A note on turbulent energy dissipation in the viscous wall region

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From simulation data it is shown that the difference between the true rate of dissipation of turbulent kinetic energy and the "isotropic dissipation" used by Reynolds-averaged modelers is less than 2% in a conventional viscous wall region, and negligible elsewhere. The difference is a contribution to viscous diffusion which is a small fraction of the total.

Corrsin¹ reminded the community that the correct form of the viscous diffusion term in the turbulent energy equation, first derived by Reynolds² in a paper presented in 1894, is not simply the customary v times the Laplacian of the diffused quantity, but contains an extra second derivative. The complete viscous term is

$$vu_{i} \frac{\overline{\partial^{2} u_{i}}}{\partial x_{j}^{2}} = v \frac{\partial^{2}}{\partial x_{j}^{2}} \left(\frac{1}{2} \overline{u_{i}^{2}}\right) - v \overline{\left(\frac{\partial u_{i}}{\partial x_{j}}\right)^{2}}$$
$$= v \left[\frac{\partial^{2}}{\partial x_{j}^{2}} \left(\frac{1}{2} \overline{u_{i}^{2}}\right) + \frac{\partial^{2} \overline{u_{i} u_{j}}}{\partial x_{i} \partial x_{j}}\right] - \frac{1}{2} v \overline{\left(\frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}}\right)^{2}}.$$

We describe the form on the second line as (thermodynamically) "correct" (see Lumley³) because the last term is the exact dissipation rate (being the time- or ensembleaveraged sum of the product of each component of fluctuating viscous stress with the fluctuating strain rate in the same plane, and thus the average rate at which work is done against viscous stress by the turbulence). The last term on the first line is the quantity called "isotropic dissipation" by turbulence modelers, who use it because its exact transport equation is simpler than that for the true dissipation. The difference between the two forms of the viscous term is the unexpected contribution to the diffusion, $\partial^2 u_i u_j / \partial x_i \partial x_j$: note that equality of the two requires only homogeneity, not isotropy.

It is commonly and correctly argued, on the basis of order-of-magnitude arguments (e.g., Hallbäck⁴), that the extra viscous diffusion term is small compared to the main, Laplacian, term, but surprisingly there seems to be no quantitative estimate in the literature. In this note we use direct simulation data to show that the extra term, and therefore the difference between the true dissipation and the "isotropic" dissipation, is negligibly small.

Apart from shock waves where $\partial^2 u^2 / \partial x^2$ is large, the only place where viscous diffusion is significant is the viscous wall region, $y^+ < 30$ say, where $y^+ \equiv \sqrt{\tau_w/\rho} y/v$. Here by far the largest contribution to $\partial^2 \overline{u_i u_i} / \partial x_i \partial x_i$ in flows obeying the boundary-layer approximation is $\partial^2 \overline{v^2} / \partial v^2$. Figure 1 shows $\overline{v^2}$ from the channel-flow simulation of Kim et al.,⁵ using law-of-the-wall scaling, and Fig. 2 shows the ratio of the extra viscous diffusion term, $v\partial^2 \overline{v^2}/\partial y^2$, to the (exact) dissipation term. The extra contribution to viscous diffusion is everywhere less than about 2% of the dissipation rate, and this figure should apply in all flows which obey the law of the wall. The quantitative figure for the difference might change if the flow deviated from the law of the wall, but it is small enough to suggest that the difference will be negligible on any smooth solid surface. Flows with unconventional surface boundary conditions (riblets, free surfaces, or the start or end of a region



FIG. 1. Simulation data⁴ for v-component intensity in channel flow.

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of roughness or transpiration) would have to be considered individually.

From the turbulence modeler's viewpoint the conclusion is that the difference between the true dissipation rate and the "isotropic" dissipation rate can be ignored for all purposes of computation and discussion.

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