that an overbar represents whatever the calculation computes. It is not possible to prove that an implicit filter commutes with differentiation, but it is a fairly reasonable assumption to make (at least to first order).

Starting from these exact equations numerous modeling approaches are possible. The familiar two equation RANS models are a simplification of Eqn (1b) from a tensor equation to a scalar equation. The primary unknown, R_{ij} must then be reconstructed from this scalar kinetic energy, *k*, using a hypothesized algebraic relation such as the eddy viscosity hypothesis. As the mathematical analysis of Germano [1] makes clear, there is no fundamental reason why more complex modeling approaches (currently used by RANS models) can not be applied to LES. The apparent natural evolution of turbulence models from simplest to more complex, suggests that two-equation transport models for LES are in fact the next logical step.

3. A TWO-EQUATION LES MODEL

The unclosed equations (1a) and (1b) can be modeled using the follow transport equations,

$$u_{i,t} + (u_i u_j)_{,j} = -(p + \tfrac{2}{3}k)_{,i} + [(\nu + \nu_T \alpha)(u_{i,j} + u_{j,i})]_{,j} \qquad (2a)$$

$$k_{,t} + (k u_j)_{,j} = [(\nu + \nu_T / \sigma_k) k_{,j}]_{,j} + \alpha P - \varepsilon \qquad (2b)$$

$$\varepsilon_{,t} + (\varepsilon u_j)_{,j} = [(\nu + \nu_T / \sigma_\varepsilon)\varepsilon_{,j}]_{,j} + \tfrac{\varepsilon}{k}[C_{\varepsilon 1} P - C_{\varepsilon 2}\varepsilon] \qquad (2c)$$

where the overbar on the velocity and pressure have been dropped for convenience. The production is given by $P = \nu_T (u_{i,j} + u_{j,i}) u_{i,j}$ and eddy viscosity is given by $\nu_T = C_\mu \frac{k^2}{\varepsilon} (\frac{k}{k + k_r})$. The constants are fairly standard k/ε constants, $C_{\varepsilon 1} = 1.55$, $\sigma_\varepsilon = 1.2$, $\sigma_k = 1.0$, $C_\mu = 0.18$. The parameter $C_{\varepsilon 2} = \frac{11}{6} f + \frac{25}{\mathrm{Re}_T} f^2$ is sensitive to the local turbulent Reynolds number $\mathrm{Re}_T = \frac{k^2}{\nu \varepsilon}$ of the modeled turbulence via the function $f = \frac{\mathrm{Re}_T}{30}\left[\sqrt{1 + \frac{60}{\mathrm{Re}_T}} - 1\right]$ as per the analysis of Perot & de Bryn Kops [2]. This varies $C_{\varepsilon 2}$ from its theoretical limits of 11/6 at high Reynolds numbers to 3/2 at low Reynolds numbers. For incompressible flow, the

pressure in Eqn (2a) is determined from the constraint $u_{j,j} = 0$.

If $\alpha = 1$, this system is the same as a classic k/ε model. The k/ε system was chosen in this work in order to reach the largest audience possible. There are very good reasons to prefer other transport equation model systems. The proposed modeling ideas can be easily generalized to these other frameworks as well. The proposed formulation assumes that the turbulent stress tensor is reconstructed using the eddy viscosity hypothesis, $R_{ij} = \frac{2}{3}k\delta_{ij} - \nu_T \alpha (u_{i,j} + u_{j,i})$. The simplest model possible is used in this work in order to focus as directly as possible on the key idea - it is possible to develop models that automatically work at any mesh resolution.

One key component of a self-adaptive turbulence model is that it must be able to backscatter energy from the unresolved (modeled) turbulence to the calculated (resolved) velocity field. A classic k/ε model is too simplified and can not backscatter energy. The additional parameter α has been added to the k/ε model above to correct this important flaw and control the energy flow. Usually α is positive (and order 1), but it can become small or negative (as will be shown shortly).

4. BACKSCATTER OF ENERGY

Figure 2 graphically shows the concept of backscatter and normal forward scatter shown on a 1D energy spectra.

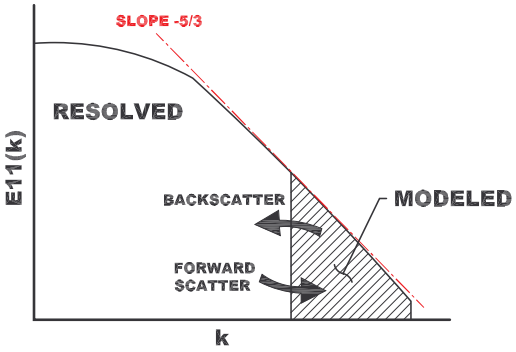

Figure 2. Illustration of backscatter and normal forward scatter.

Here the resolved turbulence is illustrated on the left and the modeled turbulence is to the right (shaded), the arrows indicate the direction of energy transfer. To further clarify the benefits of backscatter, let's assume a 128^3 isotropic turbulence simulation is performed. Because of the large mesh size, most of the turbulence would be resolved and only a small percentage would need to be modeled. However, if the simulation was set up incorrectly and most of the energy was defined to be included in the modeled and very little resolved (i.e. RANS initial condition on an LES mesh) the adaptive turbulence model should displace energy from the model and energize the resolved velocity field to correct for this error. With a feature such as backscatter to control the flow of energy, a turbulence model could indeed give accurate flow predictions, correct errors in the initial conditions and (most importantly) perform these functions without any user intervention.

The idea of allowing backscatter in a turbulence model is not a new one. It has been shown by Chasnov [3] and Carati et.al. [4] that a -5/3 power law in isotropic decay is better predicted by LES (dynamic) models that account for backscatter. Along similar lines, Piomelli [5] has recently shown that a crude form of energy backscatter (noise) to the DES model improved channel flow results.

5. ENERGY TRANSFER FUNCTION

The proposed equation for the energy transfer is

$$\alpha = 1.5 * (1.0 - C^* (\tfrac{k}{k+k_r})^2 [(\tfrac{\Delta x_i}{k_r} \tfrac{\partial \sqrt{k_r}}{\partial x_i})^2 + 0.11]^{-1}) \qquad (3)$$

where k_r is the resolved turbulent kinetic energy (at a certain location and time), k is the modeled turbulent kinetic energy, and $C^* = 0.28$. The quantity $(\Delta x_i \frac{\partial \sqrt{k_r}}{\partial x_i})^2 / k_r = \{(\Delta x \frac{\partial \sqrt{k_r}}{\partial x})^2 + (\Delta y \frac{\partial \sqrt{k_r}}{\partial y})^2 + (\Delta z \frac{\partial \sqrt{k_r}}{\partial z})^2\} / k_r$ is a dimensionless measure of the error (similar to what is sometimes used in mesh adaptation). In this formulation the resolved turbulent kinetic energy $k_r = \frac{1}{2}(u_1^2 + u_2^2 + u_3^2)$ is the indicator function that is being used to estimate the mesh resolution. If the flow is DNS or over-resolved (such as a RANS initial condition on a LES mesh) then this quantity is small, its inverse is large (but limited away from infinity by the empirically determined 0.11 term) and the model tends to backscatter energy. In contrast, normal energy transfer (from resolved scales to the modeled scales) occurs in the regions of the flow where the gradient length scales are comparable to the mesh size. On very coarse meshes, RANS like behavior should be recovered. In this limit, k_r, is expected to be small. This means that $\alpha \to 1.5 - 0.42/(0.8 + 0.11) \approx 1.04$ and the standard RANS model is very closely recovered in the RANS limit.

When $\alpha < 1$, the eddy viscosity in Eqn (2a) is essentially negative. Negative viscosity amplifies

existing resolved velocity fluctuations and amplifies small wavelength modes (those closest to the mesh resolution) the most rapidly. This is a very reasonable model for backscatter. It is not injecting energy via some random forcing of the resolved flow, rather it works to enhance the existing instabilities and modes. Moreover, the energy transfer is local in spectral space. It tends to take energy from the model (which has most of its energy at scales just below the mesh resolution) and preferentially delivers it to the resolved flow at almost the same length scale (but just above the mesh resolution).

When $\alpha < 0$, the model is backscattering energy. The parameter α is not a model constant, rather it is a field that varies in space. α also appears in Eqn (2b), the k-equation, so that the total kinetic energy is a conserved quantity and can only disappear via dissipation to heat. Its presence is not necessary in the scale equation, Eqn (2c).

The particular form of the energy transfer function was developed and tuned solely to obtain the correct limits. Many other functional expressions and/or indicator quantities are certainly possible. The goal of this paper is not to advocate for this particular function but to demonstrate that self-adaptive turbulence models are possible, and this particularly function serves this purpose adequately.

6. HIGH RE ISOTROPIC DECAY RESULTS

The initial turbulent Reynolds number, $\mathrm{Re} = k^2/\nu\varepsilon$, for this test case is 640. The DNS data was performed on a 768^3 mesh using a Fourier spectral method and is given by the large circles. Simulations were performed using mesh resolutions from 1^3 to 256^3. In each case the model is identical and only the mesh size is changed. The simulations show that at any mesh resolution, the model predicts the decay of the turbulence well. The lowest mesh resolution is clearly a RANS or URANS simulation and the largest mesh, 256^3, is an LES simulation.

The intermediate resolutions might be considered URANS, VLES, or LES. It might be hypothesized that all these solutions give the same result because the resolved flow is damped to zero rapidly and all the solutions give essentially the RANS decay result. This is shown not to be the case. The ratio of the modeled kinetic energy to the total kinetic energy is shown in Figure 3 with one curve for each of the mesh resolutions. The 1^3 solution is the top curve, with all its energy contained in the model (giving a ratio of 1.0) and the 256^3 simulation is the bottom line, with the smallest ratio of modeled kinetic energy (< 10%). Note that these curves are relatively constant and decrease slightly in time (as the simulation proceeds).

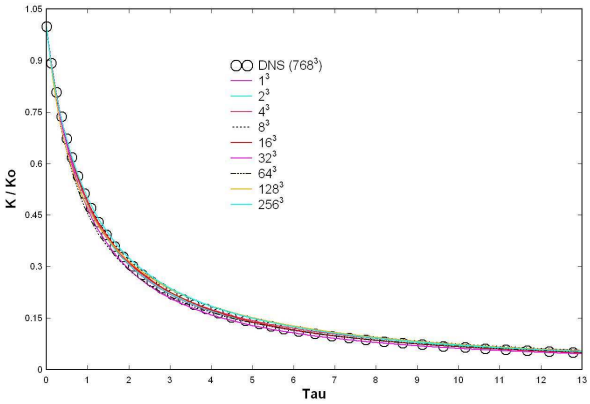

Figure 3. Total kinetic energy predictions for isotropic decay (Re = 640).

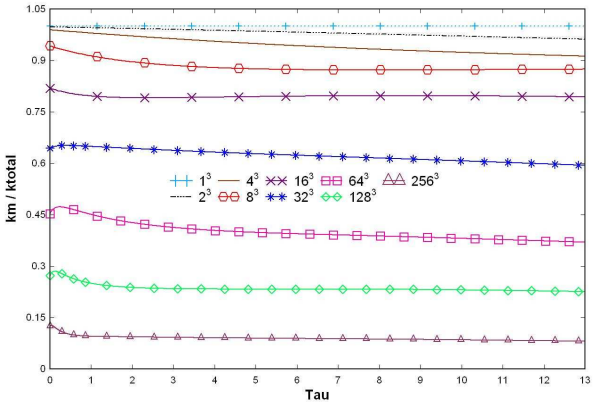

Figure 4. Ratio of modeled kinetic energy to total kinetic energy (Re = 640).

Even though, the equation system looks like a classic RANS model – it is not. The analysis of Germano [1] shows that this two-equation model is actually a 'universal' turbulence model applicable at any mesh resolution. There is no tendency for the solutions to move towards the RANS solution (a ratio of 1.0). The LES solutions stay entirely unsteady and three-dimensional. The slight decrease in this ratio over time is the correct behavior. It is due to the fact that over time the Reynolds number of the flow is slowly decreasing and the mesh can (and does) resolve a larger percentage of the turbulent fluctuations.

7. INITIAL CONDITION PERTURBATIONS

A truly adaptive model should be able to obtain the correct behavior from incorrect initial conditions. For example, it is of considerable interest to see if a 64^3 mesh initialized with a RANS solution can, over time, develop into a full LES simulation. In order to test the model in this way, the initial conditions were either smoothed or sharpened using a filtering operation. The filter used to alter the initial conditions was a nearest neighbor averaging procedure,

$$u_{ijk}^{filtered} = \beta u_{ijk} + (1-\beta)(u_{i+1jk} + u_{i-1jk} + u_{ij+1k} + u_{ij-1k} + u_{ijk+1} + u_{ijk-1})\frac{1}{6}.$$

For smoothing $\beta = 0.0$ was used. This replaces the value at a mesh point by the average of its nearest neighbors. This type of filter removes energy primarily from the highly oscillatory modes with wavelengths close to the mesh size. In spectral terms it damps the spectra in the region just above the cutoff wave number. The affect is shown in Figure 5, which shows the original initial spectra for the 64^3 simulation (normal), and the spectra for the smoothed and sharpened initial conditions. Sharpening is performed by using $\beta = 1.5$. This adds energy to the existing high frequency modes.

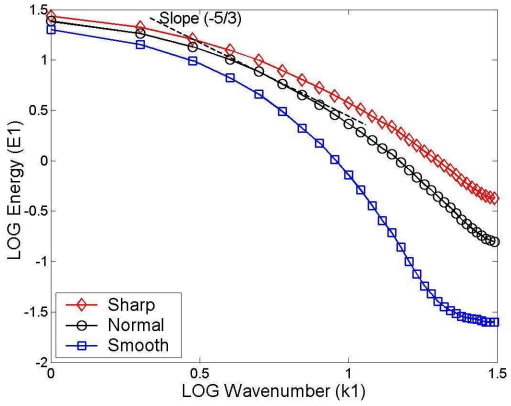

Figure 5: 1D energy spectra for 64^3 simulation, Re=640 (Sharp, Normal, Smooth) .

Figure 6 shows the affect of smoothing and sharpening the initial conditions on the kinetic energy ratio. When smoothing is used, energy is removed from the resolved modes. In order to keep the total kinetic energy the same, the model now must start with more energy. The ratio therefore starts higher than before.

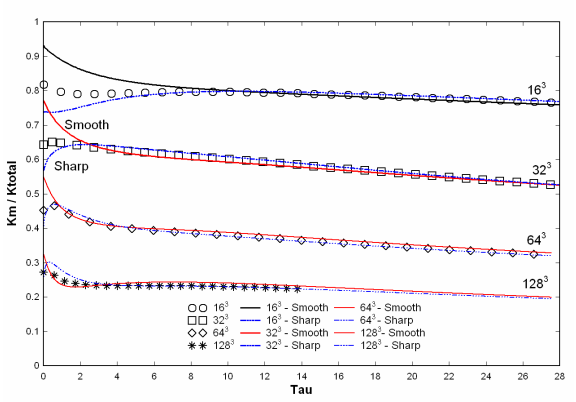

Figure 6: Ratio of modeled kinetic energy to total kinetic energy, Re=640 (perturbed initial conditions).

As time proceeds the model achieves the same ratio irrespective of the initial conditions. At early times, the smoothed solution has less error and therefore backscatters somewhat more than the unperturbed initial condition. This removes energy from the model and makes the ratio decrease faster, so that it approaches its original state. A similar (but opposite) process happens when the spectra is sharpened. In this case, the model senses that the mesh can not support the input resolved fluctuations, the eddy viscosity is increased by α ,and damping of the resolved modes occurs with the resulting energy transfer to the model. Note that the rate at which the model adjusts to incorrect initial conditions depends on the mesh resolution. The higher mesh resolutions adjust much more quickly. It is hypothesized that the time it takes to transfer the energy scales on the timescale of the turbulence at the cutoff (transfer) lengthscale (k/ε).

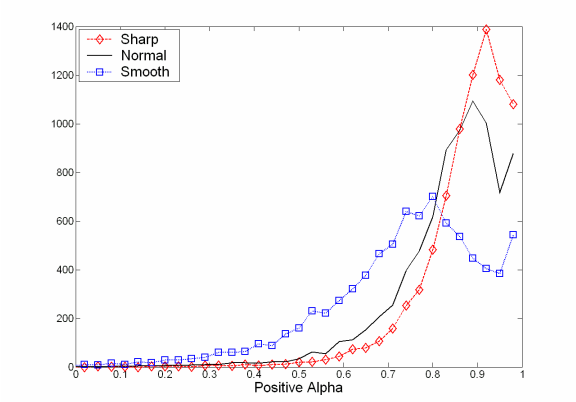

Figure 7. Histogram of energy transfer function (α) positive values.

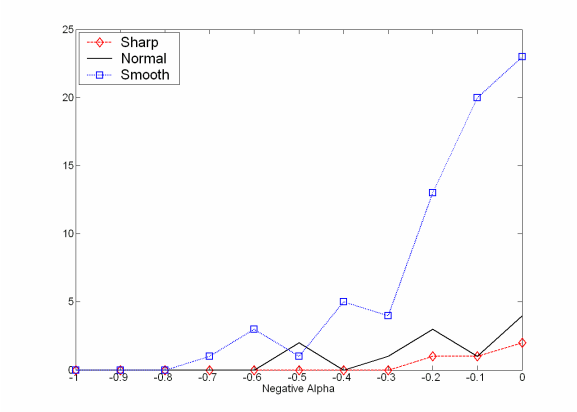

Figure 8. Histogram of energy transfer function (α) negative values.

It had been stated earlier that the energy transfer function (α) was usually positive and order 1. In order to visualize exactly what happens during a simulation with correct initial conditions and with perturbed initial conditions (sharp, smooth) Figure 7 shows a histogram of alpha (positive values) and Figure 8 shows a histogram of alpha (negative values). Figure 7 shows that with the correct initial conditions (solid line) alpha obtains an average value of around 0.87 for this mesh resolution. Figure 8 is an enlarged view of the negative alpha values from negative one to zero. The histograms also include values of alpha for the smooth (squares) and sharpened (diamonds) initial conditions whose spectra were shown in Figure 5. As mentioned previously, smoothing the velocity field removes energy from the resolved flow and places too much energy into the model. To compensate for this error Figure 7 shows that alpha takes on a smaller average value and Figure 8 shows a drastic increase in negative values (more backscatter). By having more negative values alpha is correctly displacing energy from the model to the resolved field to correct for the wrong initial condition.

The opposite effect is shown for the sharpened case. The simulation has been started incorrectly with too much energy in the resolved field. Here it is seen that alpha obtains a slightly larger value than 0.87 to ensure more normal forward transfer of energy. Because of this shift to the right, there is considerably less (almost non-existent) negative values. This guarantees almost no backscatter and has an increase in forward scatter to correct the initial conditions.

8. SCALING

In classic LES models the lengthscale is assumed to be proportional to the mesh size, Δ, and the gradients scale like $u_{i,j} \sim \varepsilon^{1/3}\Delta^{-2/3}$. Figure 9 looks at this scaling behavior (in log scale) at a fixed time t=0.5 The modeled lengthscale is given by $L_m = \frac{k^{3/2}}{\varepsilon}$. At very small values of Δx (large numbers of mesh points) it is obvious that the lengthscale is proportional to the grid spacing as would be expected with classic LES. Mesh resolutions smaller than 64^3 exhibit a more interesting behavior because now the lengthscale is no longer proportional to the mesh.

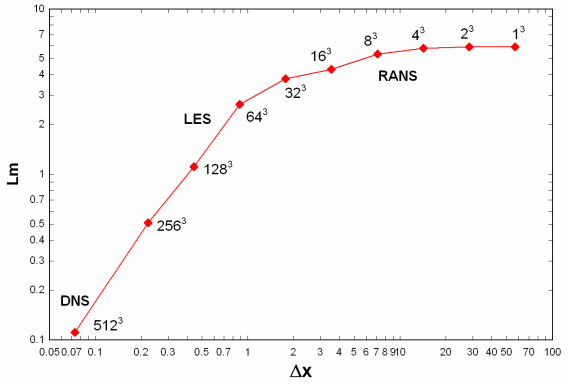

Figure 9: Modeled lengthscale vs. grid size.

This is a transition region that one could call VLES, but not full LES. Carati et. al. [4] ran LES experiments for isotropic decaying turbulence and determined that the smallest mesh size for a LES simulation was 48^3, this is in excellent agreement with Figure 4.

9. DISCUSSION

A turbulence model has been shown to incorporate RANS equations to model the subgrid scale stresses for a LES simulation. The model automatically adjusts to the mesh, and without user intervention performs RANS, LES, or DNS. The model gives good predictions for isotropic decaying turbulence at any mesh resolution. While a slightly modified two-equation model was utilized it is important to note that any RANS transport equation could be used in this manner.

ACKNOWLEDGEMENTS

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