The Utility of Mimetic Methods for Computing Geophysical Flows

Flow Problems in Oil & Gas Industry

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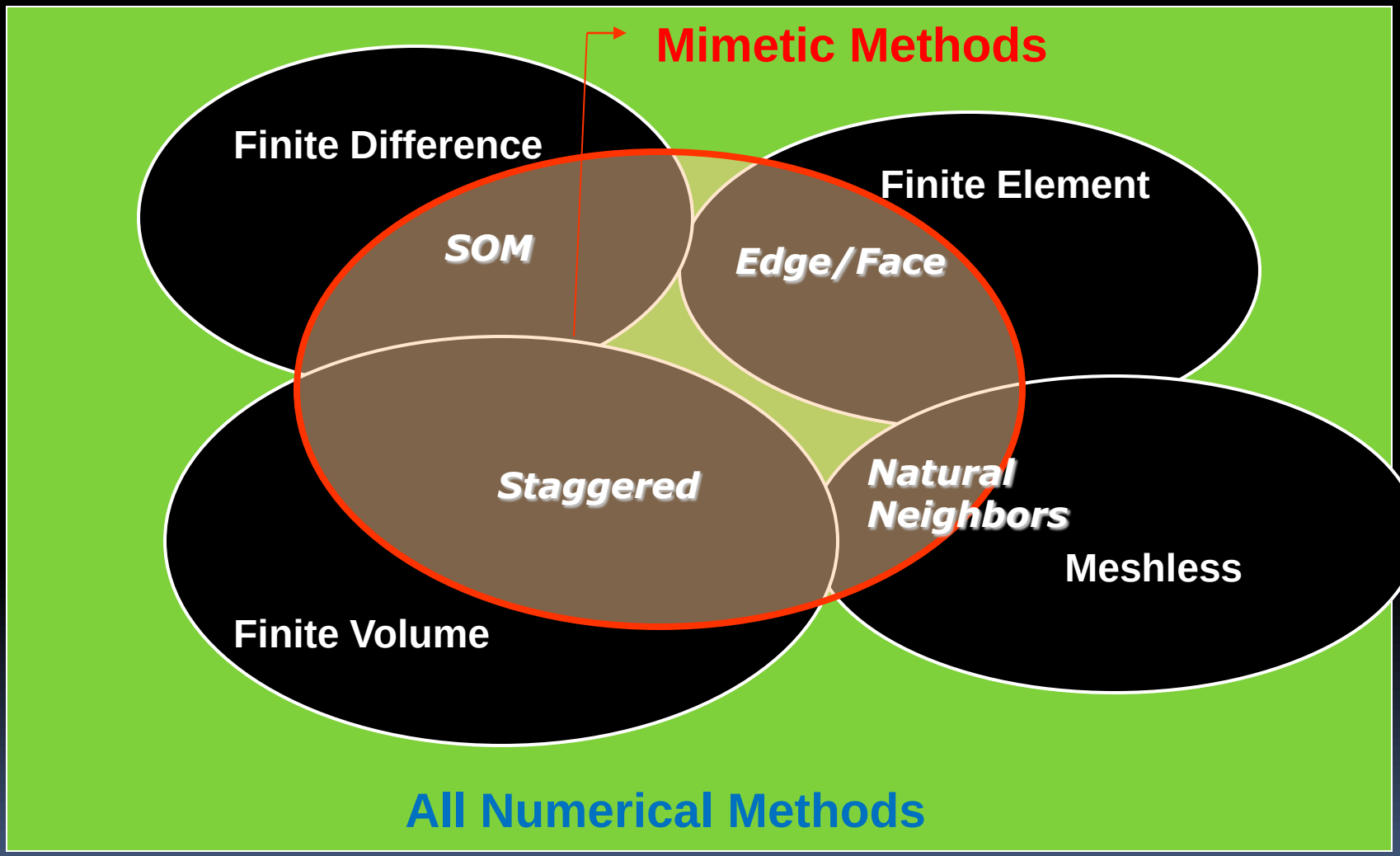
Mimetic Methods



Mimetic Description

All Methods: Accuracy, Stability, Convergence

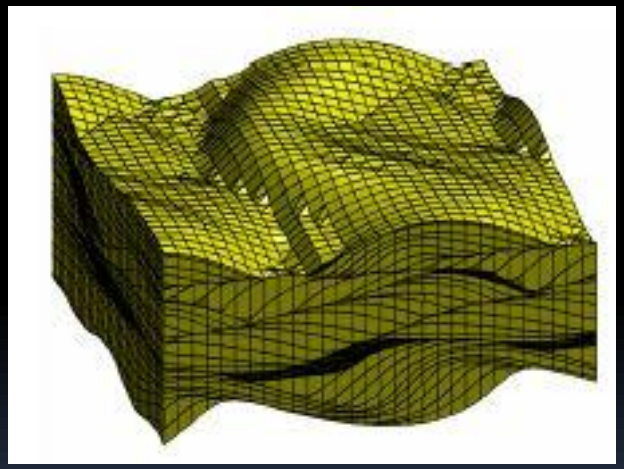
Mimetic Methods: Also have:

- Secondary conservation statements
- Wave propagation properties
- Satisfy constraints (incompressibility)
- Don't have modes.
- They **mimic** the PDE properties.

Get the Physics and Mathematics of PDES right by using **Exact Discretization**

Why it Matters

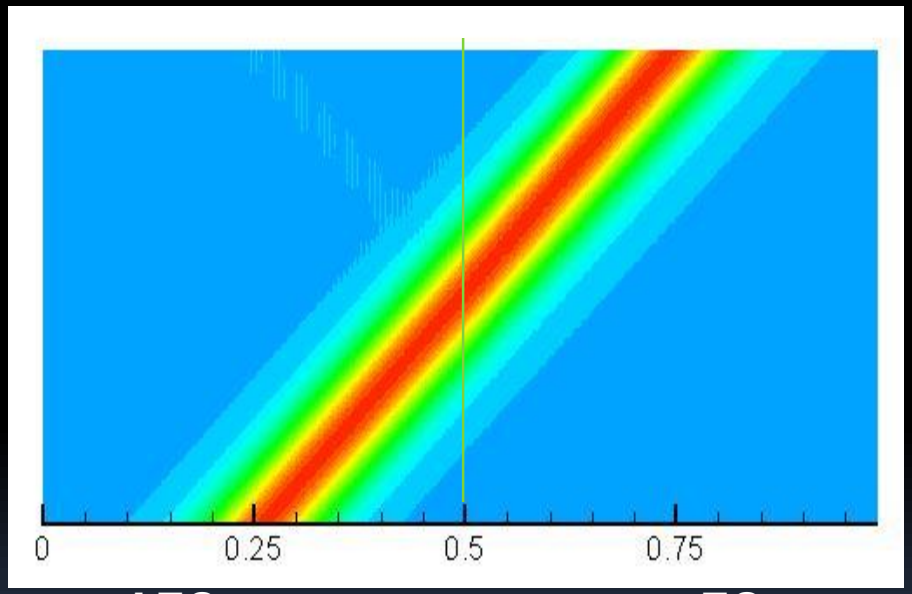
- Very large jumps in material properties
- Very high aspect ratio meshes
- Complex physics



Often nobody will notice (quickly) if the computational solution is wrong.

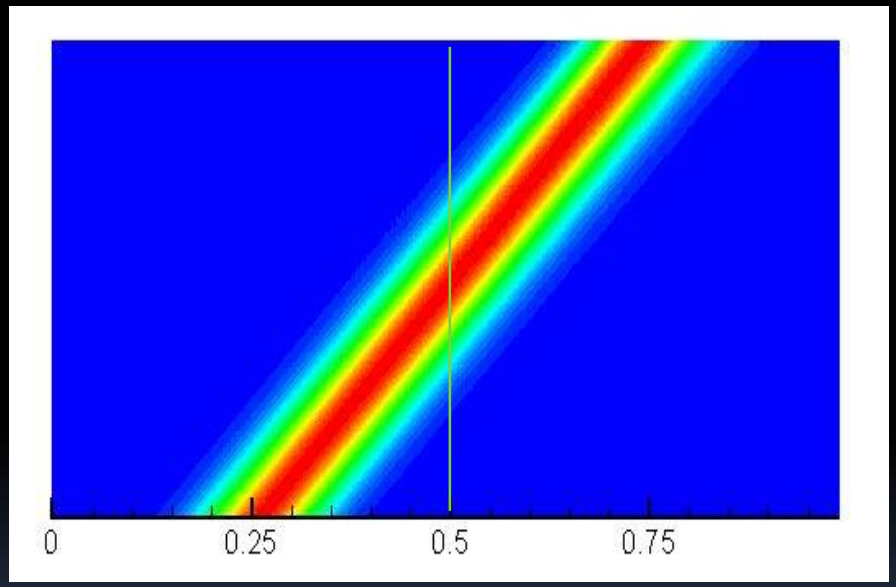
Mimetic Advection $\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = 0$

Jason Frank, CWI/University of Amsterdam
Change in mesh size (3x)



150 **50**

Central



Box Method

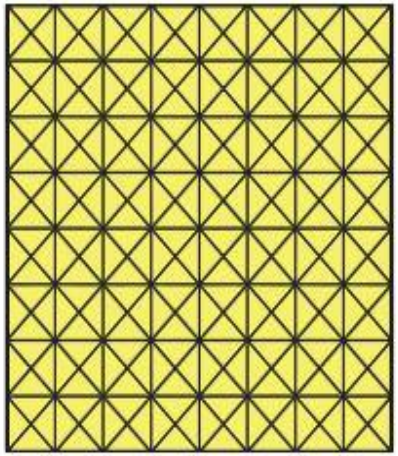
Very small timestep.

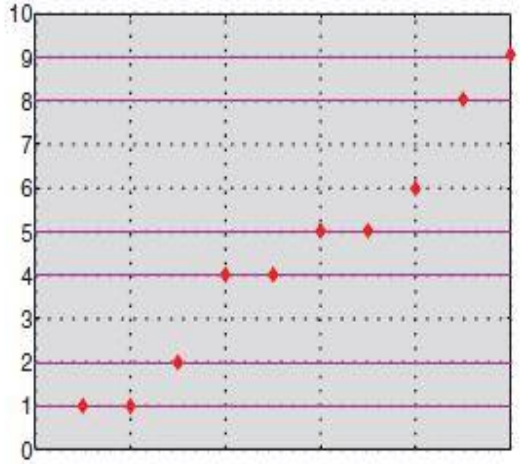
Mimetic Eigenvalues

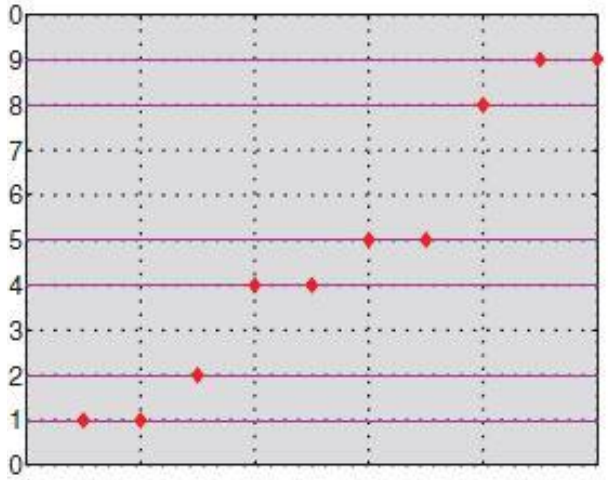
Costabel/Dauge, Arnold*, Gerritsma

$$\nabla \times \nabla \times \mathbf{v} - \nabla(\nabla \cdot \mathbf{v}) = \lambda \mathbf{v} \qquad \lambda = m^2 + n^2$$

1,1,2,4,4,5,5,8,..







Linear FE

$\mathbf{v}_{node}$

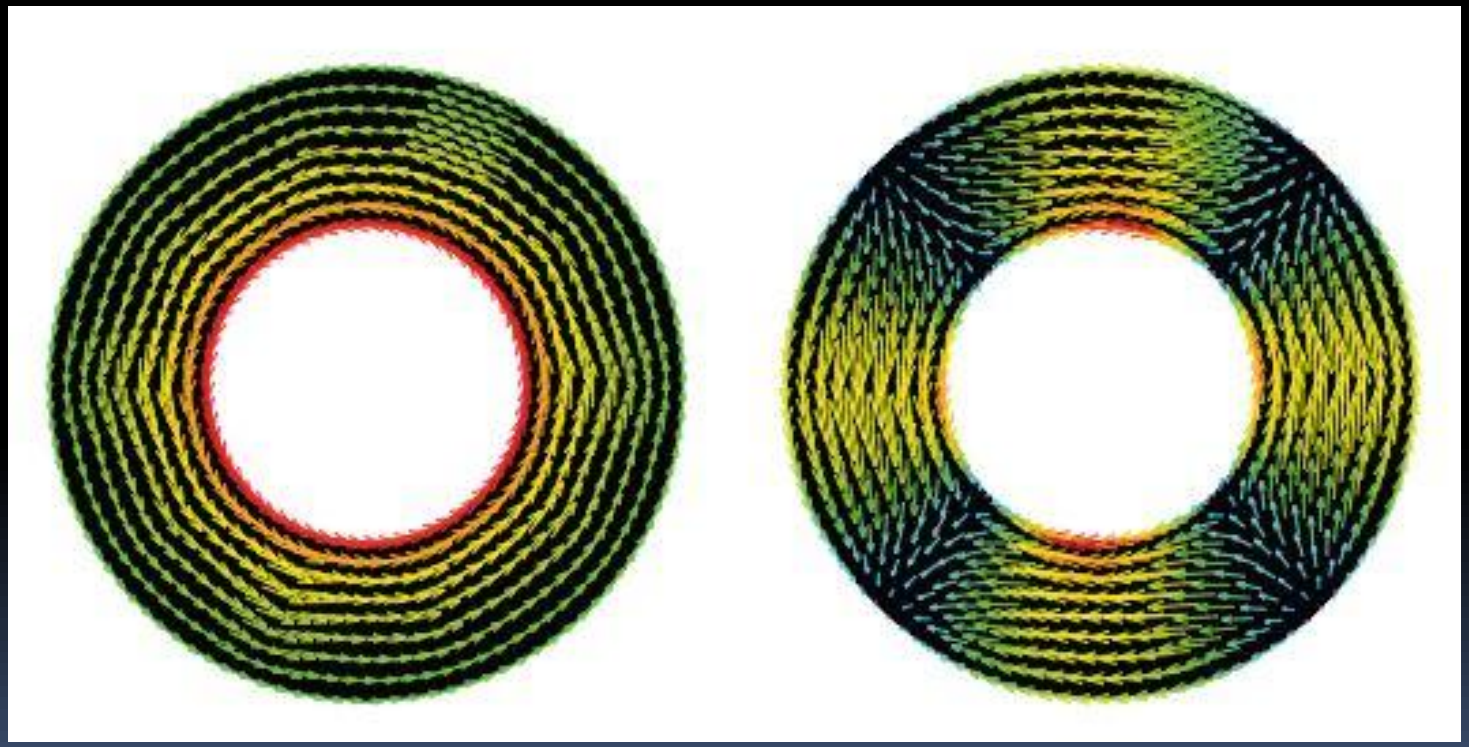
Raviart-Thomas FE

$u = \mathbf{v} \cdot \mathbf{t}_{edge}$

***Finite Element Exterior Calculus: From Hodge Theory to Numerical Stability**

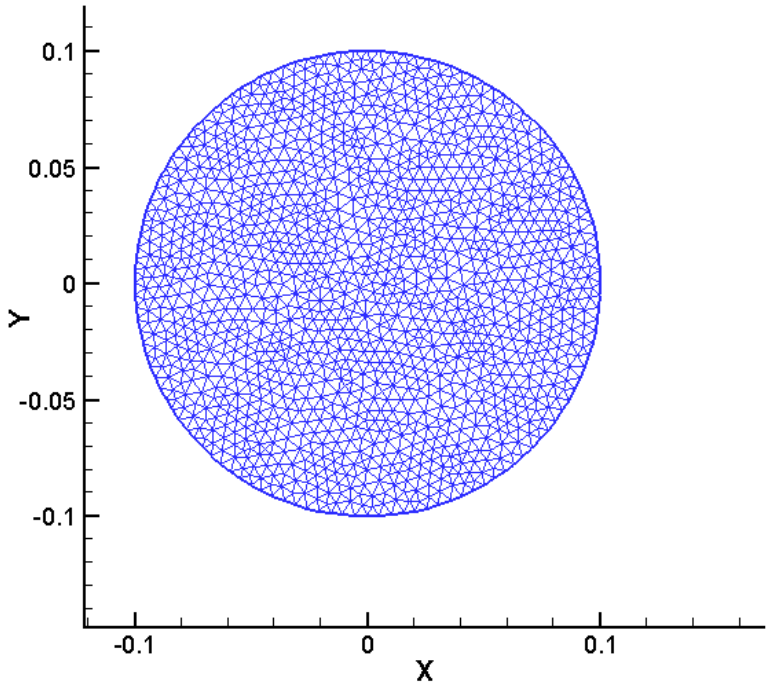
Mimetic Vector Fields

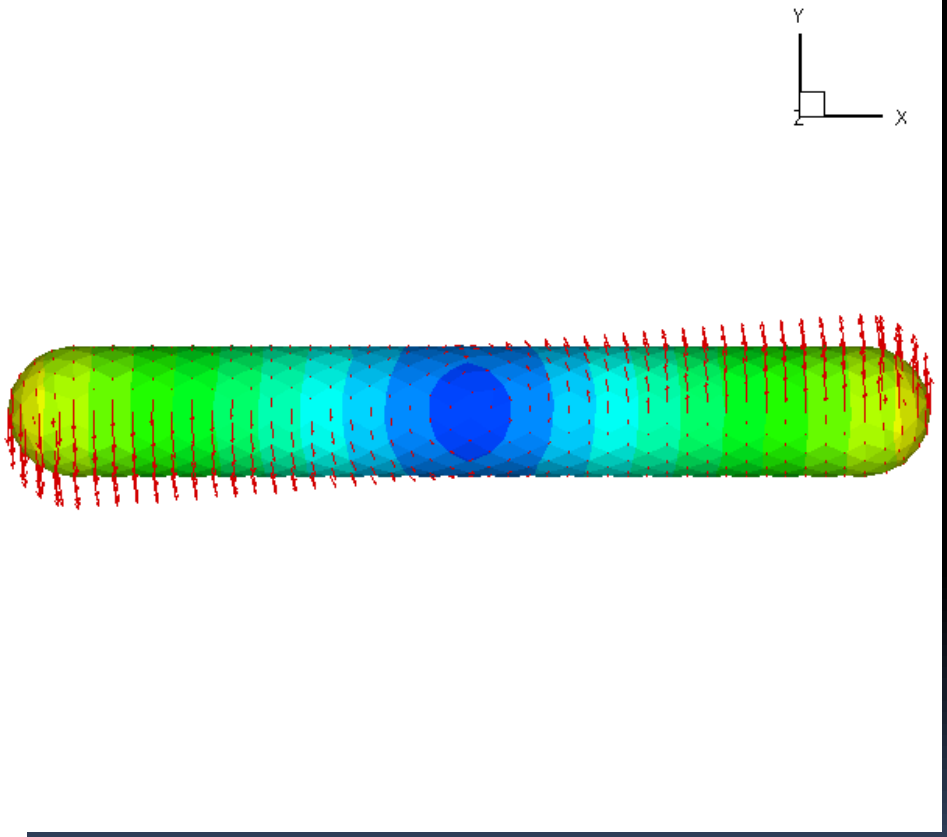
$$\nabla \times \nabla \times \mathbf{v} - \nabla(\nabla \cdot \mathbf{v}) = f$$



Linear FE **Raviart-Thomas FE**

Mimetic Physics





Discrete Calculus Approach

How to always construct a mimetic method.

Separate Discretization from Approximation

PDE -> LA
LA -> square LA

- **Do ALL discretization exactly.**
- **This means that the calculus and the physics remain exact.**

- **All approximations = interpolation problems.**
- **Numerical approximation only in material laws (which are engineering approximations already)**

Example PDE

Physical Equation (Heat Equation)

$$\frac{\partial(\rho c T)}{\partial t} = \nabla \cdot k \nabla T$$

Components of the Physical Equation

Physics

$\frac{\partial i}{\partial t} + \nabla \cdot \mathbf{q} = 0$ Conservation of Energy

Math

$\mathbf{g} = \nabla T$ Definition of Gradient

Material Approximation

$\mathbf{q} = -k\mathbf{g}$ Fourier's Law

$i = \rho c T$ Perfectly Caloric Material

Mimetic Discretization

Exact Discretization of Physics and Calculus.

$$\int_{\tilde{c}} idV\,|^{n+1} - \int_{\tilde{c}} idV\,|^{n} + \sum_{\tilde{f}} \int dt \int_{\tilde{f}} \mathbf{q}\cdot\mathbf{n}\, dA_{\tilde{f}} = 0 \quad \Rightarrow \quad I_{\tilde{c}}^{n+1} - I_{\tilde{c}}^{n} + \mathbf{D}Q_{\tilde{f}} = 0$$

$$\int_{e} \mathbf{g}\cdot d\mathbf{l} = T_{n2} - T_{n1} \quad \Rightarrow \quad g_e = \mathbf{G}T_n$$

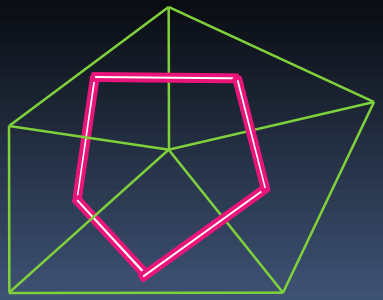
Numerically Exact

Numerical Approximation of Constitutive Eqns.

$$Q_{\tilde{f}} = -M_1 g_e \quad \Rightarrow \quad Q_{\tilde{f}} = -k\frac{A_{\tilde{f}}}{L_e} g_e$$

$$I_{\tilde{c}} = M_2 T_n \quad \Rightarrow \quad I_{\tilde{c}} = \rho c V_{\tilde{c}} T_n$$

Numerically Approximate



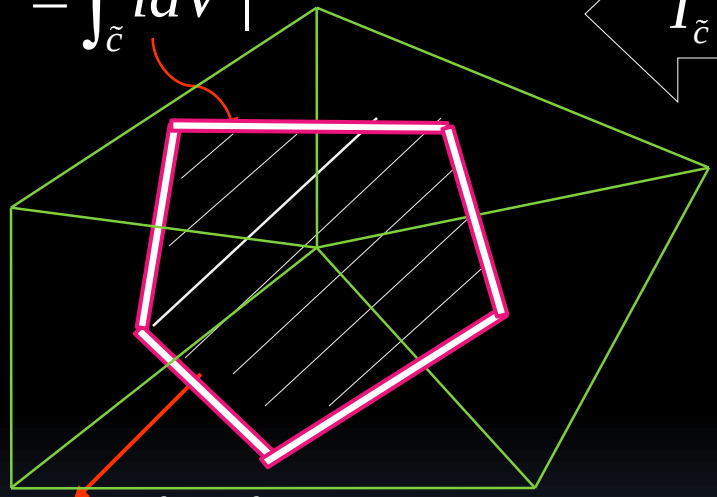
Dual Mesh Viewpoint

$$I_{\tilde{c}}^{n+1} - I_{\tilde{c}}^{n} + \mathbf{D}Q_{\tilde{f}} = 0$$

$$g_e = \mathbf{G}T_n$$

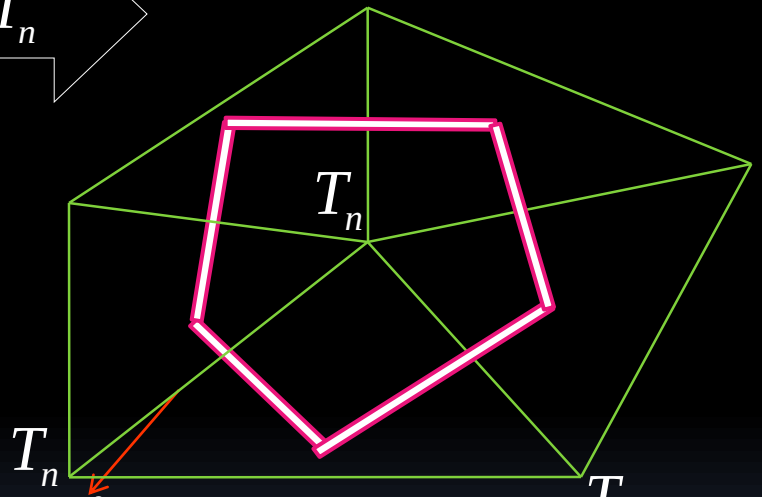
$$I_{\tilde{c}}^{n+1} = \int_{\tilde{c}} i dV \,|^{n+1}$$

$$I_{\tilde{c}} = \rho c V_{\tilde{c}} T_n$$



$$Q_{\tilde{f}} = \int dt \int_{\tilde{f}} \mathbf{q} \cdot \mathbf{n} dA_{\tilde{f}}$$

$$Q_{\tilde{f}} = -k \frac{A_{\tilde{f}}}{L_e} g_e$$

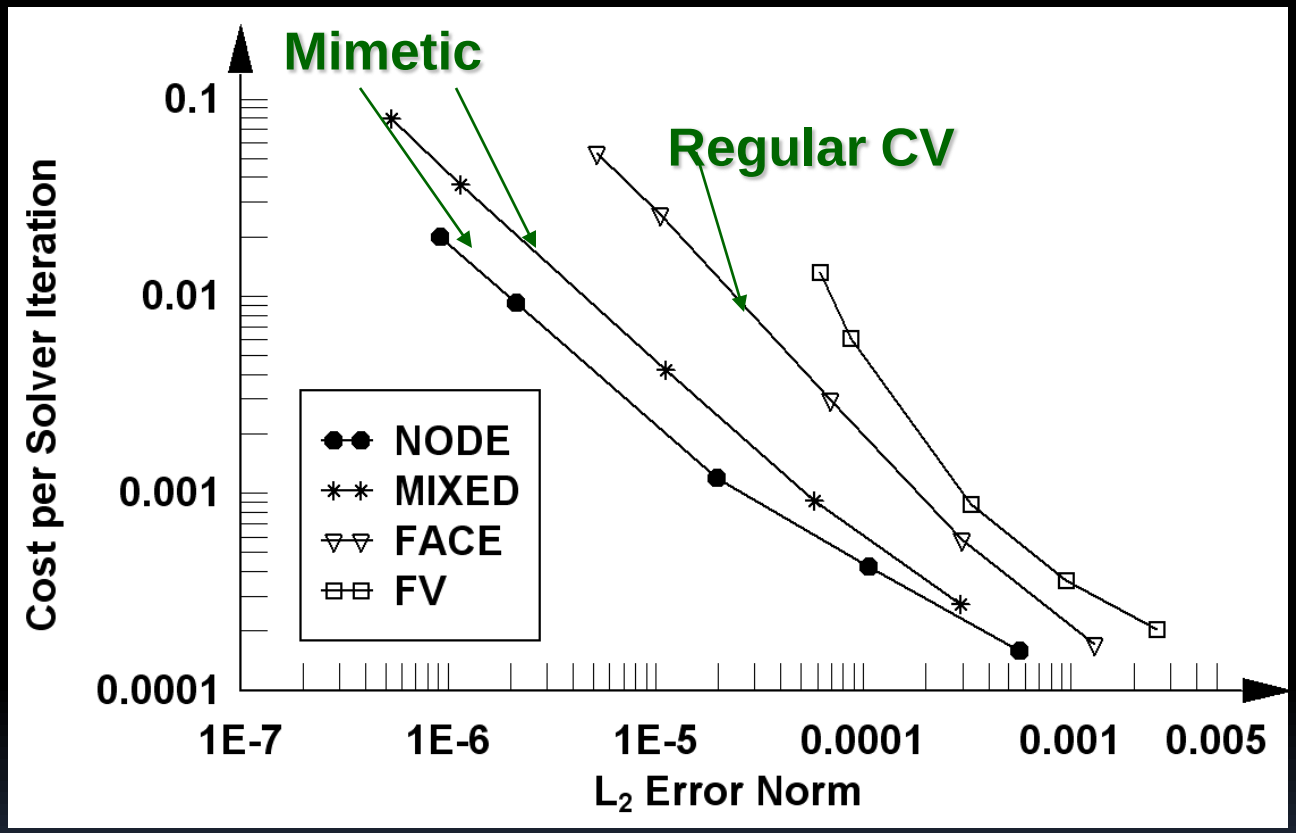


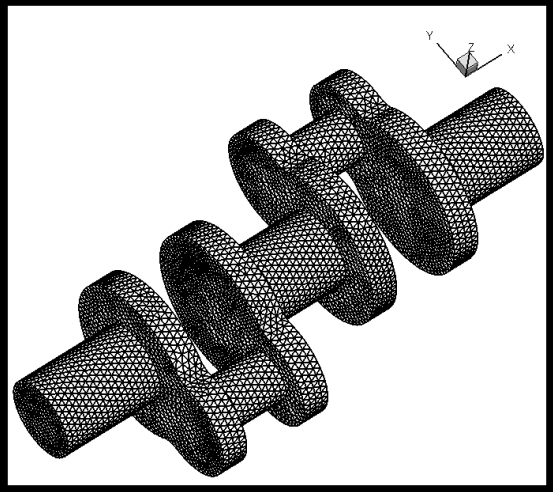
$$\int_e \mathbf{g} \cdot d\mathbf{l} = g_e$$

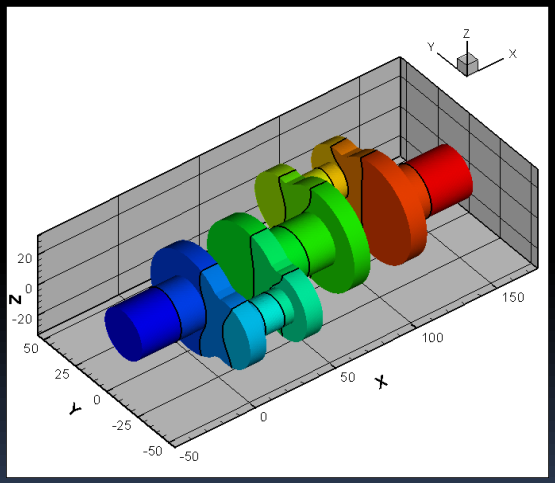
Dual Mesh

Primary Mesh

Results







- Log scale

Summary

- Interesting Developments in Numerical Methods.
- Some of it happening here in the Netherlands
- These methods are very useful for complex physics.